

Searching for gravitational wave signals in LIGO data.



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IndIGO Consortium www.gw-indigo.org

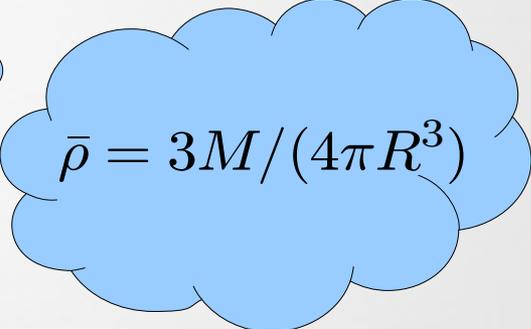
Frequencies of astronomical gravitational sources

Earth based detectors: 10 – 1000 Hz

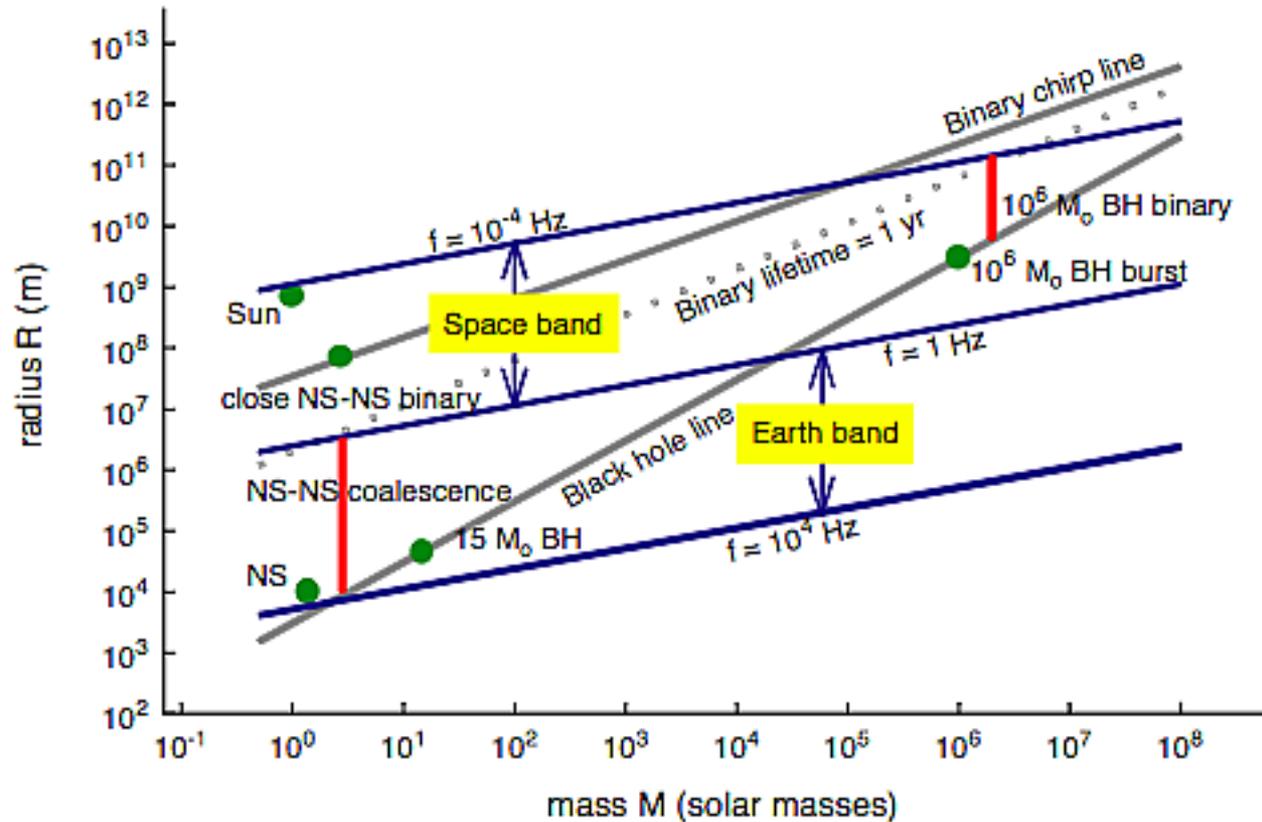
- The signals for which the best waveforms are available have narrowly defined frequencies
 - » In some cases, existing motion dominates. Pulsar spins.
 - » In most cases, one can relate this to the natural frequency of a self gravitating object.

$$f_0 = \sqrt{G\bar{\rho}/4\pi},$$

- For a NS
 - » $F = 2$ KHz
- For a stellar mass blackhole
 - » $F = 1$ kHz
- For a SMBH in the centre of our galaxy
 - » $F = 4$ mHz.


$$\bar{\rho} = 3M/(4\pi R^3)$$

Gravitational Dynamics



LIGO Observatory Facilities



LIGO Hanford Observatory [LHO]

26 km north of Richland, WA

2 km + 4 km interferometers in same vacuum envelope

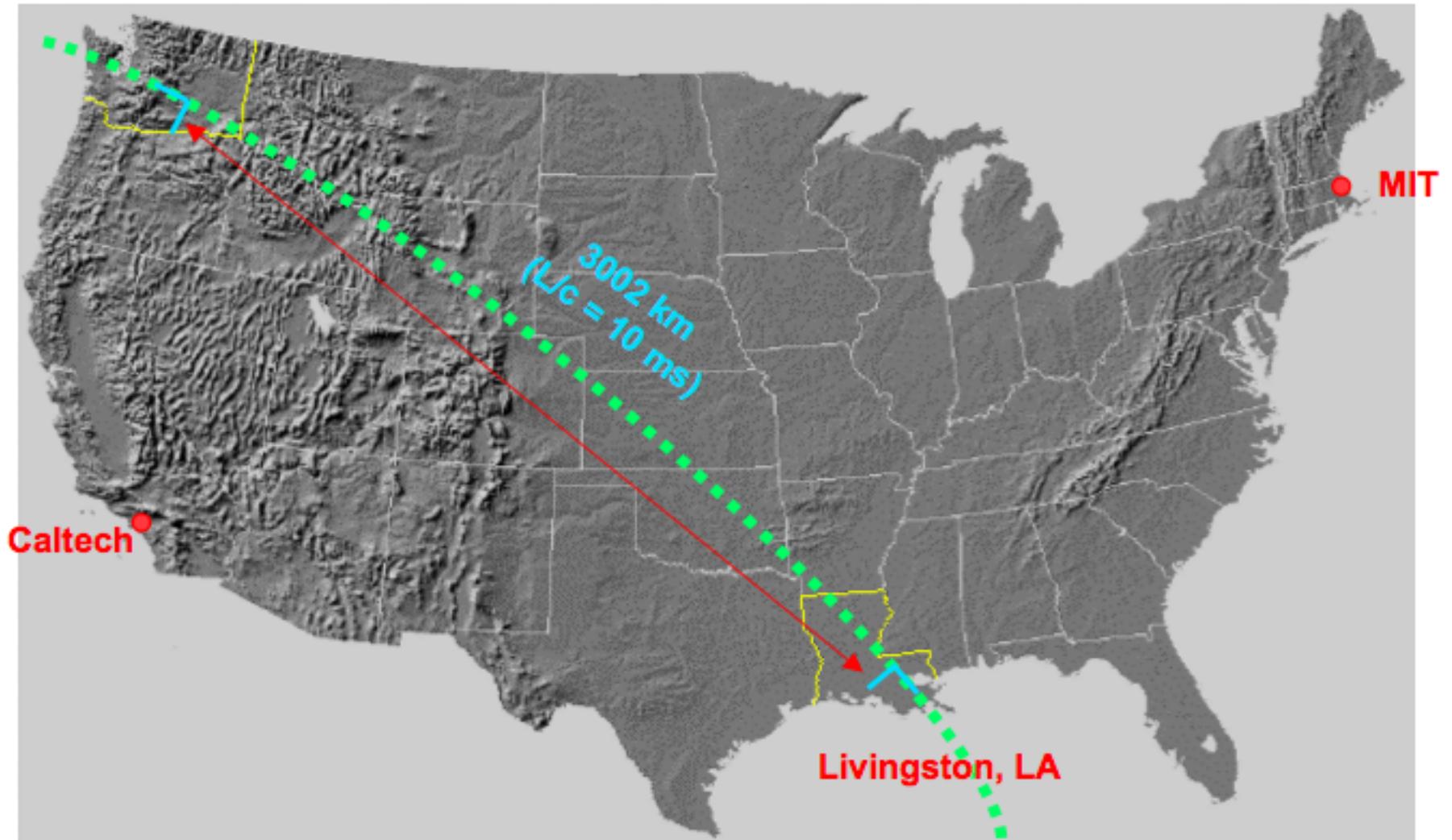


LIGO Livingston Observatory [LLO]

42 km east of Baton Rouge, LA

Single 4 km interferometer

Hanford, WA



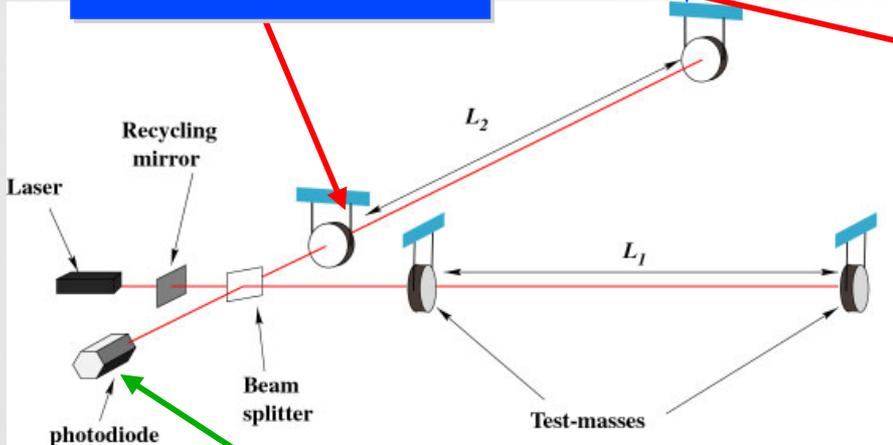
The LIGO Interferometers

- Broad-band detector to measure distortion of spatial geometry due to passing gravitational wave from astrophysical sources

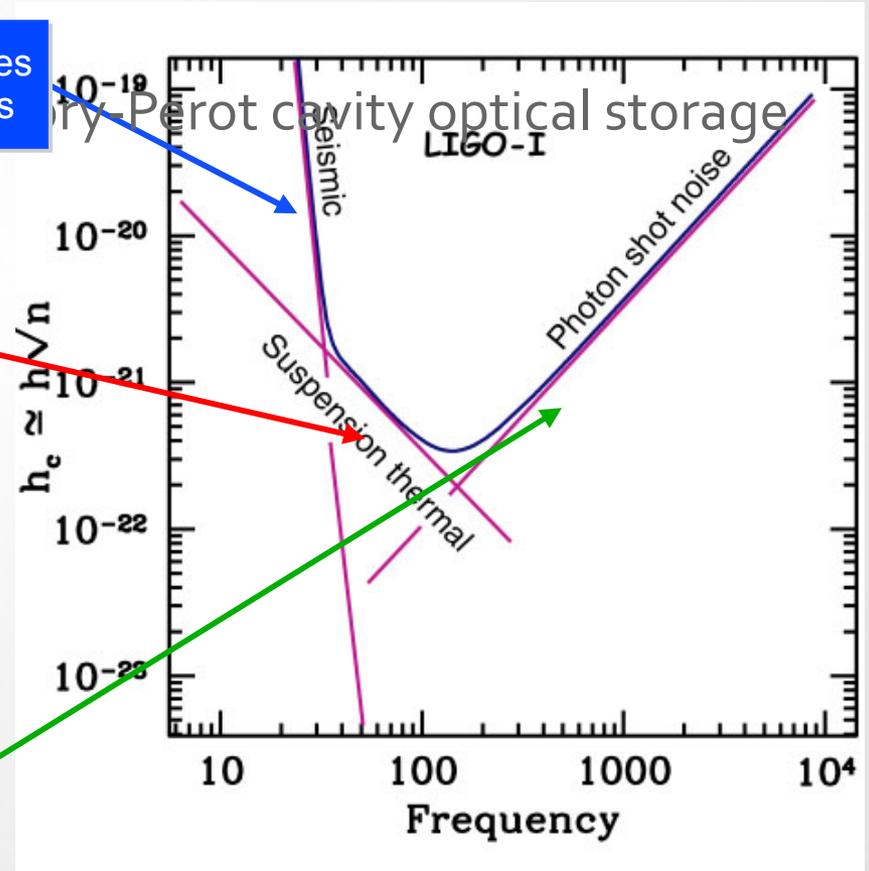
» Michelson interferometer with two arms

Ground motion couples into motion of mirrors

Thermal excitations of mirror suspensions



Counting statistics of photons at photo diode



Current LIGO Sensitivity

LIGO is operating at design sensitivity in S5



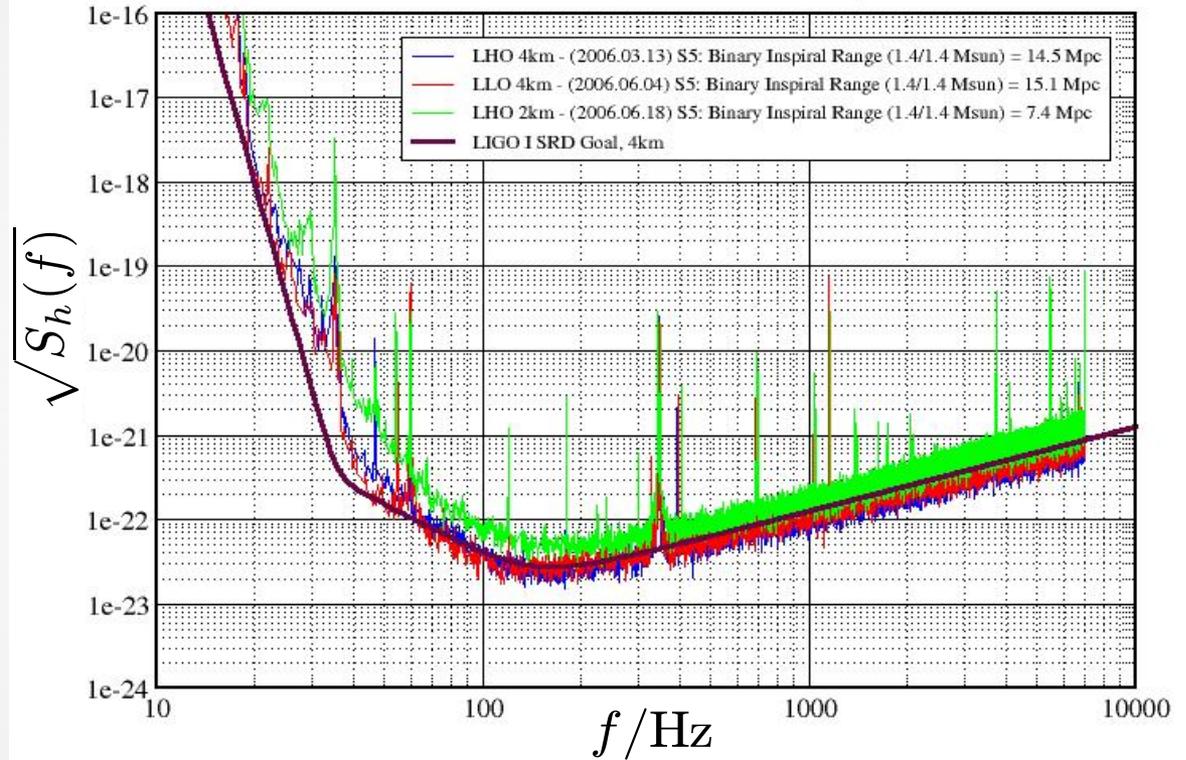
LIGO Livingston, LA



LIGO Hanford, WA

Strain Sensitivity for the LIGO 4km Interferometers

S5 Performance - June 2006 LIGO-G060293-01-Z

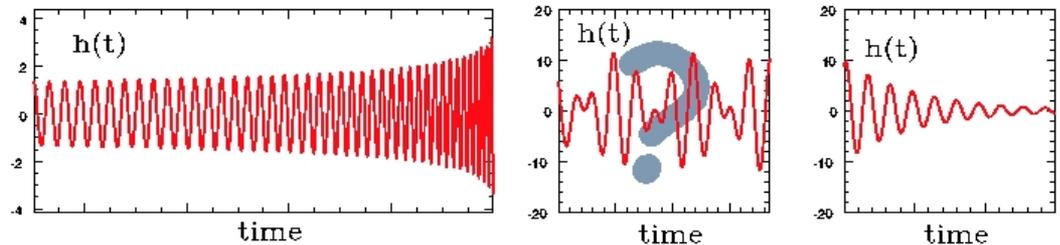
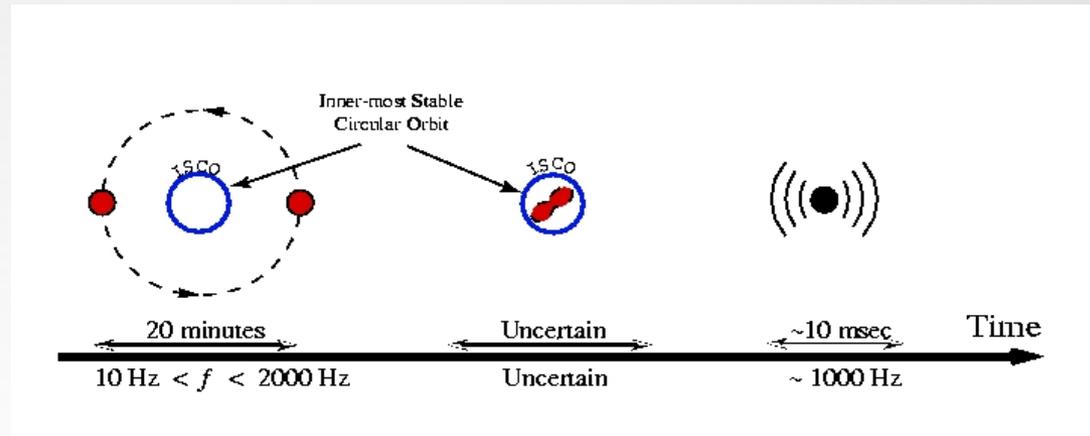


$$\langle n(f)n(f') \rangle = \frac{1}{2} S_h(f) \delta(f - f')$$

Binary Coalescence Waveforms

LIGO is sensitive to NS / BH inspirals $M \leq 100 M_{\odot}$

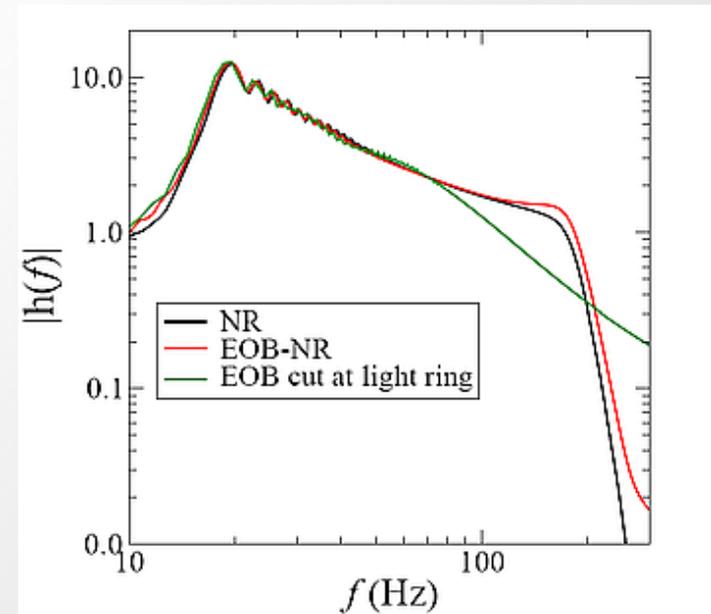
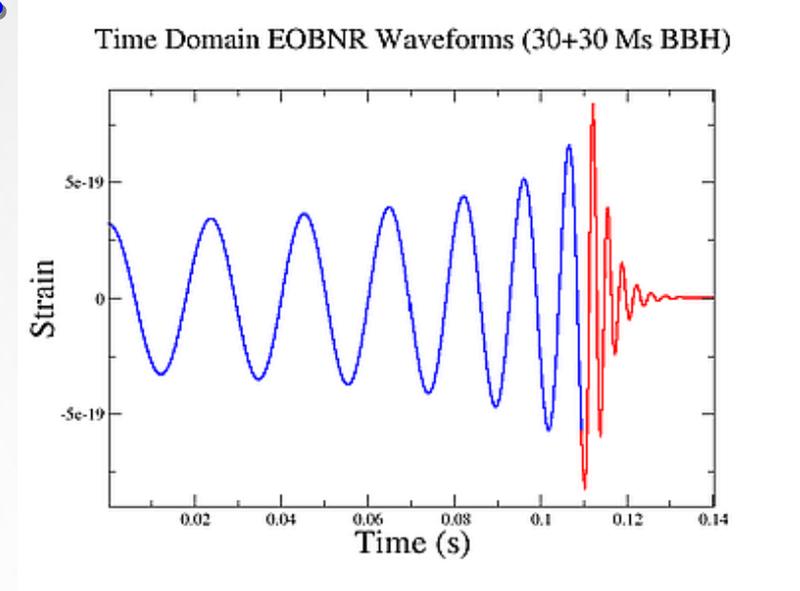
- Assume inspiral signals are (reasonably) well modeled
 - » standard matched filtering technique
- Post-Newtonian templates accurate for low mass systems in LIGO band but at higher masses post-Newtonian approximation breaks down.
- At still higher masses, inspiral searches transition into burst searches
- EOB waveforms hold good beyond ISCO upto $r \sim 3 M_{\odot}$ (more bandwidth for high mass systems)



Inspiral Merger Ringdown Templates

Numerical Relativity Inspired Waveforms

- The Effective-One-Body method provides complete analytic IMR waveforms
- The waveforms are **tuned** to agree with NR simulations
 - Tuned to simulations of non-spinning binaries with mass ratios 1:1 – 4:1.
 - A pseudo- 4PN parameter tunes the late inspiral-merger evolution.
 - Ringdown frequencies depend on the final mass and spin of the remnant black hole. These are fit to agree with simulations
- Good agreement with a set of comparable mass NR simulations and test mass limit simulations (phase difference $\sim 8\%$ of a GW cycle)



Matched Filtering with EOBNR templates

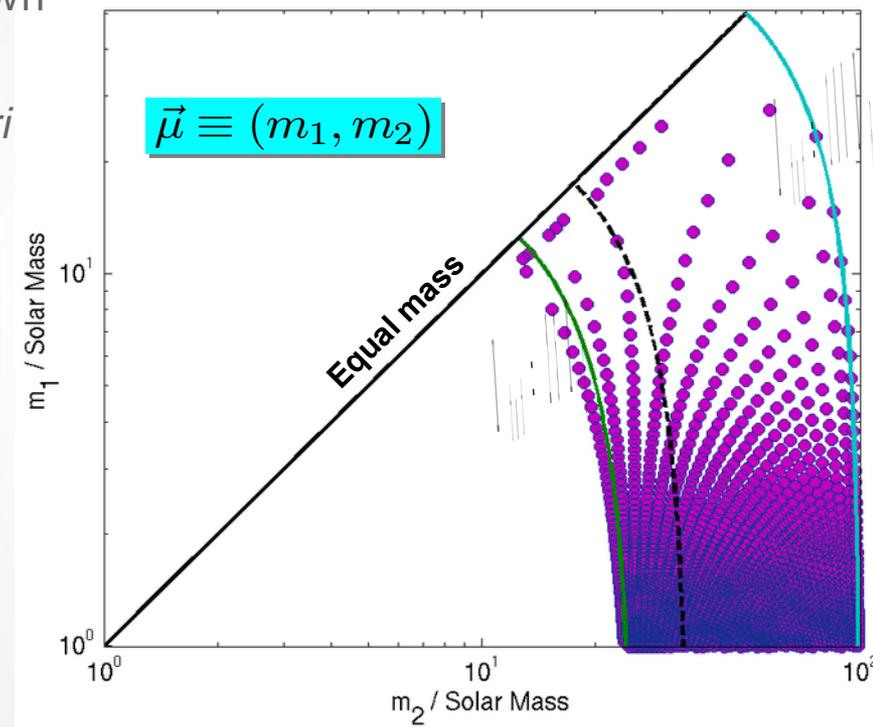
- Detector output contains noise and a possible inspiral signal $x(t) = n(t) + h(t; \vec{\mu})$
 - » Assume signal's functional form is known accurately : EOBNR waveforms
 - » Signal's parameter is not known *a-priori*

- Matched filter

$$\rho(t; \mu_i) = 4 \left| \int_{f_l}^{f_u} \tilde{x}(f) \frac{\tilde{h}^*(f; \mu_i)}{S_h(f)} e^{2\pi i f t} df \right|$$

data **filter**

- Maximize filter output over all parameters $\Lambda = \max_{t, \vec{\mu}} [\rho]$
 - » This is the detection statistic.
 - » Threshold with care



H1: Jul 2006 02:33:44
UTC

EOB templates for high-mass systems

- EOB templates are well suited for high mass CBC search

- » Higher bandwidth for high mass systems

- Larger band-width is good for inspiral search

- » Inspiral Horizon Distance increases (improvement in mass reach)

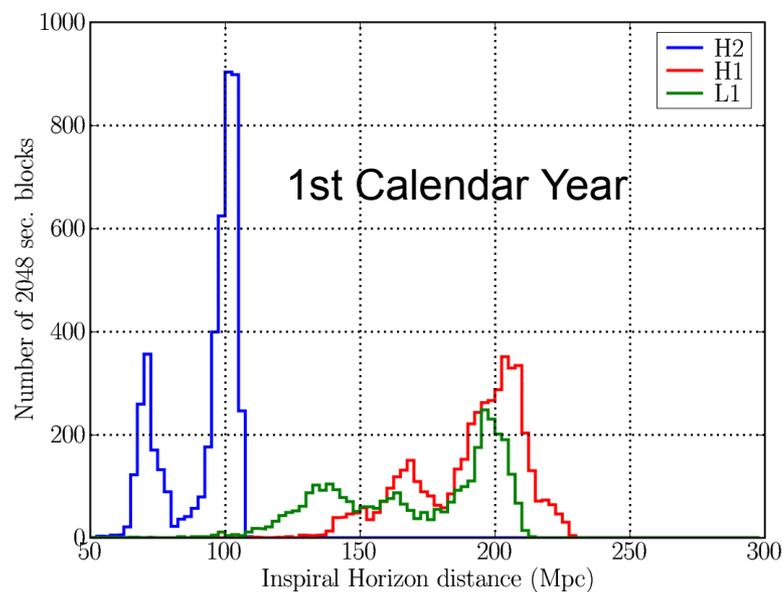
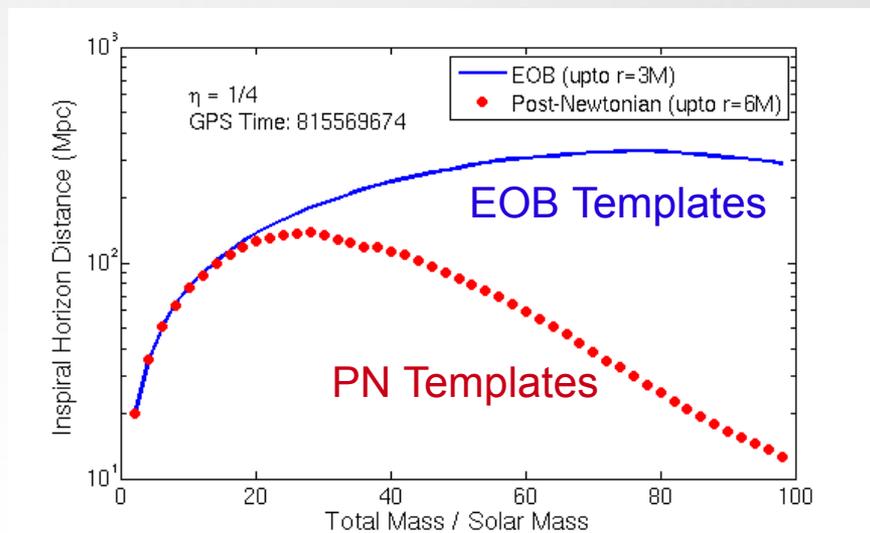
- » Useful for signal based consistency tests to reject false alarms

$$d \propto \frac{A(m_1, m_2)}{\rho} \times \int_{f_l}^{f_u} \frac{|\tilde{h}(f)|^2}{S_h(f)} df$$

- Optimally oriented binary

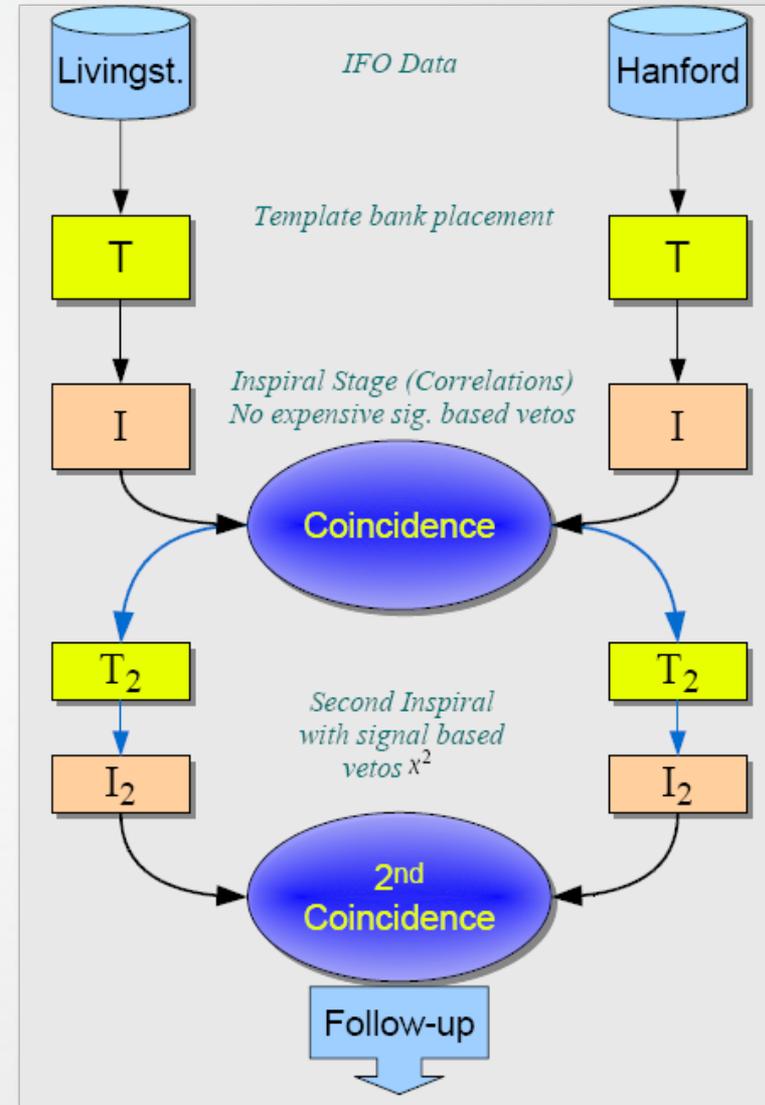
- » snr fixed at 8

- » $h(f)^2 \sim f^{-7/3}$



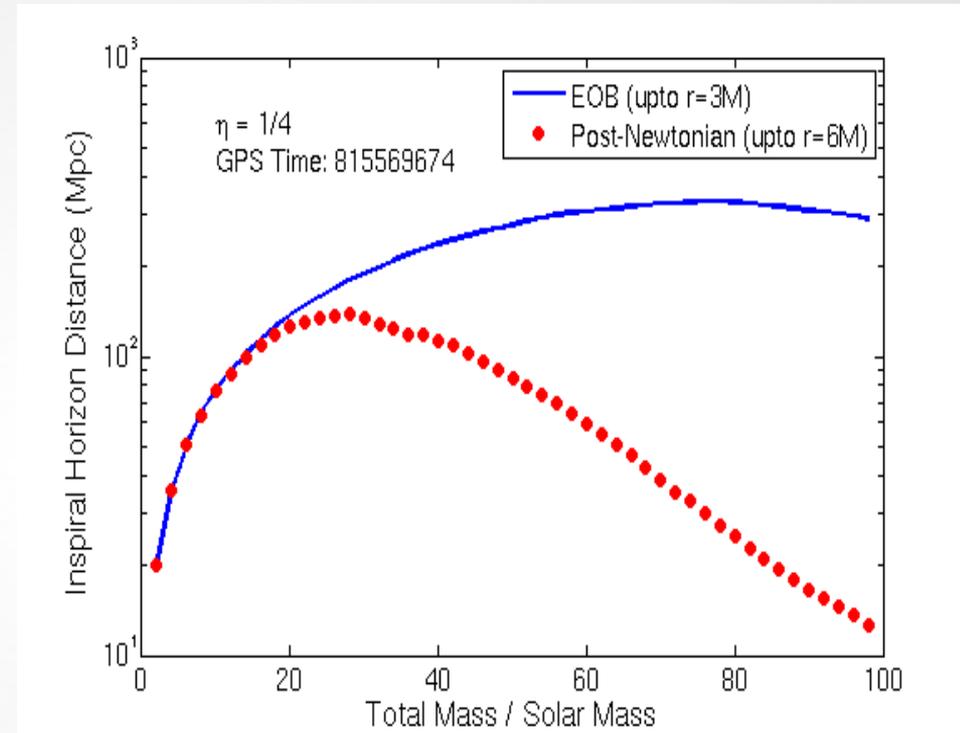
Monte-Carlo Injection Studies

- A large number of software injections are injected in data which are then parsed through the data-analysis pipeline.
 - » Helps us tune the optimum value of the pipeline parameters
 - » Understand pathologies of the data and/or detection algorithm (from the missed injection)
 - » Quantify the efficiency of the pipeline as a function of distance to the compact binary systems.
 - » Pipeline has many tunable parameters



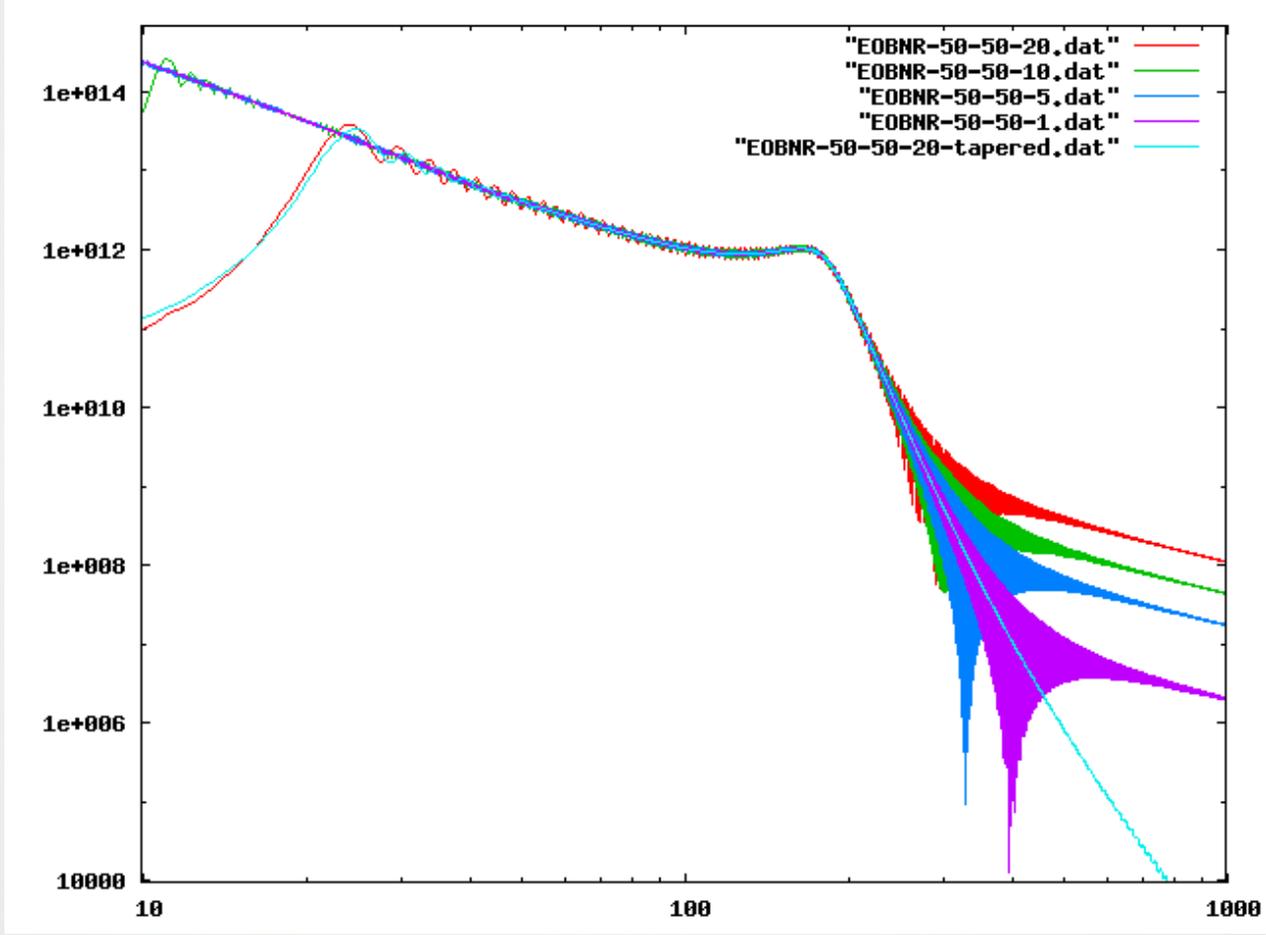
About the Search

- We have fast-tracked the analysis of the final 6 months of S5
- Inspiral-Merger-Ringdown (IMR) templates to model the entire in-band gravitational wave signal
 - High mass waveforms can be very short (~100 ms). Merger and ringdown are a large part of the in band signal.
 - Effective-One-Body (EOB) model tuned to Numerical Relativity (NR) simulations = EOBNR waveforms



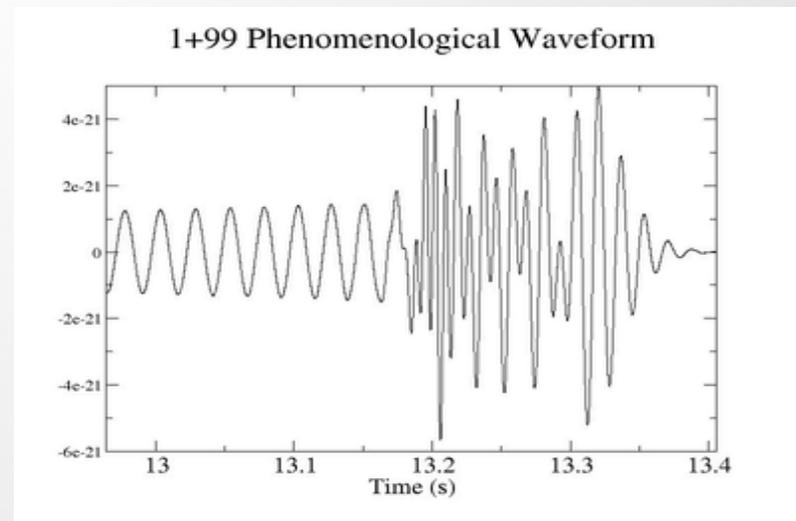
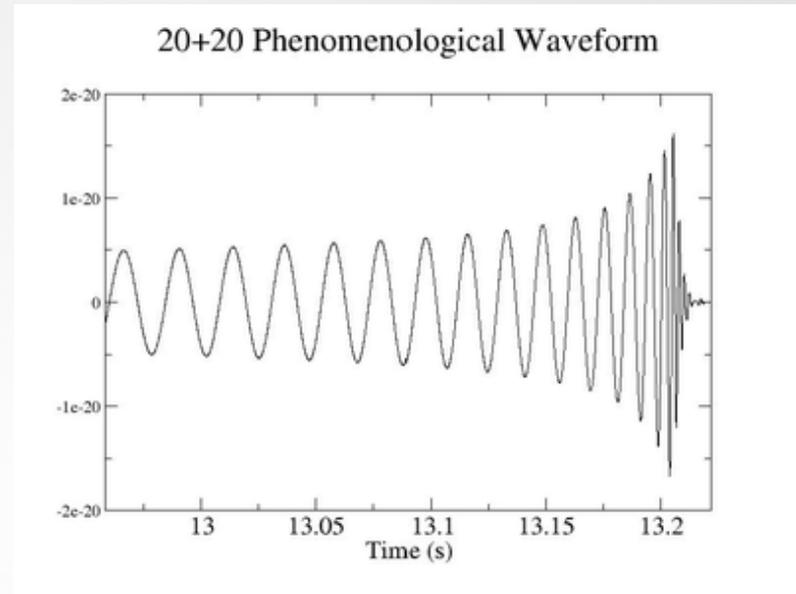
Could detect high mass binaries out to several hundred Mpc

Tapering the waveforms cleans up the spurious power at high frequencies



Phenomenological IMR Injections

- A good test to see if our templates can detect similar, but not identical waveforms.
- Another family of analytic IMR waveforms that is tuned to NR simulations
 - Constructed in the frequency domain
 - Tuned to NR simulations with mass ratios 1:1 – 4:1
 - Not intended to be used for mass ratios beyond 4:1
- Nevertheless, we injected up to 99:1 to test our pipeline
- Very asymmetric injections were recovered with poor chi-square and parameter estimation

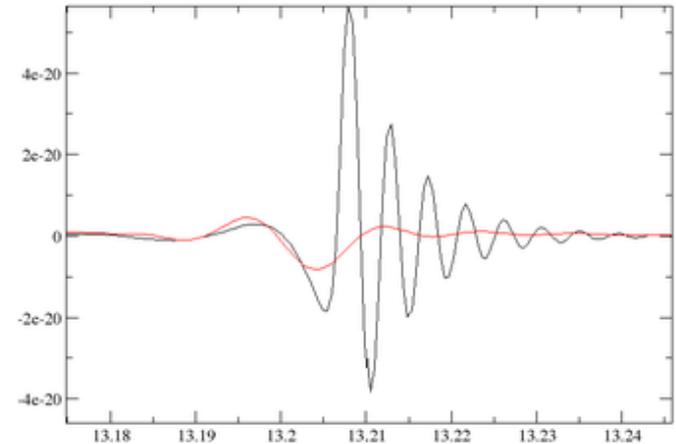


Spinning Kludge Waveform Injections

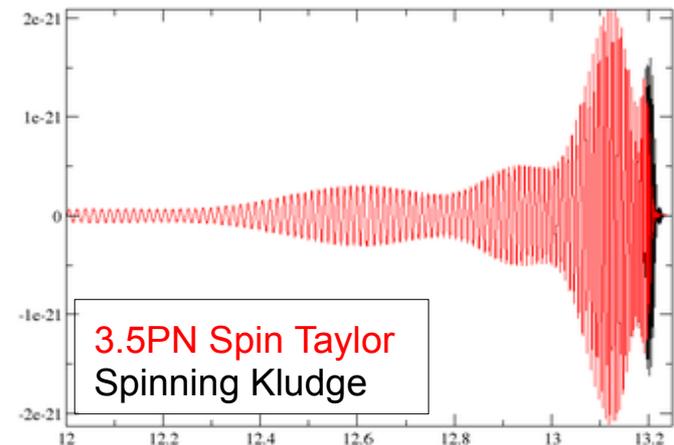
Ad-hoc ringdown attached at the end ...

- Can our non-spinning templates detect spinning IMR waveforms?
- We don't know what these waveforms will look like
- Kludge waveforms are constructed from:
 - 3.5PN Spin Taylor inspiralMerger
 - and ringdown are attached in an ad hoc manner

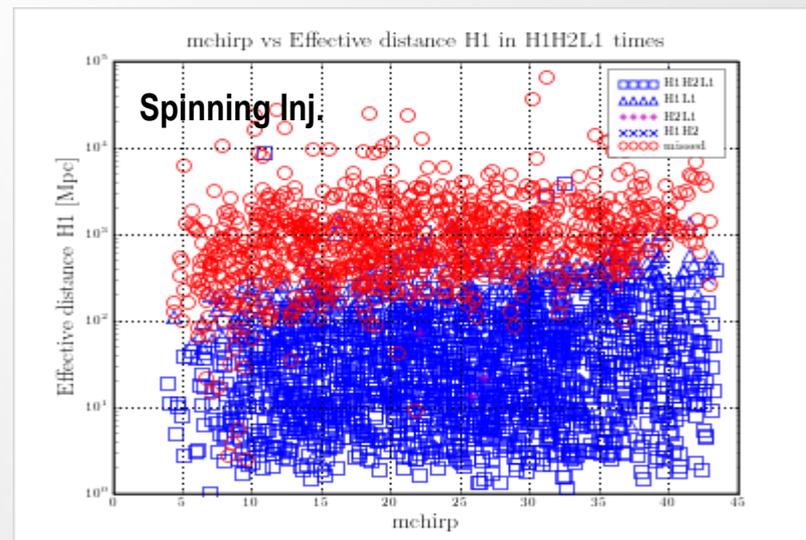
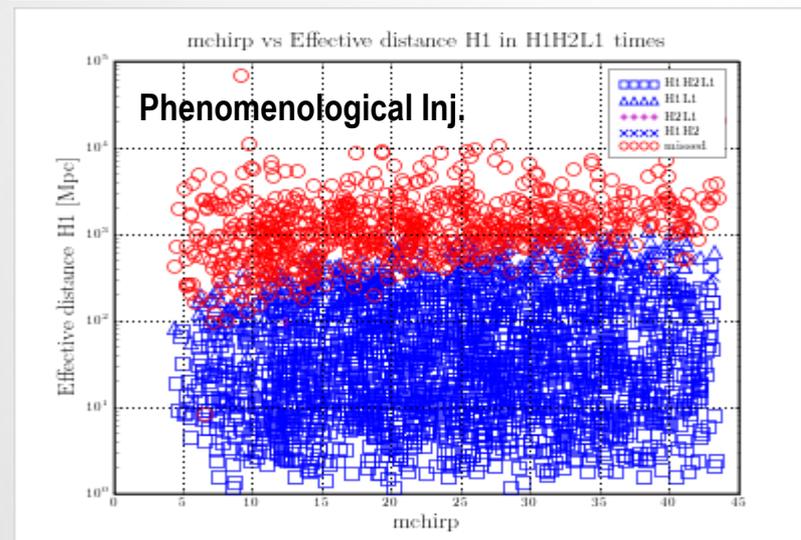
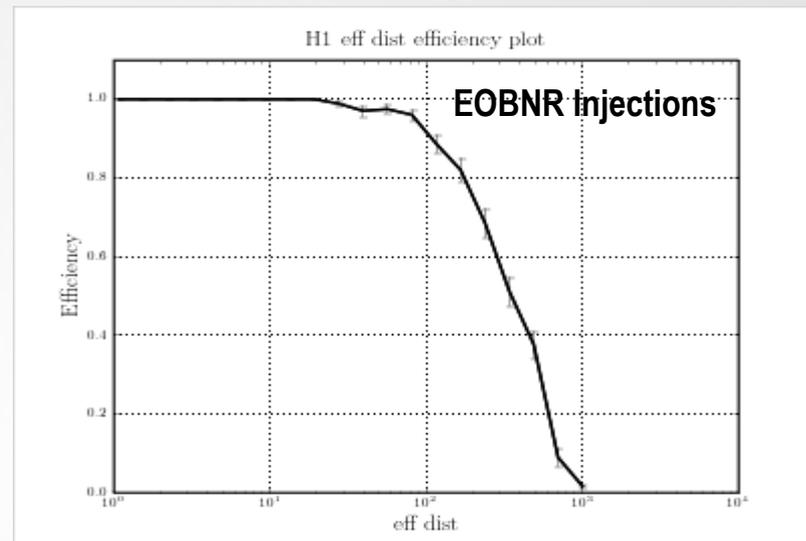
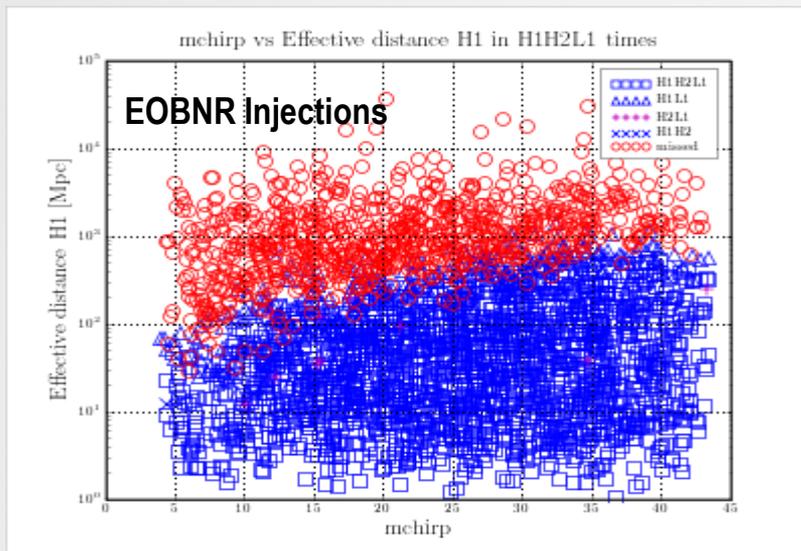
50+50 M_{\odot} Spinning Injections



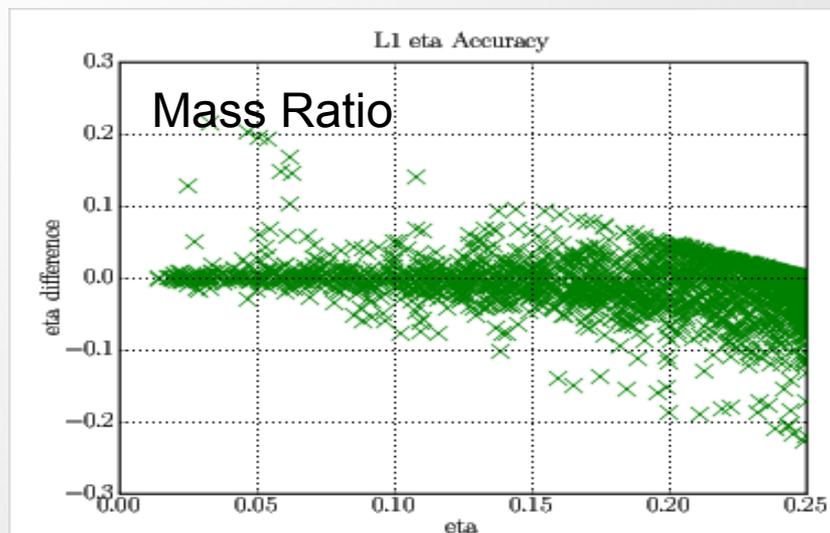
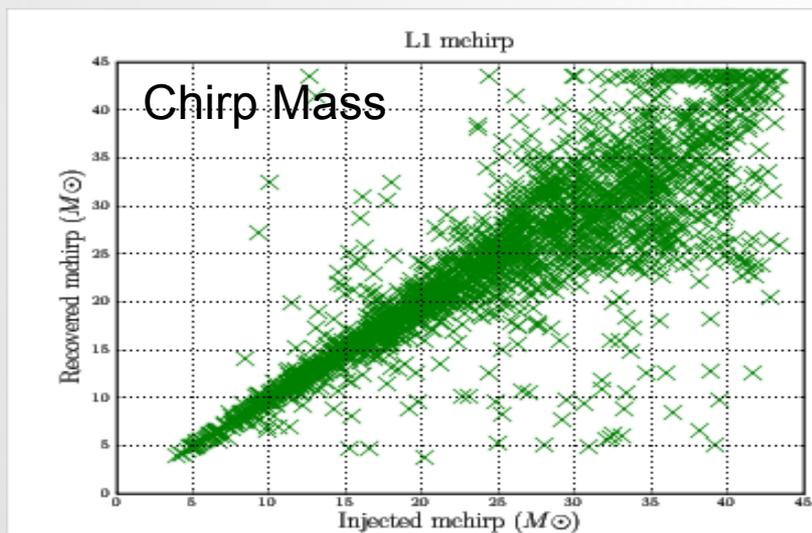
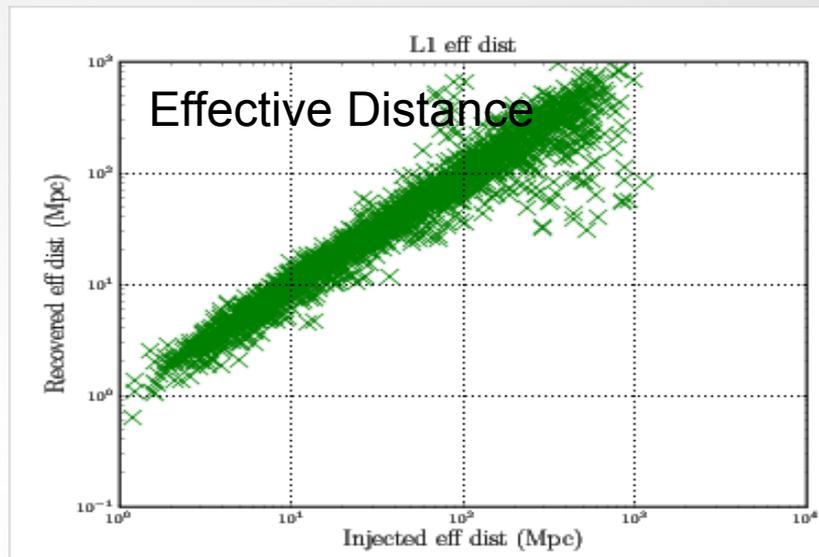
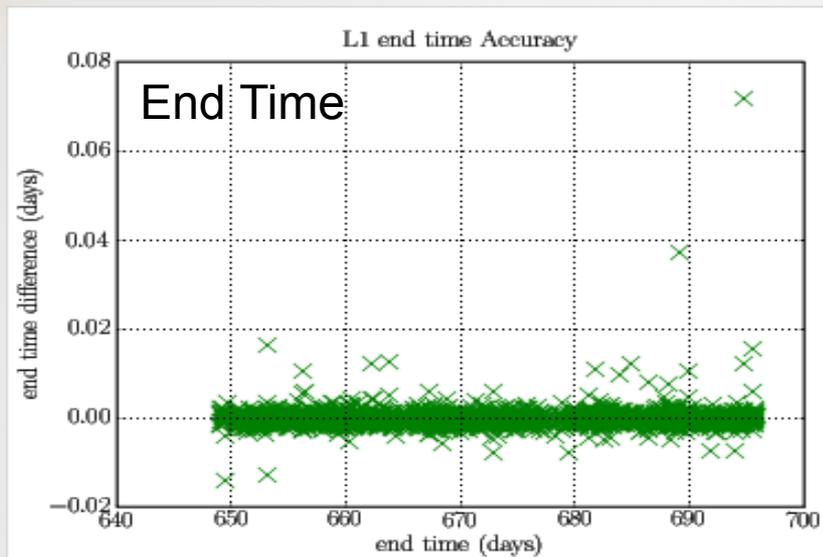
1+25 M_{\odot} Spinning Injections



Detection Efficiency



Parameter Accuracy: EOBNR Injections

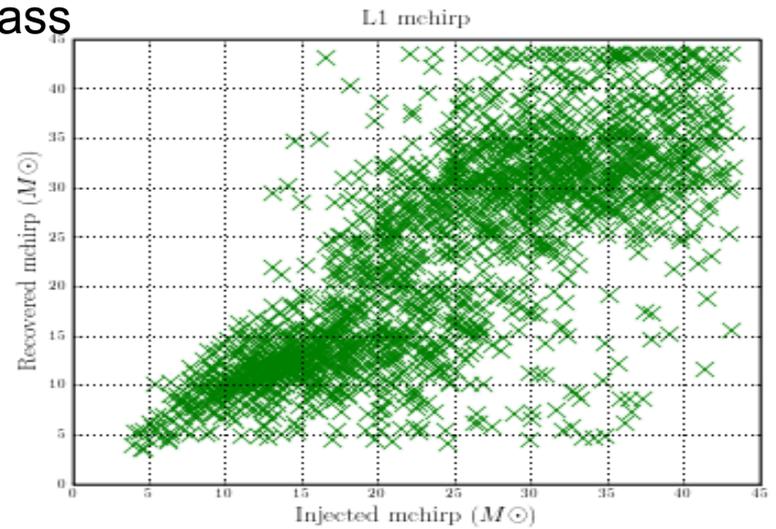
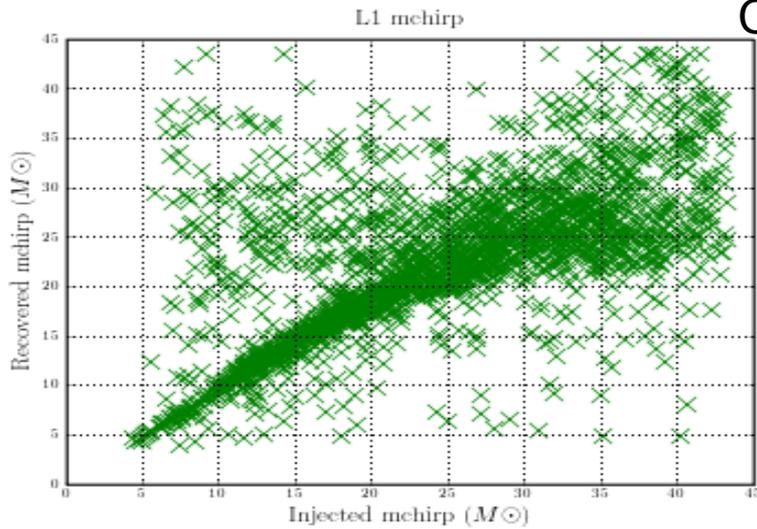


Parameter Accuracy: Other Injections

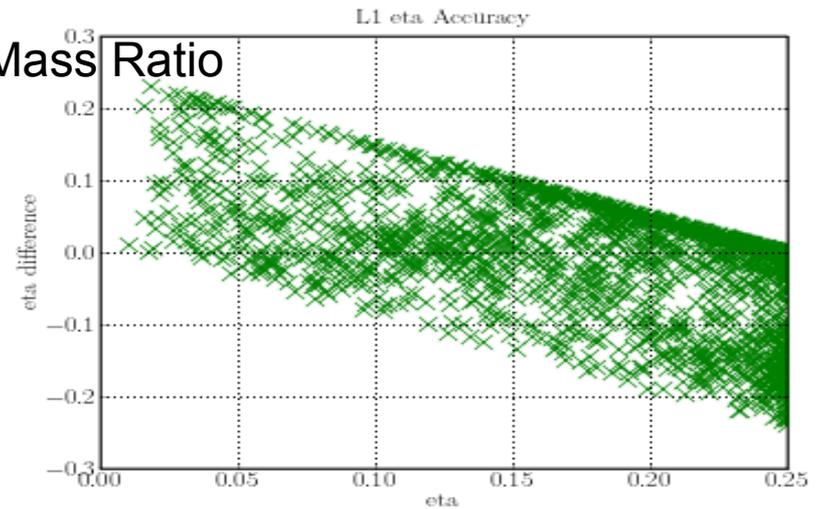
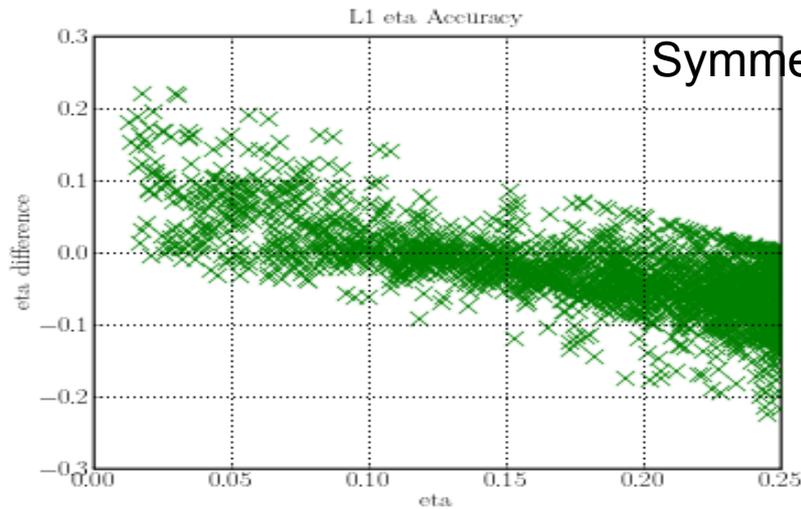
Phenomenological Injections

Spinning Injections

Chirp Mass

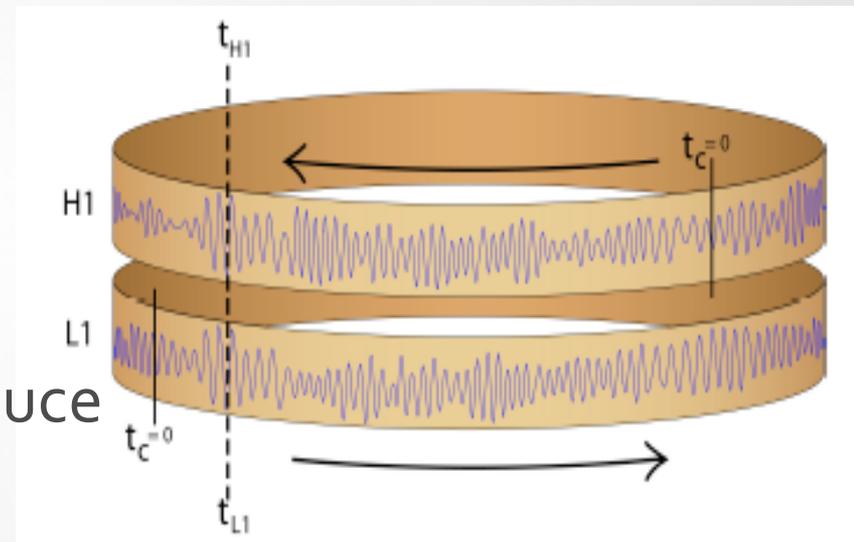


Symmetric Mass Ratio



Signal based veto

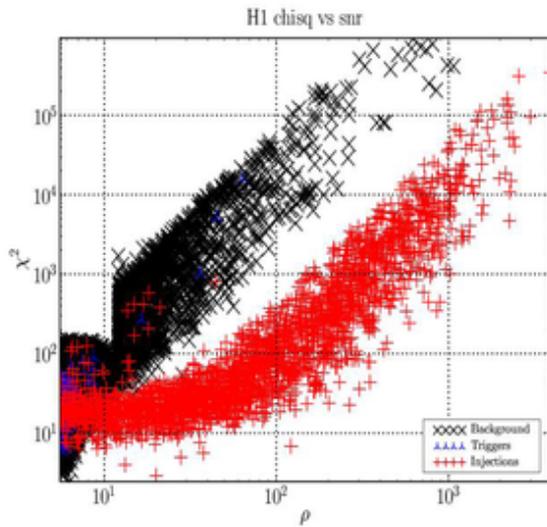
- Triggers from separate instruments are slid *wrt* each other
 - » Accidental coincidences
 - » Could not have arisen due to true gravitational wave event
 - » Estimate of our background
- Signal based vetoes help us reduce accidental coincidences
 - » Separate background from injections
 - » Improve confidence of detection
 - » Monte-Carlo injections help us tune the pipeline to reject accidental coincidences



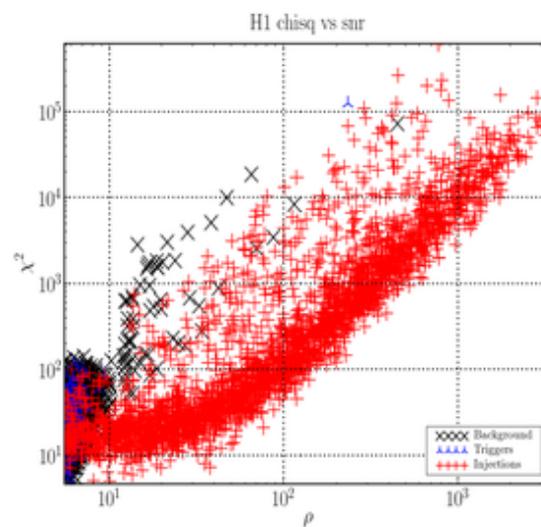
Graphics: Becky Tucker

Chi-square Signal-Based Veto

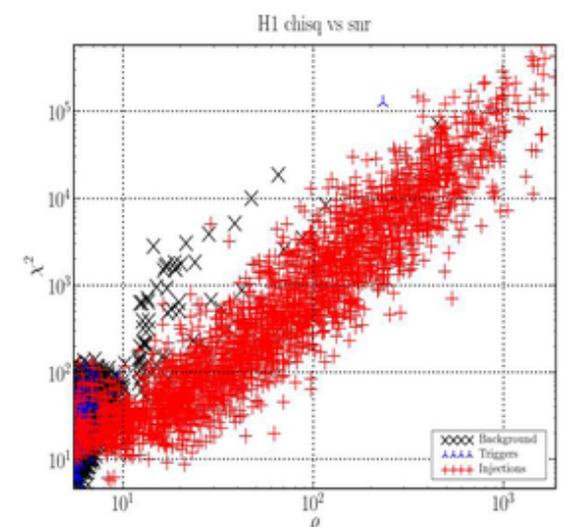
EOBNR Injections



Phenom. Injections



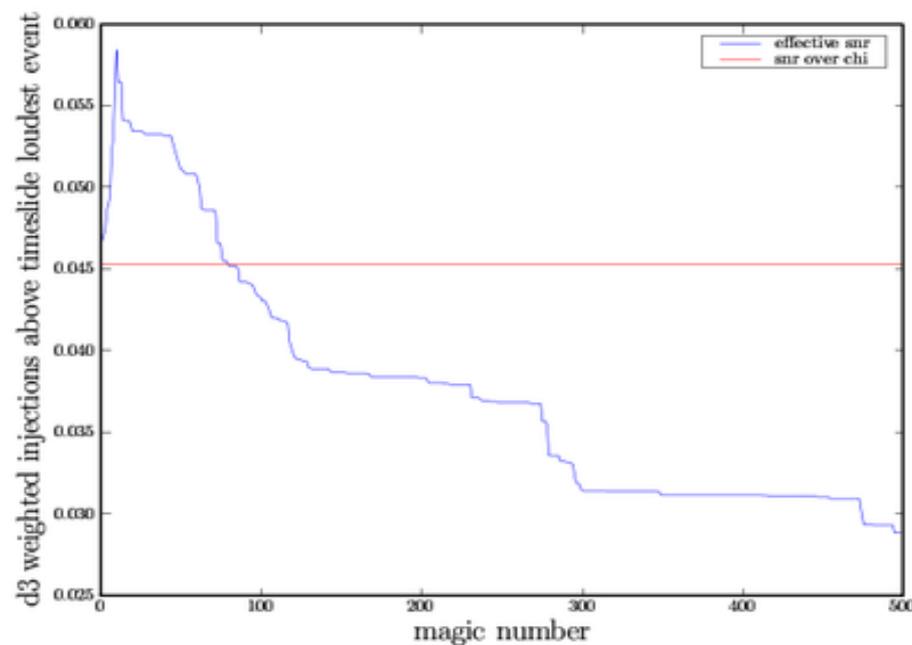
Kludge Injections



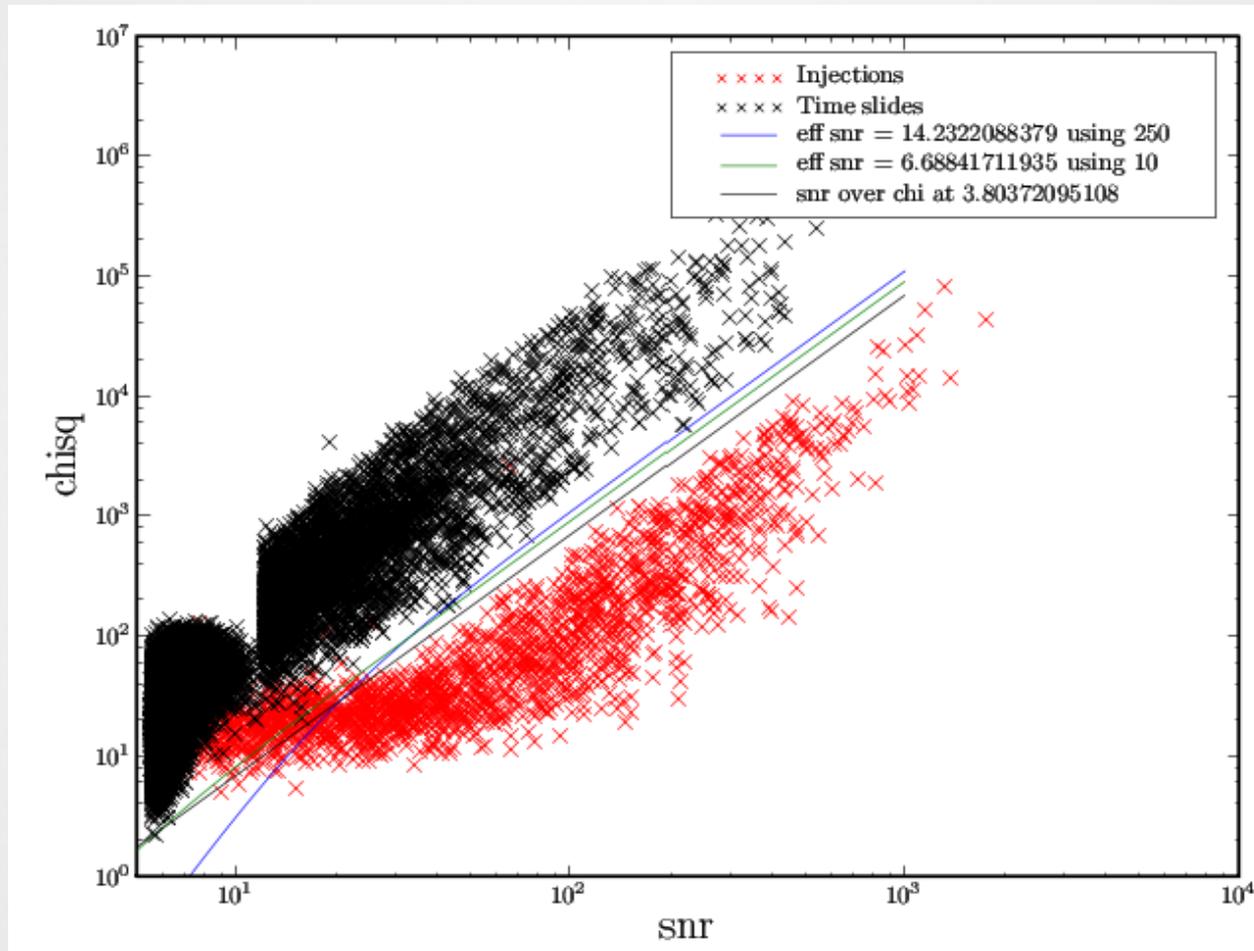
Tuning: Detection Statistic

- We want as many injections as possible to have ρ_{eff} louder than our loudest background trigger.
- We try to maximize our D_{eff}^3 weighted efficiency
 - » Most of the population will be at large effective distance (hence small SNR)
 - » These are the signals that we need to separate from the background.

$$\rho_{eff} = \frac{\rho}{\left(\frac{\chi^2}{2p-2}\right)\left(1 + \frac{\rho^2}{\rho_c}\right)^{1/4}}$$

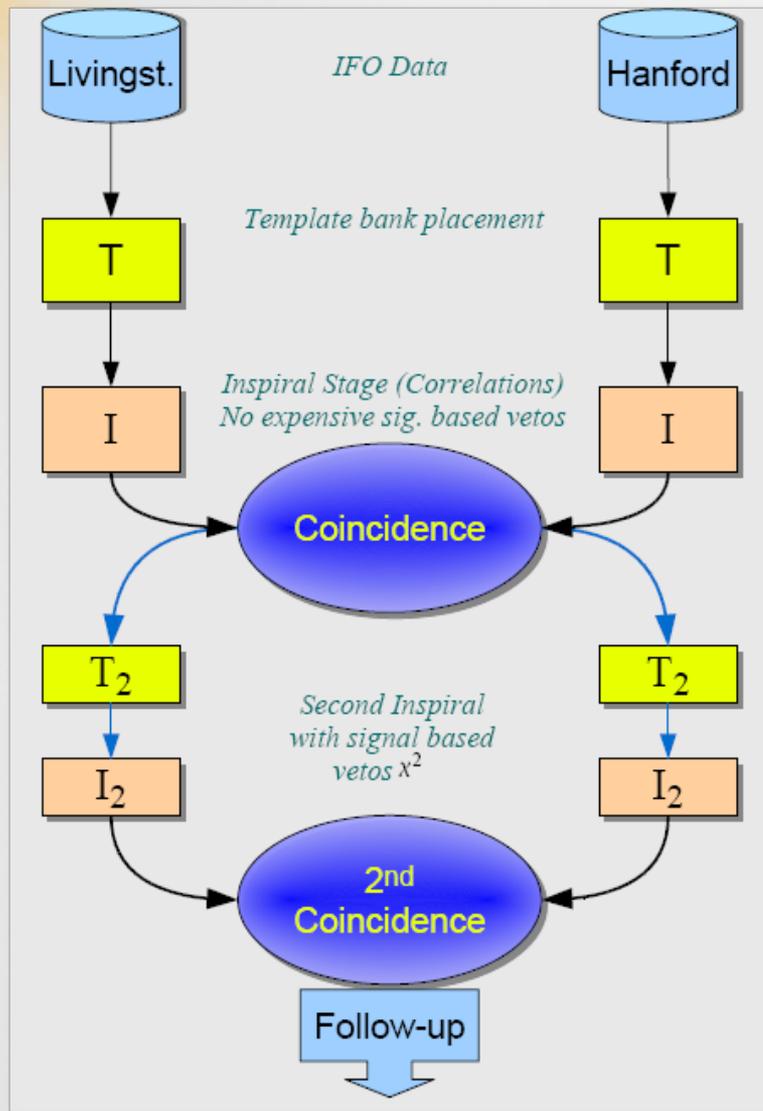


Tuning detection statistic



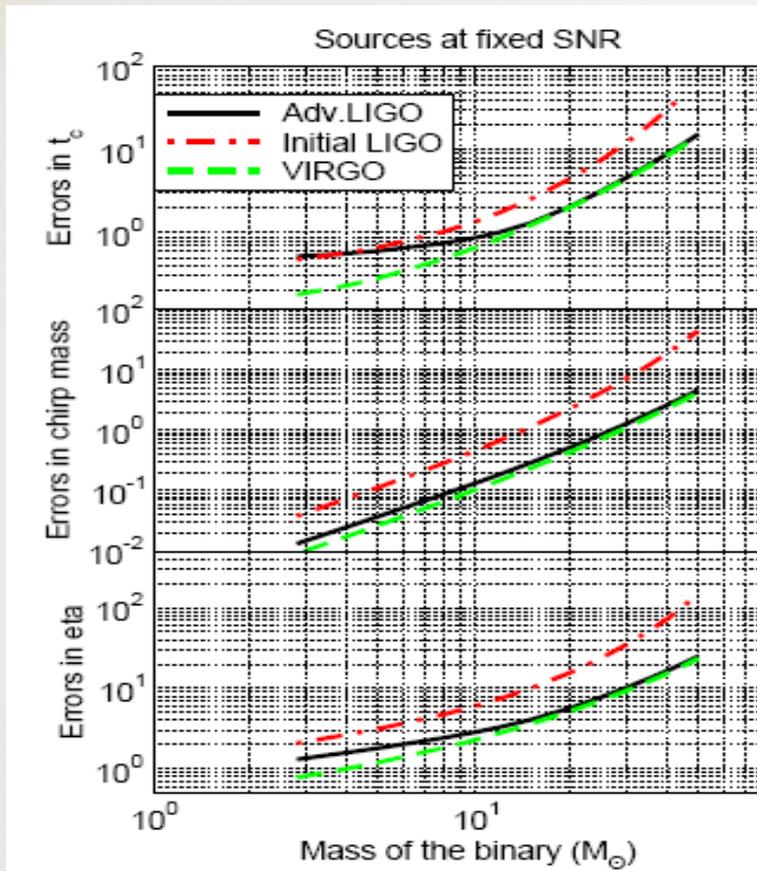
E-thinca : A new coincidence algorithm

Sengupta, Robinson and Sathyaprakash, 2008
Sengupta, Gupchup and Robinson, 2008



- » Coincidence windows replaced by error ellipsoids associated with each trigger
- » Error ellipsoids determined by the metric in the space of parameters
- » One tunable parameter:
 - *ellipsoid scaling factor, e_p*
- » Introduces parameter dependence,
 - but also uses information about correlation
 - Volume of ellipsoid $\sim 30x$ less than equivalent standard windows

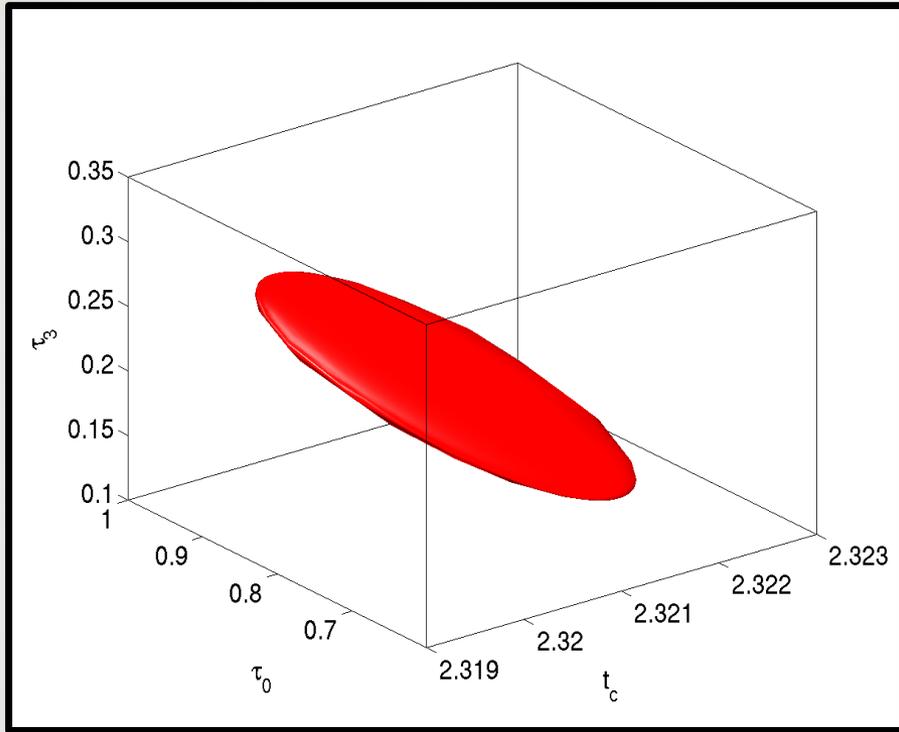
Motivations for parameter-dependent approach



K.G. Arun, B. R Iyer, B.S. Sathyaprakash, P.R. Sundararajan, 2004

- » Error in measurement of parameters vary widely across BBH parameter space.
-
- » Suggests fixed-window coincidence method is not optimal for BBH searches.
-
- » Motivated the development of analysis using parameter-dependent windows.

Ellipsoid model for the triggers



Metric codes in correlation between parameters.

- » Ellipsoids
 - Position vector of the centre
 - Shape matrix \mathcal{G}

- » Mathematical definition of the ellipsoids

$$\mathcal{E}(\vec{r}, \mathcal{G}) = \{ \vec{x} \in \Lambda \mid (\vec{x} - \vec{r})^T \mathcal{G} (\vec{x} - \vec{r}) \leq 1 \}$$

- » Where,

$$\mathcal{G} = \frac{\Gamma}{1 - e_p}$$

Contact Function

- » Contact function to test overlap of ellipsoids

$$F_{(i,j)} = \max_{0 \leq \lambda \leq 1} \mathcal{F}_{(i,j)}$$

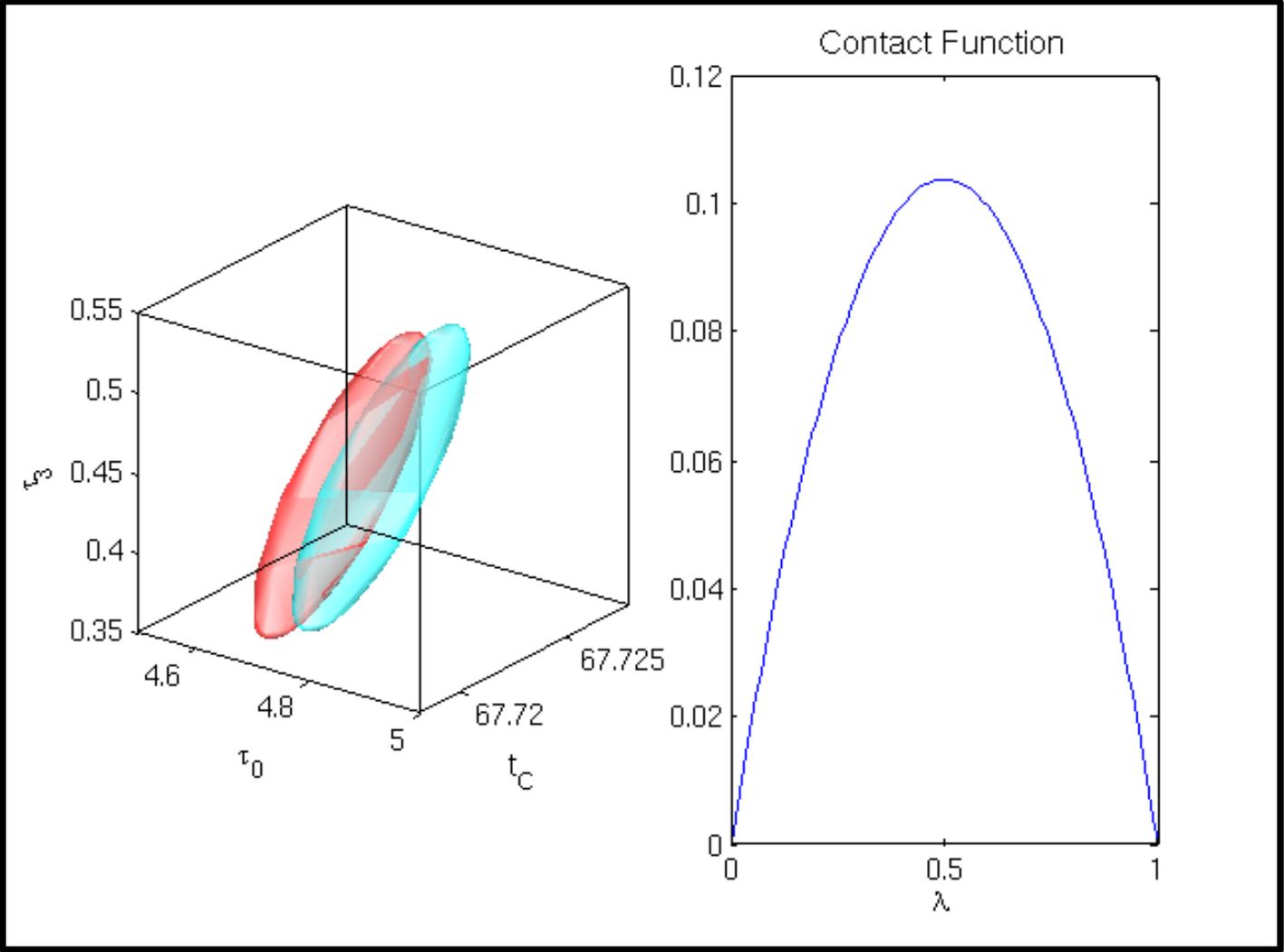
where,

$$\mathcal{F}_{(i,j)} = \left\{ \lambda(1 - \lambda) \vec{r}_{(i,j)}^T [\lambda \mathcal{G}_j^{-1} + (1 - \lambda) \mathcal{G}_i^{-1}]^{-1} \vec{r}_{(i,j)} \right\}$$

Perram & Werthiem, 1985,
Journal of Computational Physics

- » Function is bound between $0 \leq \lambda \leq 1$
- » Second derivative of contact function $\mathcal{F}_{(i,j)}$ w.r.t parameter λ is negative definite. This implies a unique maximum in the above interval.
- » Ellipsoids labeled i and j are deemed to overlap if $F_{(i,j)}$ is less than 1.

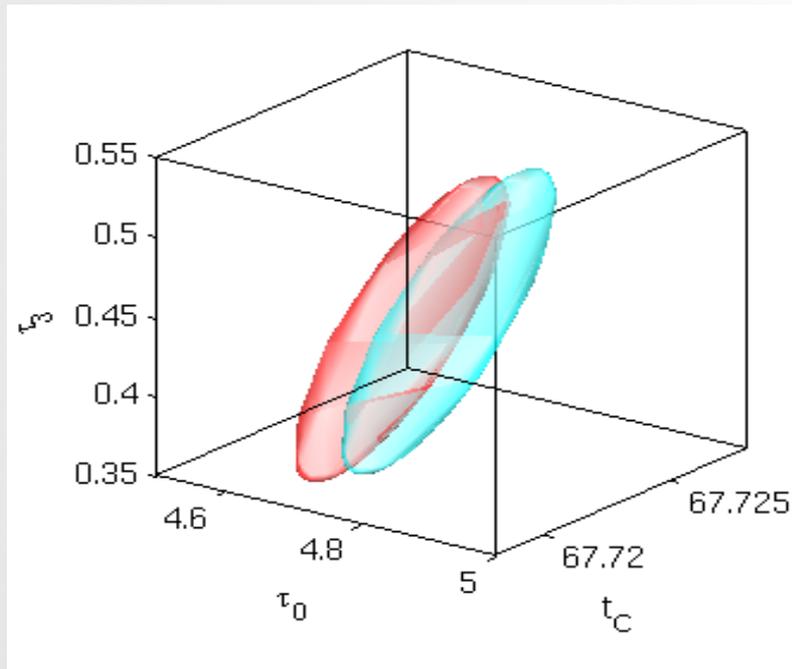
Overlap of ellipsoids-2



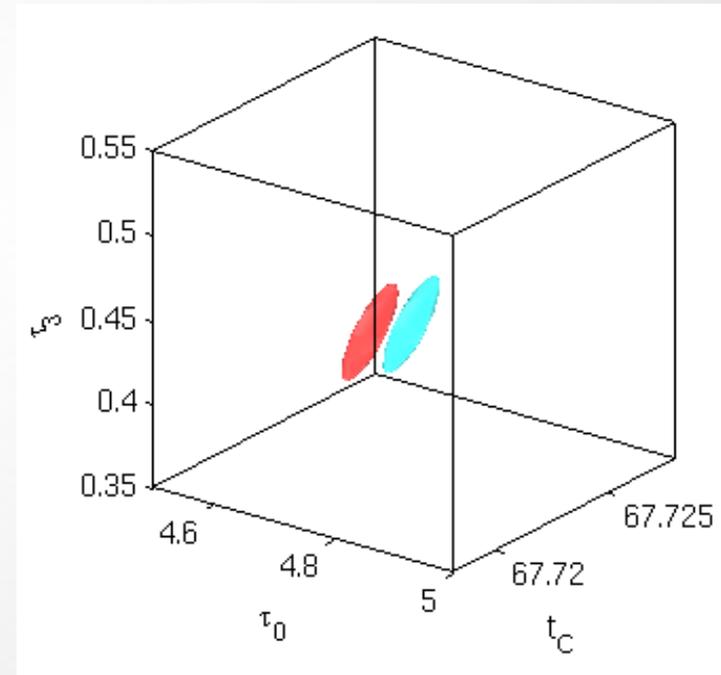
Ellipsoid coincident analysis method

If ellipsoids of triggers from different IFO's overlap, they are deemed coincident

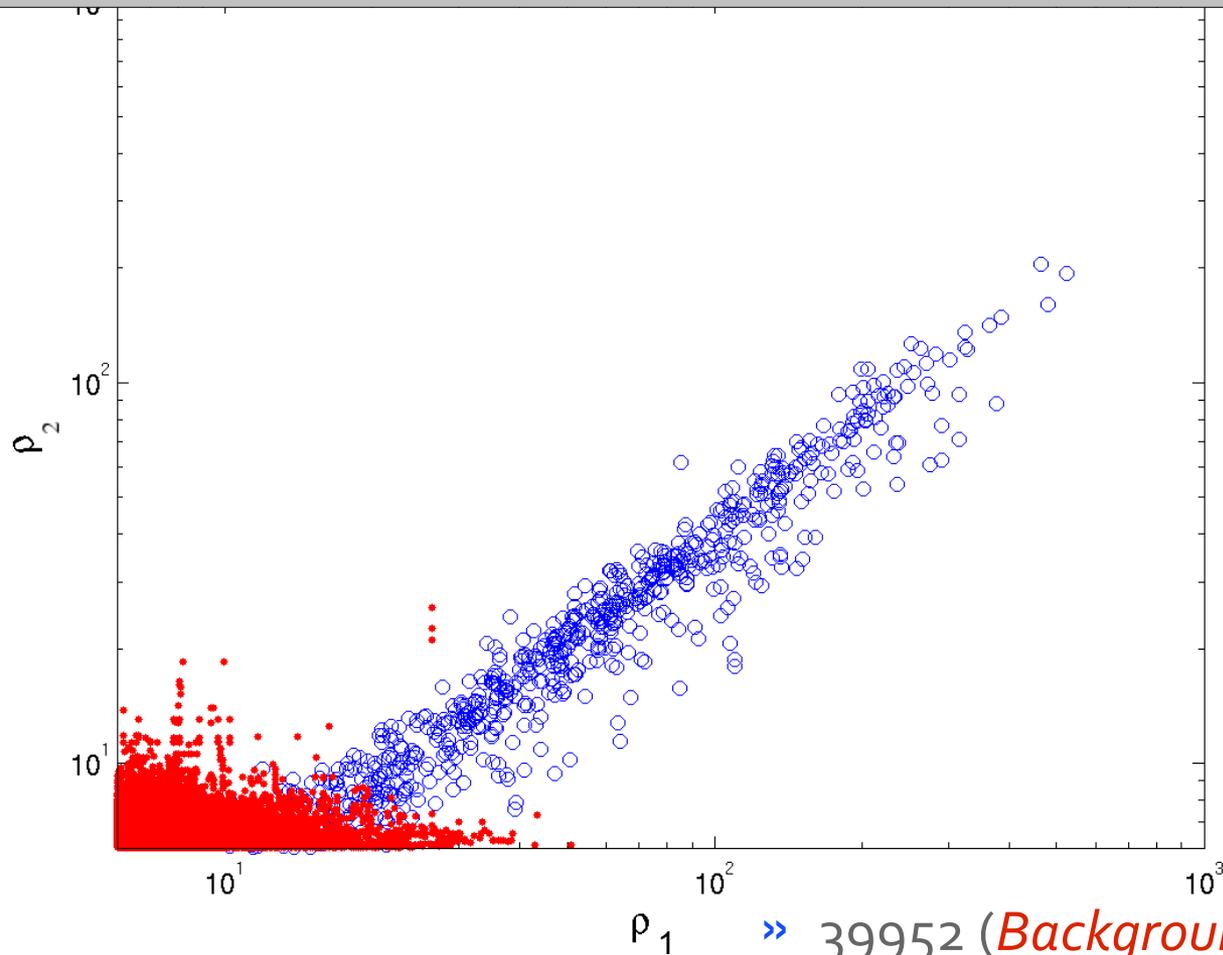
Coincident



Not coincident



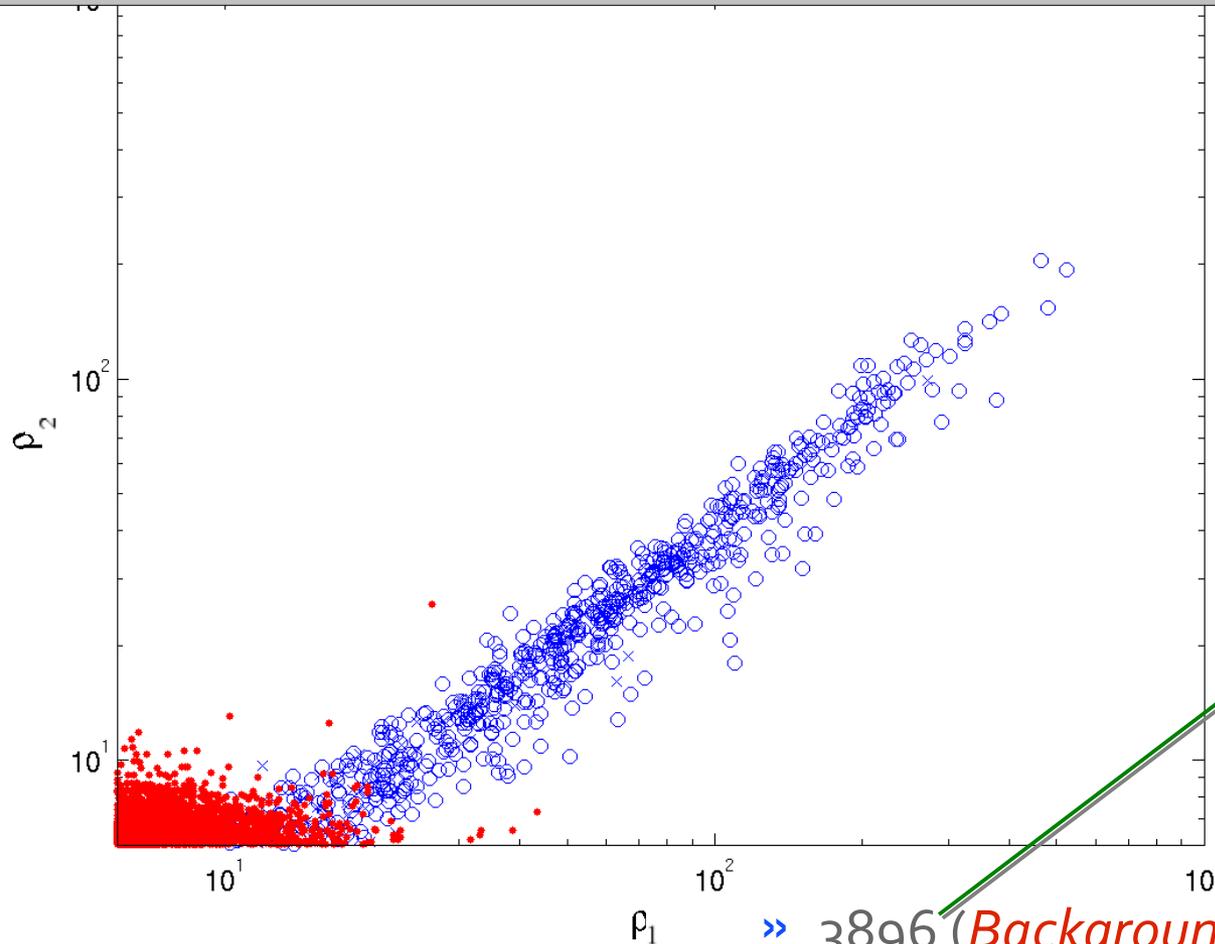
Time Slides and injections using standard coincidence method



» 39952 (*Background*)

» 883 found out of 1925 (*Injections*)

Time Slides an injections using ellipsoid coincidence method
(ellipsoid scaling factor = 0.5)

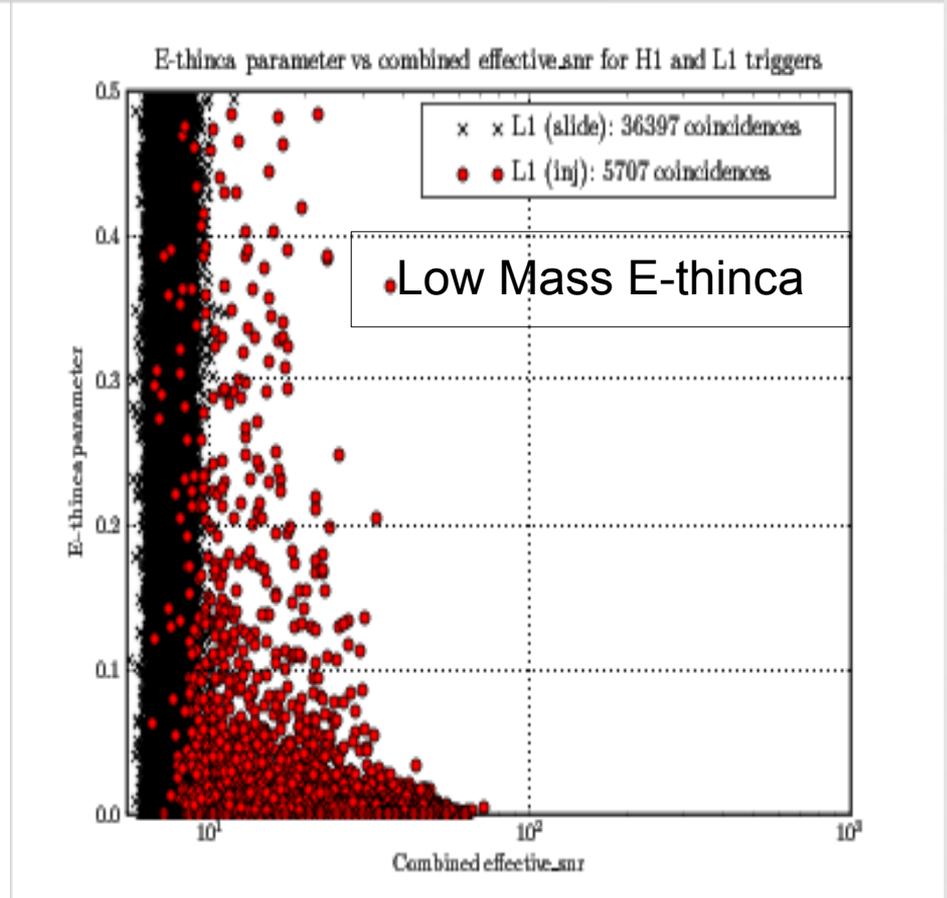
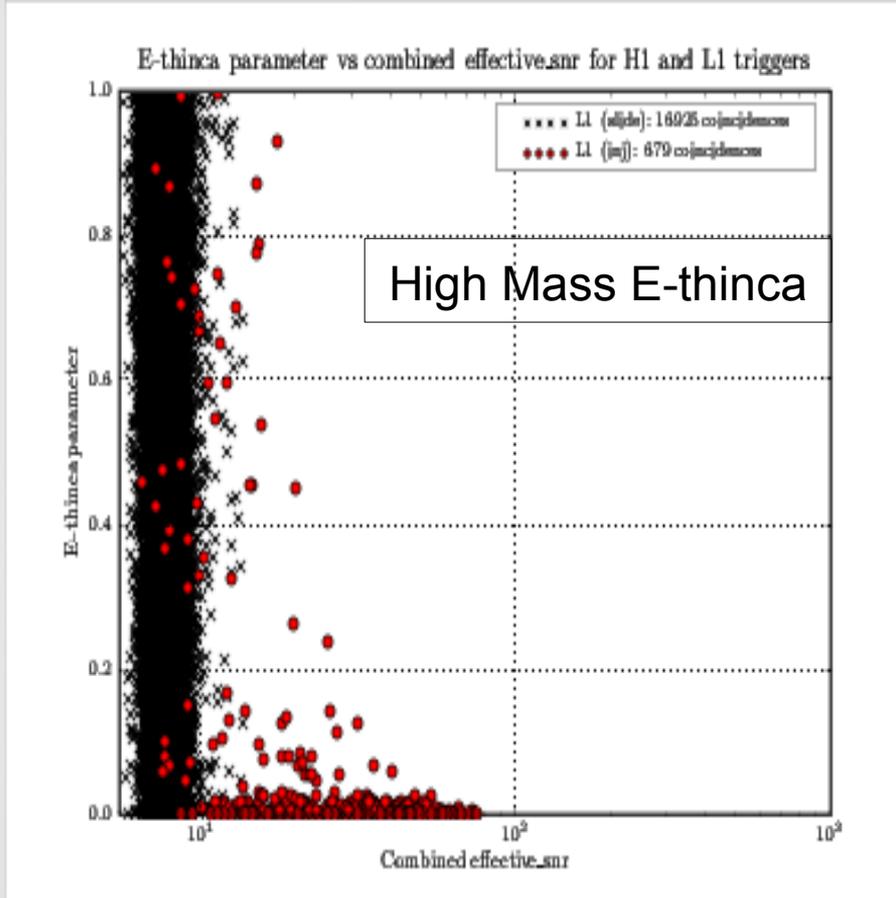


X10
reduction

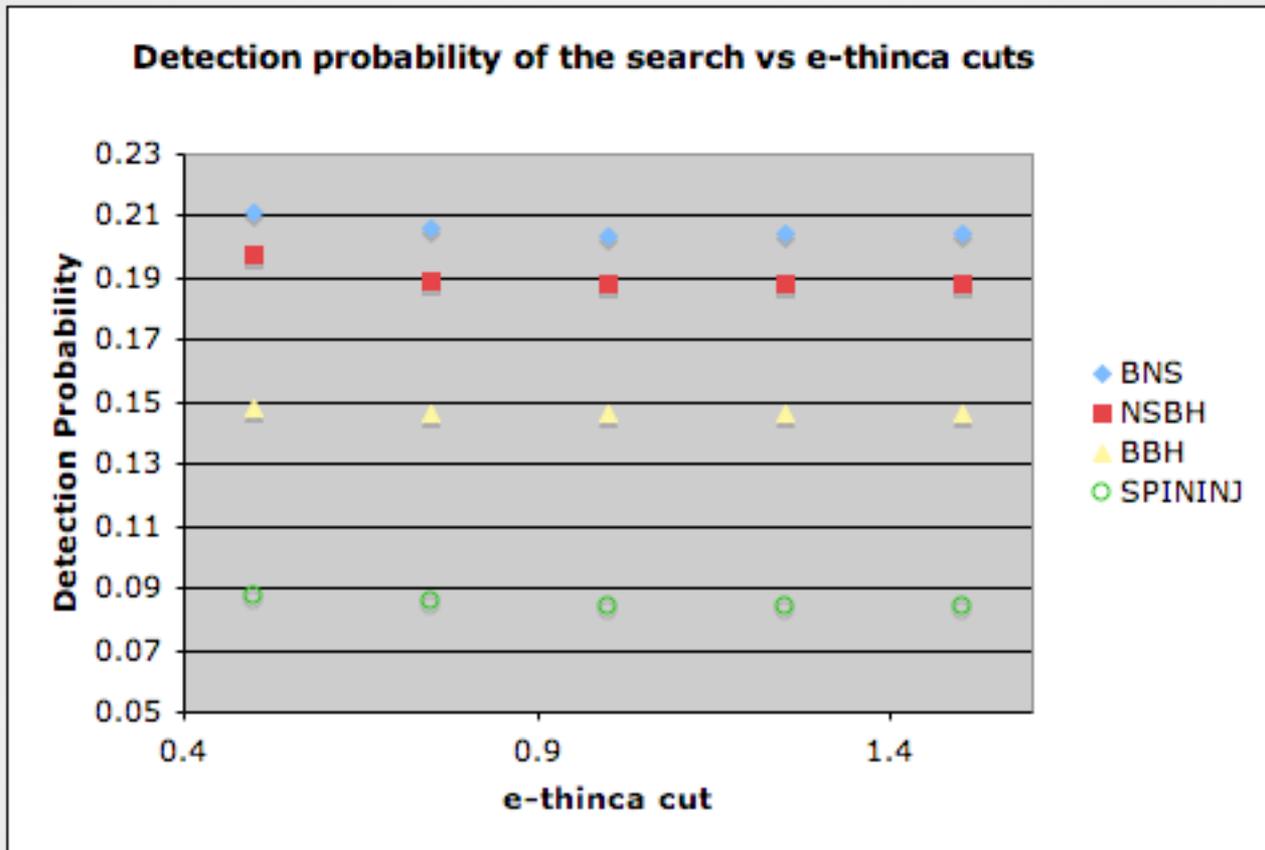
» 3896 (*Background*)

» 878 found out of 1925
(*Injections*)

Tuning: E-thinca parameter



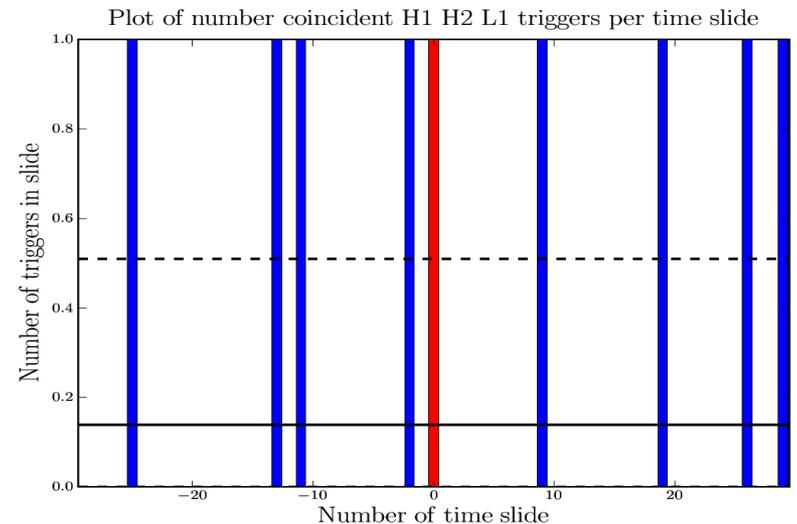
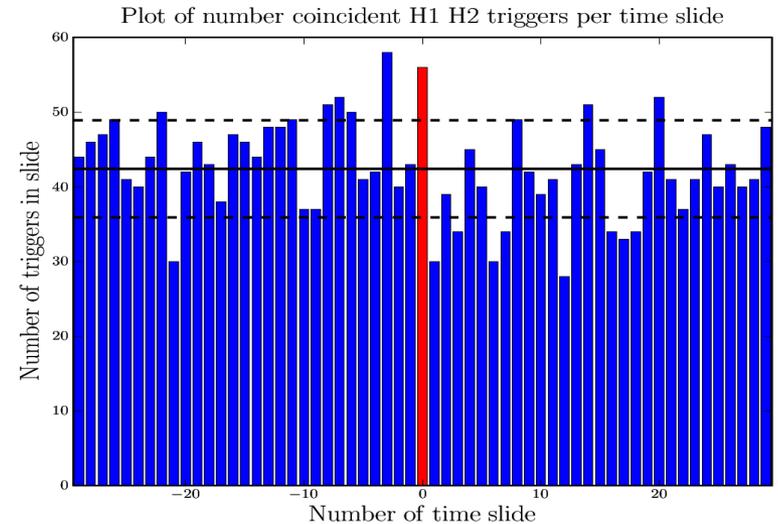
Tuning the e-thinca parameter



Fixed false alarm probability

H1H2 Time Slides

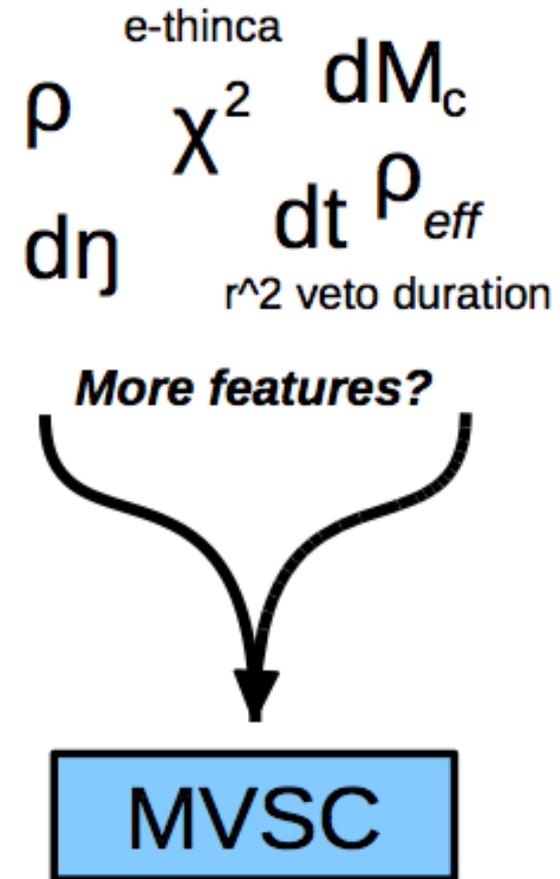
- To estimate the background, we slide data from different interferometers in time
- Sliding H1 and H2 relative to one another underestimates the rate of coincident glitches
- This causes an underestimate of the H1H2 and H1H2L1 backgrounds
- H1 and H2 should be slid together
- This will fix the H1H2L1 background, although the H1H2 background cannot be estimated

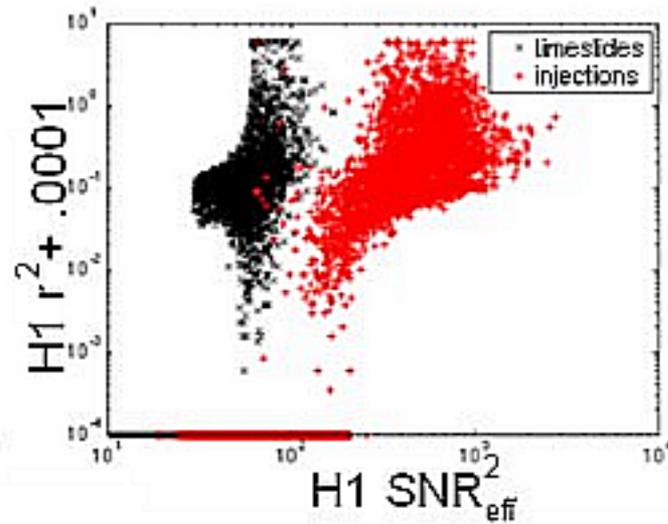
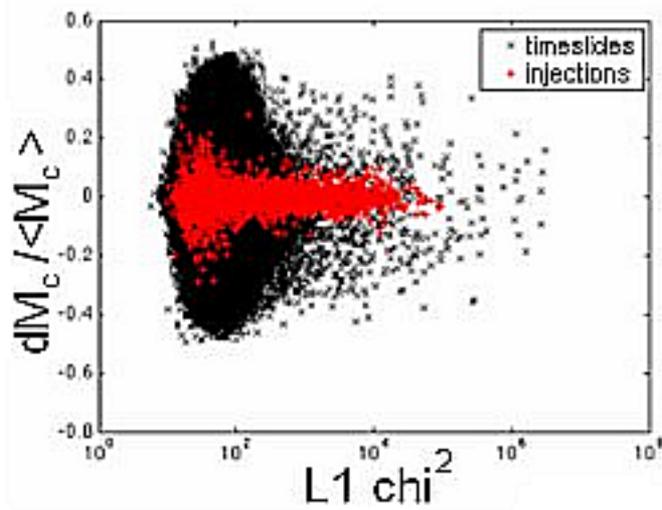
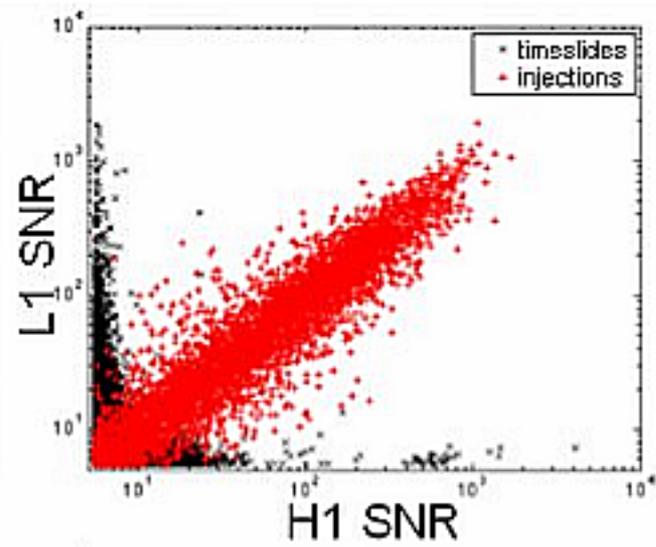
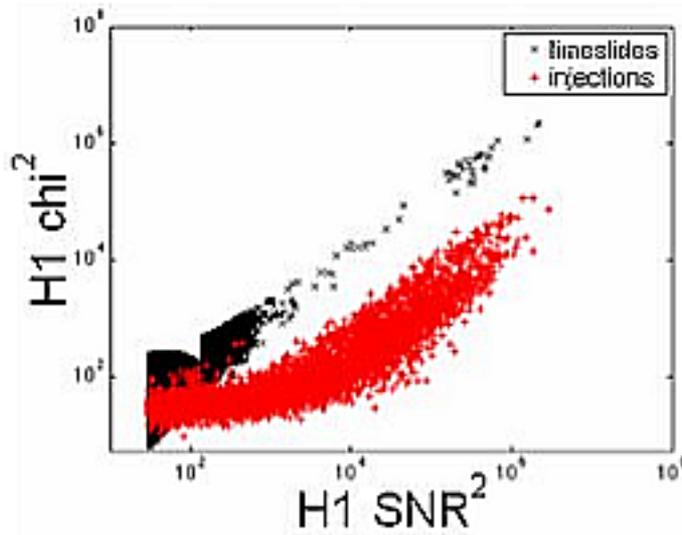


Multi-Variate Statistical Classifier (MVSC)

Hodge, Sengupta, Weinstein 2010

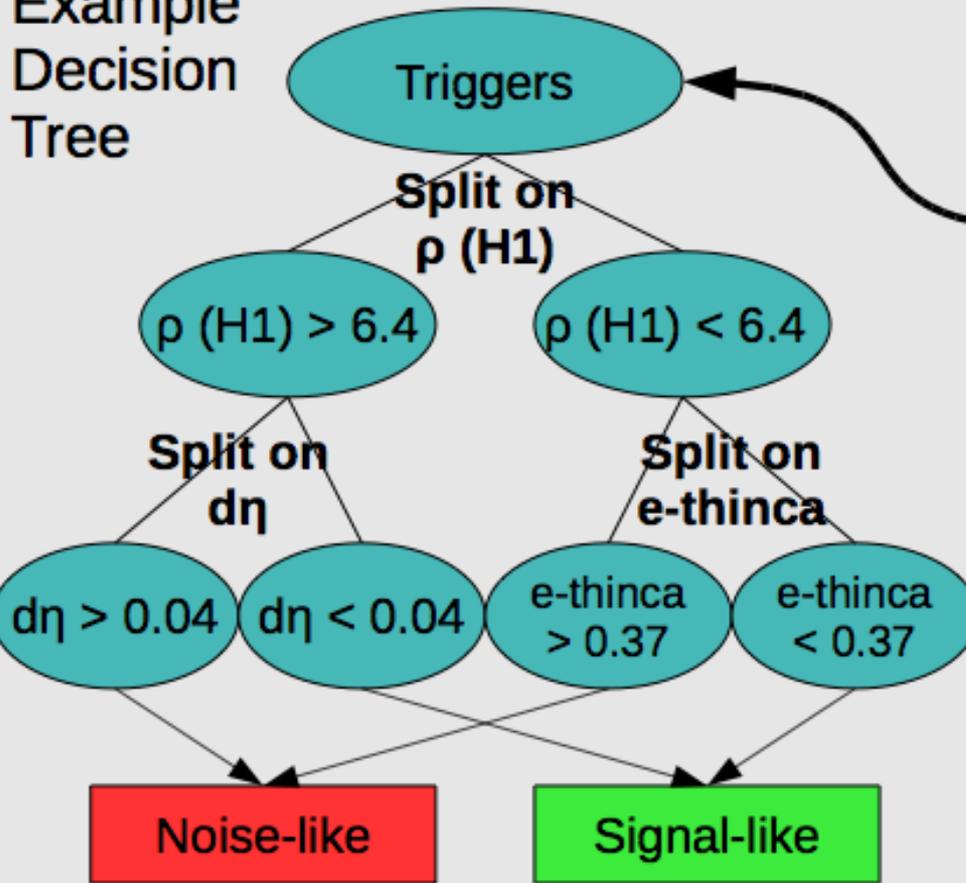
- MVSC combines many different features of a trigger into a single number between 0 and 1, with 0 being noise-like and 1 being signal-like.
- Generation of the MVSC statistic is fully automated
 - Uses a machine-learning technique known as a **random forest of bagged decision trees**





Forest of decision trees

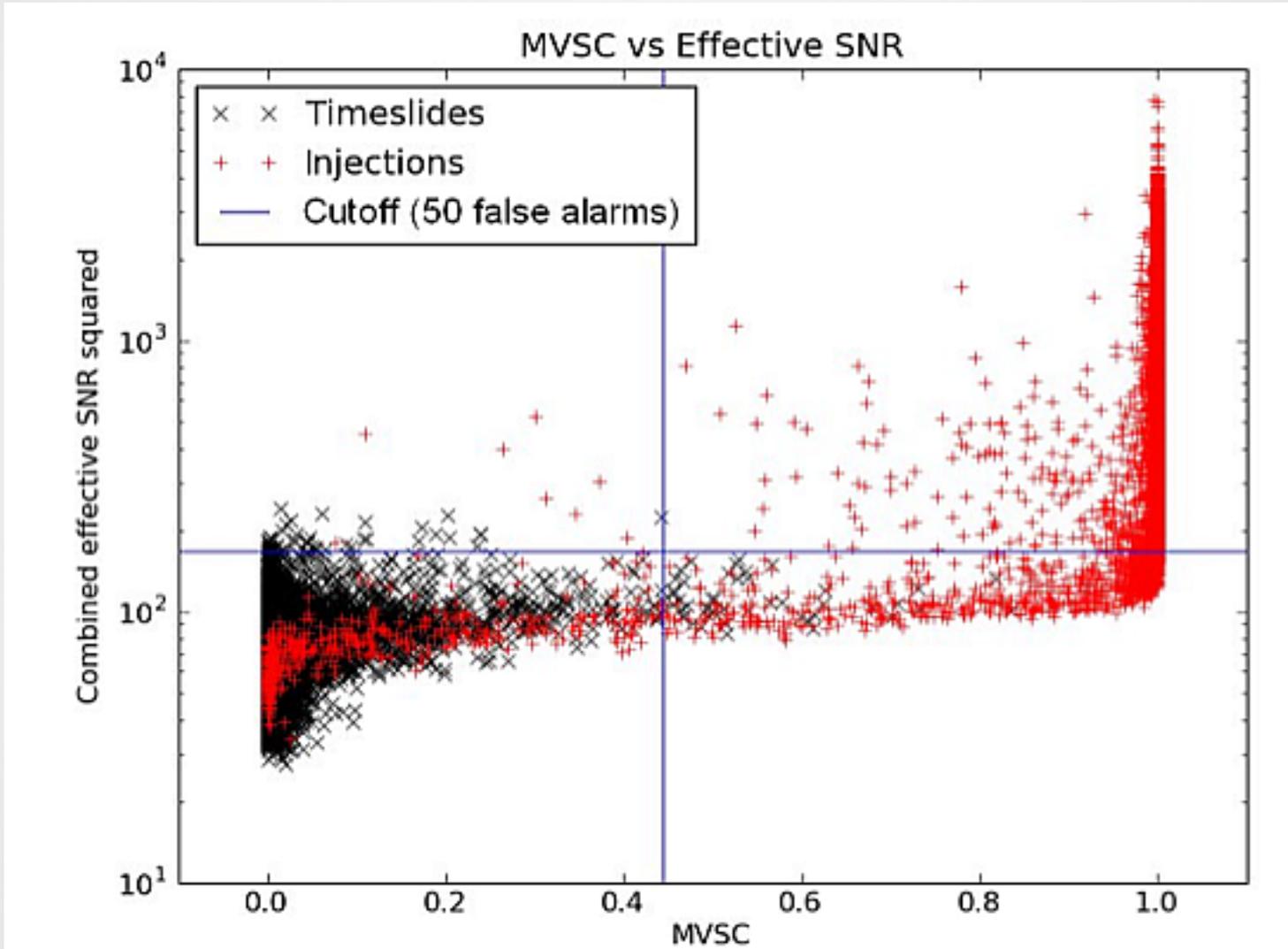
Example
Decision
Tree



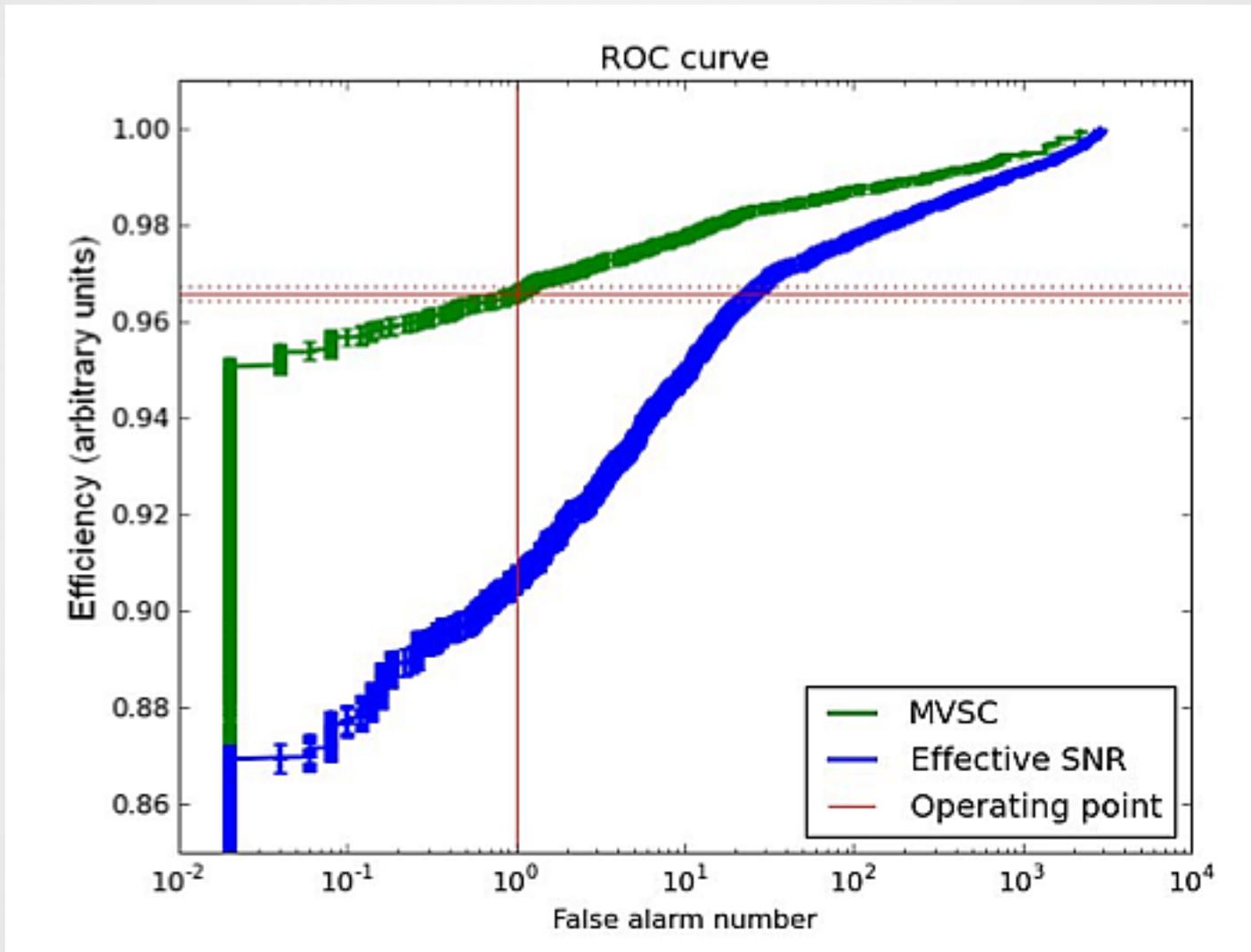
Bagging = Bootstrap Aggregating

- For a training set of size N , randomly choose N events, with replacement.
- For each tree, choose only a few features (ie ρ (H1), $d\eta$, e-thinca) to use in that tree.
- Generate many decision trees, have them decide by vote.

MVSC improves the low SNR sensitivity of our searches



MVSC improves the low SNR sensitivity of our searches



Singular value decomposition of templates

Cannon, Hanna, Keppel et al. 2010

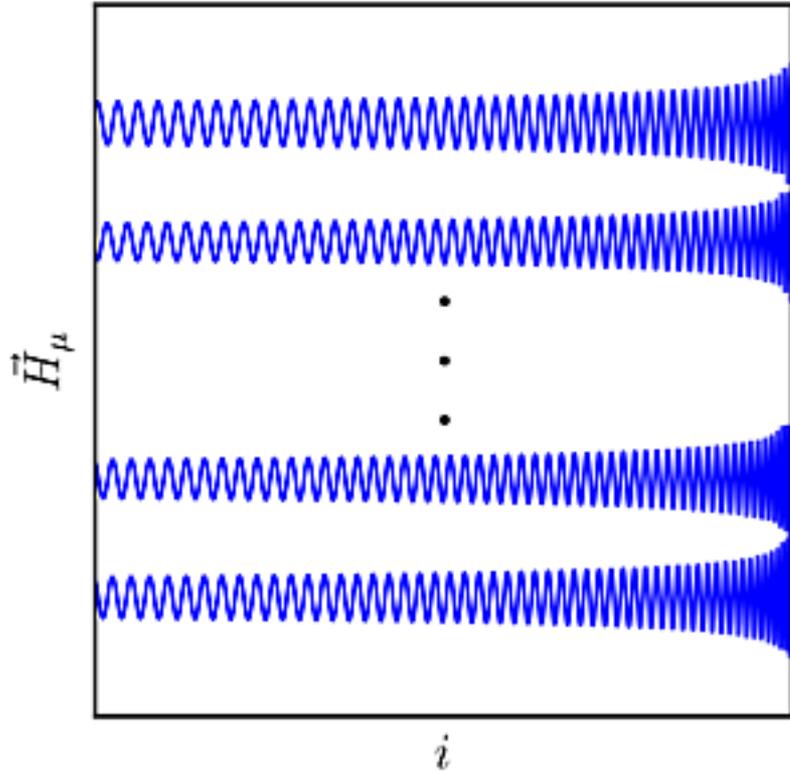
- Singular value decomposition
 - » Factorization of the signal matrix of size $\mu \times j$

$$A_{\mu j} = \sum_{\nu=1}^N V_{\mu\nu} \sigma_{\nu} U_{\nu j}$$

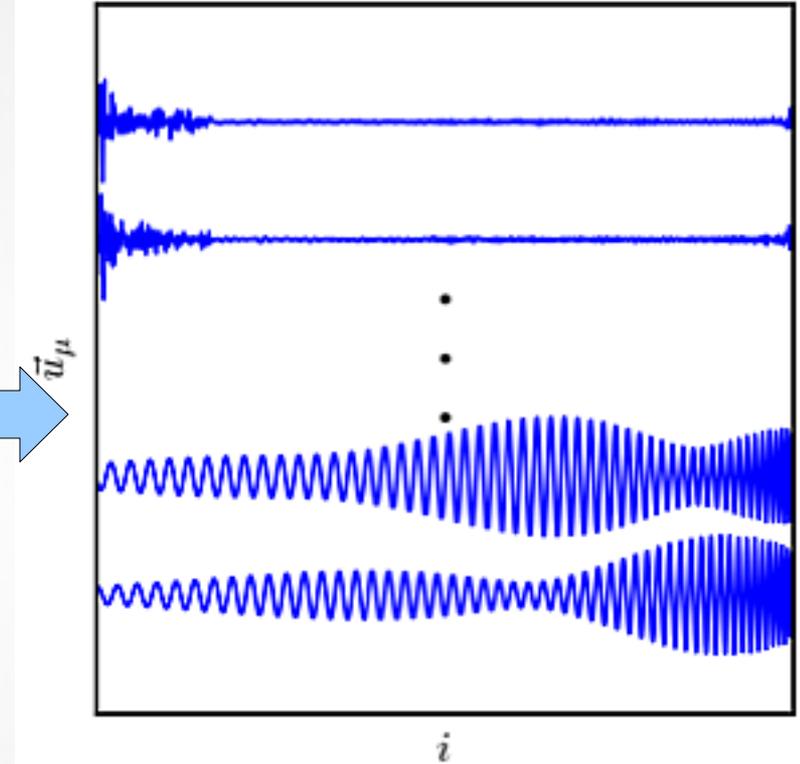
- » Matched filtering reduces to

$$\rho_{\alpha} = H_{\mu j} \circ s_j$$

- » The eigen values rapidly decrease as a result of which only a few eigen vectors (or SVD'd templates) are required to reconstruct



SVD

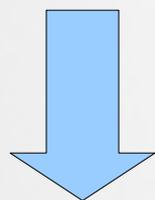


$$A_{\mu j} = \sum_{\nu=1}^N V_{\mu\nu} \sigma_\nu U_{\nu j}$$

Signal to noise re-construction

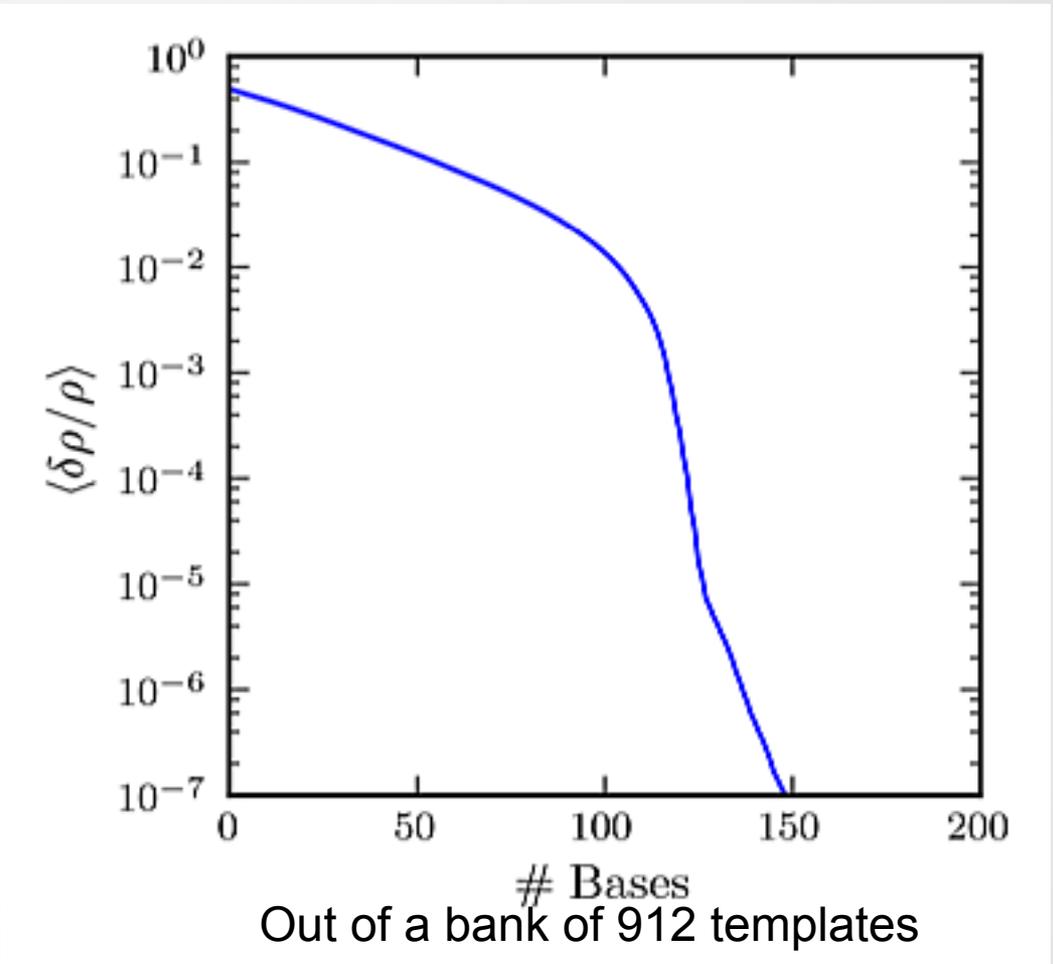
Only a few eigen-vectors can reconstruct the statistic accurately

$$A_{\mu j} = \sum_{\nu=1}^N V_{\mu\nu} \sigma_{\nu} U_{\nu j}$$



$$\rho_{\alpha} = H_{\mu j} \circ S_j$$

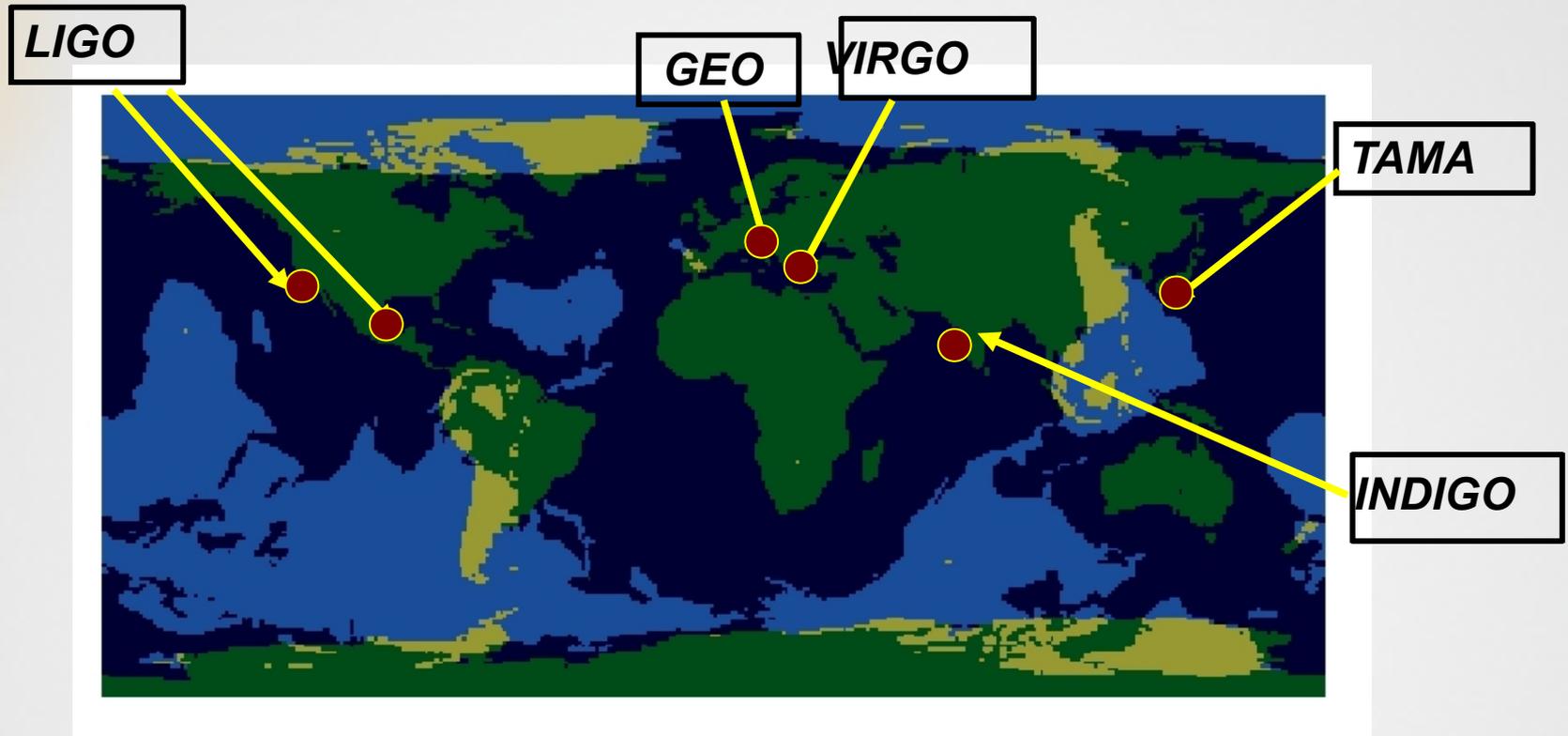
X10 speed-up !!!!



GW detectors worldwide

Improve confidence, coverage and collaboration

- GEO
- VIRGO
 - Current status
 - Low frequency: seismic isolation
 - advanced Virgo
- LIGO-VIRGO collaboration (LVC)
 - Manpower
 - Analysis tools
 - Joint projects
- LCGT, ET
- Space based GW detectors
 - PathFinder
 - LISA, DECIGO, BBO

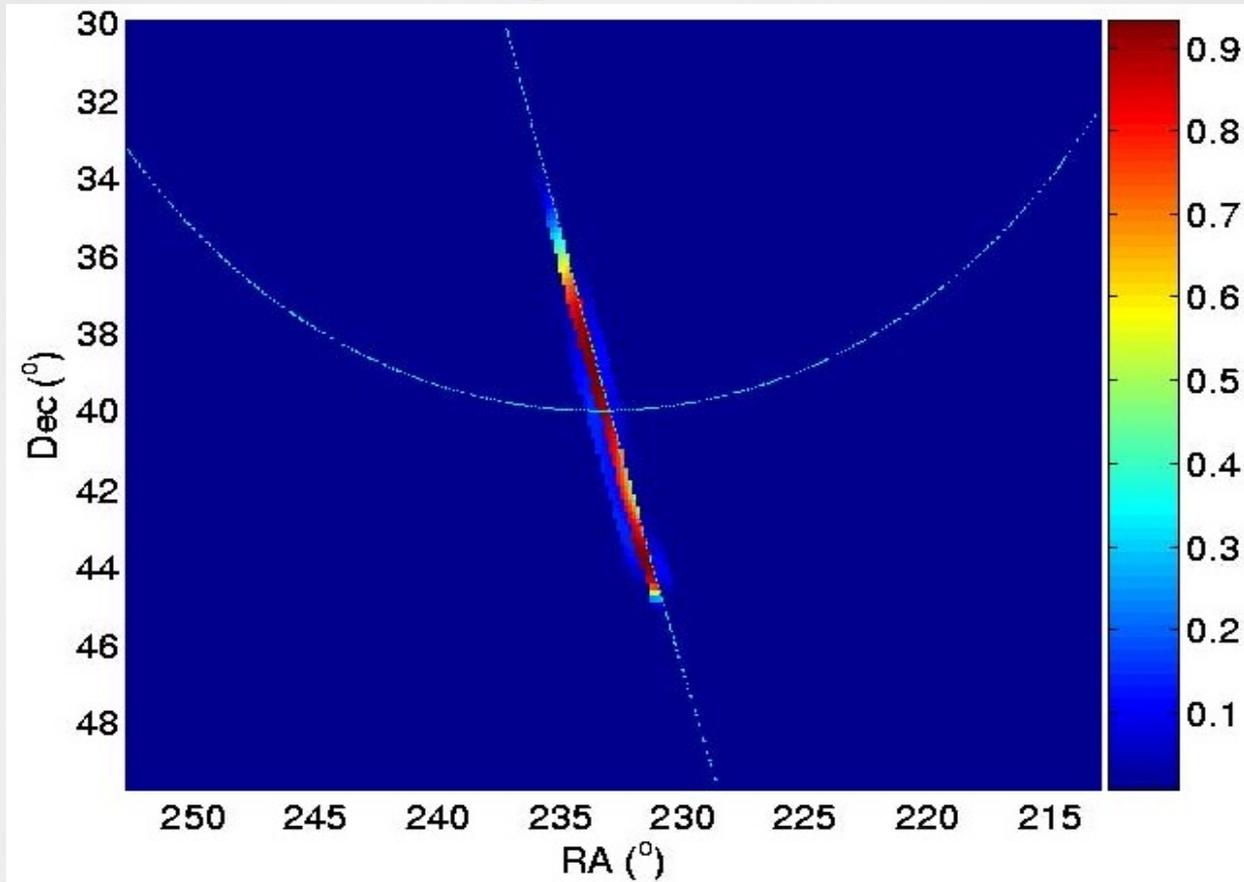


All interferometers run simultaneously and detect gravitational wave signal within a few msec.

Locate source by triangulation

Decompose the polarisation of gravitational waves.

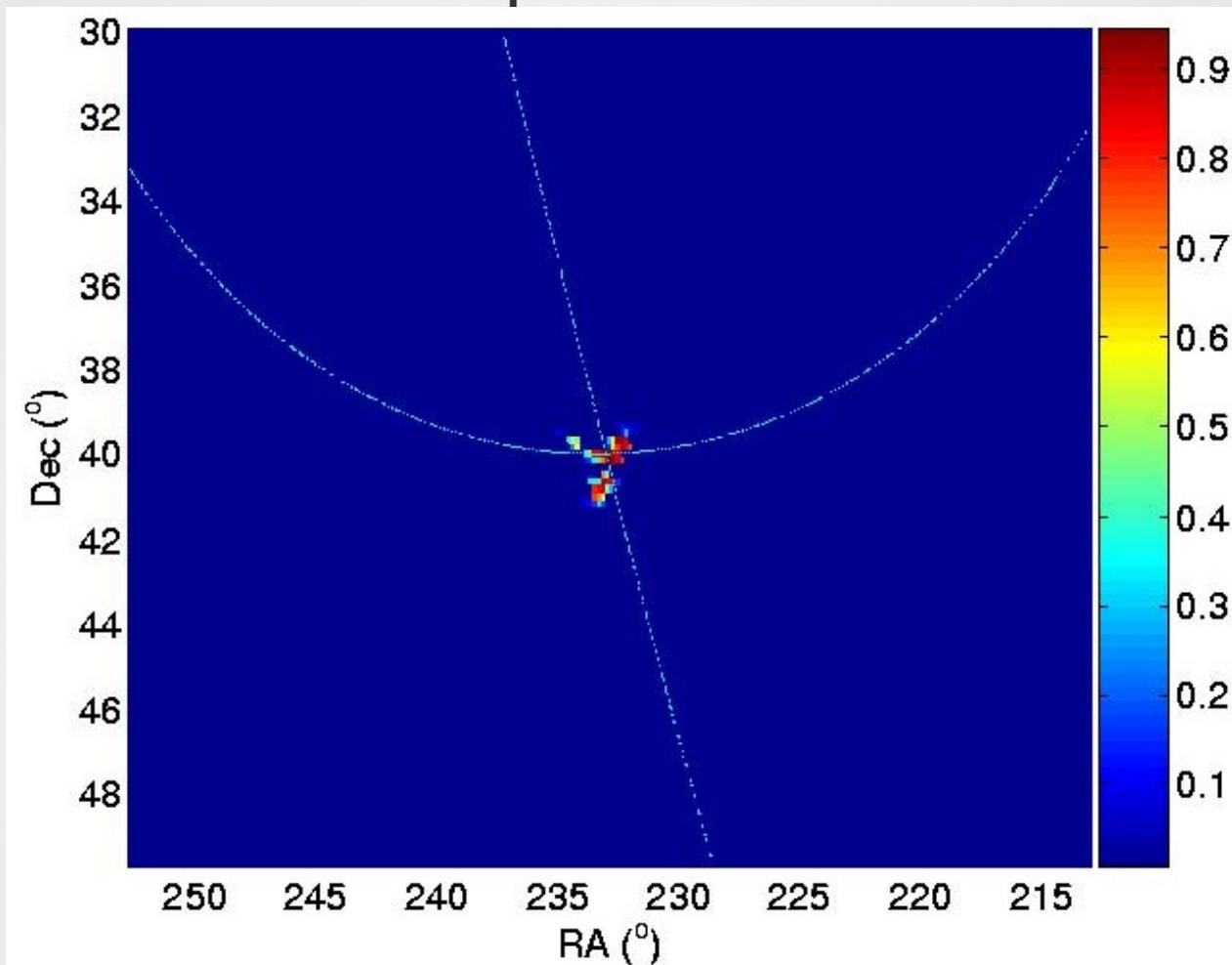
Source Localization using a network of 2 detectors



BBH ($10 M_{\odot}$) ringdown at 1 Mpc injected towards the minimum of L1 sensitivity

Waveforms obtained from Lazarus's numerical simulations. Duration about 7 ms
central frequency 500 Hz. Optimum 3 detector SNR is 85.

Adding GEO-600 to the picture



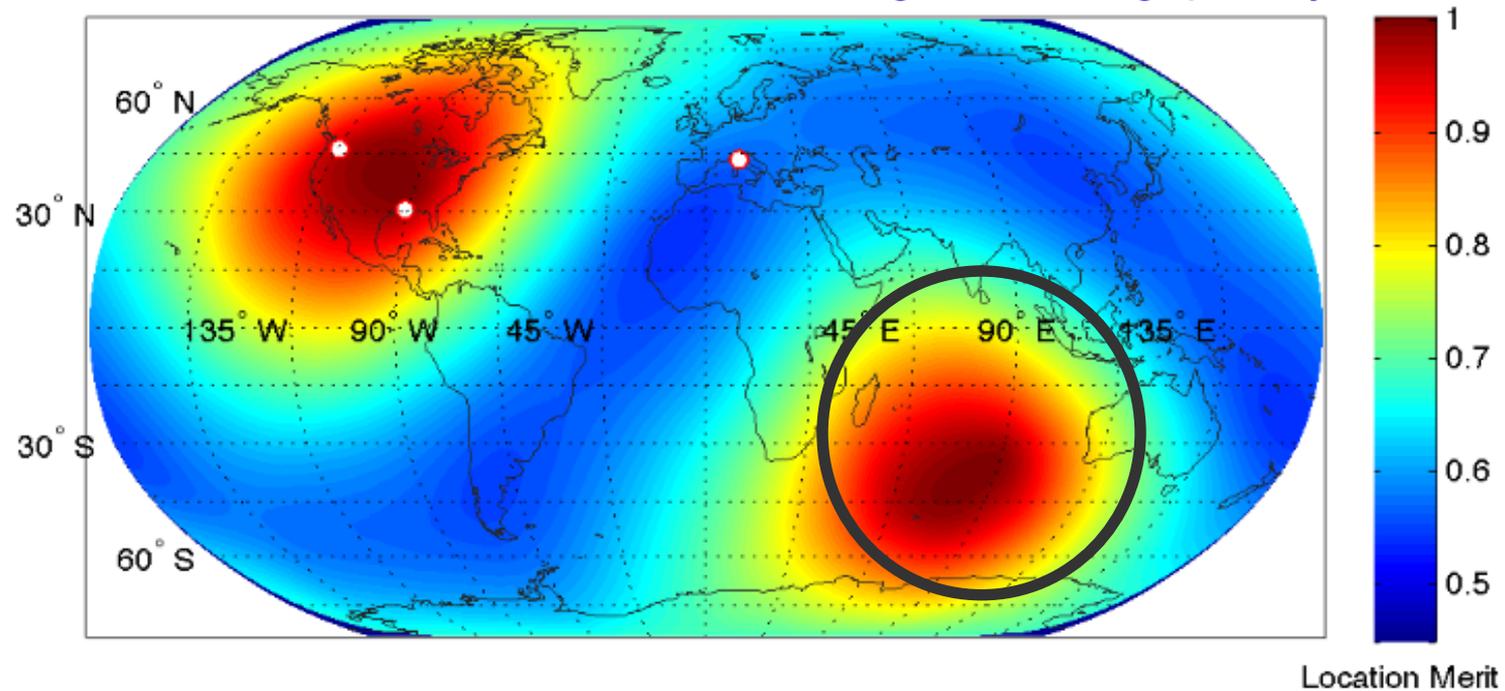
Despite its smaller size, GEO 600 is designed to have similar sensitivity as the LIGO instruments at higher frequencies. Implications for target of opportunity searches

Optimum location of a new detector

- Given an existing network of LIGO-Virgo detectors,
 - » where should one put an additional detector ?
 - » What should be its orientation angle ?
- Define a figure of merit
 - » How would this new detector augment the 'coherent' detection of binary inspiral signals ? *Searle, Scott, McLelland and Finn*
 - » In other words, coherent volume *Sengupta, Maghami, Shreshtha and M*
 - » and 'coincident' signals ?
 - » Usual simplifications
 - Uniform distribution of such sources
 - Ignore differences in sensitivity

Improvement to the coherent detection

Maghami, Sengupta, Ajith, Mitra 2010



- 6% difference between best and worst sites (70% for coincident)
- varying orientation changes detection by 2% (9% for coincident)
- fifth detector has almost no effect in coherent search but 25% fewer for coincident search