State of art in Gravitational Wave Source modelling

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Motivation

- Prior knowledge always helps very much to dig out signal inside noise.
- The more accurate the waveforms are the better is the detection & parameter estimation.
- Inspiralling compact binaries are the most promising sources of Gravitational Waves (GWs) due to its prior modellability within the frame work of General Relativity (GR)

The objective of this talk is to review the progresses made in modelling inspiralling compact binary and discuss the outstanding problems and active researches at present.

Modelling coalescing compact binaries: Important issues

IMR

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Coalescence = inspiral [PN methods] + merger[Numerical Relativity] + ringdown[BH perturbation theory]

Two important aspects:

- Compute the waveforms with as much accuracy as possible (to the maximum order possible if one is using Perturbative methods and improving the numerical accuracy in numerical relativity.
- Include all possible physical effects into the waveform model (e.g: Spins, eccentricity)

Inspiral Waveforms using post-Newtonian approximation

PN modelling of nonspinning binaries on circular orbits: Phasing formula

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\psi(f)},$$

$$\begin{aligned}
\alpha_0 &= 1, \text{ Peters\&Mathews, 1963} \\
\alpha_1 &= 0, \\
\alpha_2 &= \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\eta\right), \text{ Blanchet\&Schaefer, 1992} \\
\alpha_3 &= -16\pi, \text{ Blanchet\&Schaefer1993, Wiseman93} \\
\alpha_4 &= 10\left(\frac{3058673}{1016064} + \frac{5429}{1008}\eta + \frac{617}{144}\eta^2\right) \text{ (BDIWW, 1994)}, \\
\alpha_5 &= \pi\left(\frac{38645}{756} + \frac{38645}{252}\log\left(\frac{v}{v_{\rm lso}}\right) + \frac{5}{3}\eta\left[1 + 3\log\left(\frac{v}{v_{\rm lso}}\right)\right]\right) \text{ (Blanchet, 1996)}, \\
\alpha_6 &= \left(\frac{11583231236531}{4694215680} - \frac{640\,\pi^2}{3} - \frac{6848\,\gamma}{21}\right) \\
&+\eta\left(-\frac{15335597827}{3048192} + \frac{2255\,\pi^2}{12} - \frac{1760\,\theta}{3} + \frac{12320\,\lambda}{9}\right) \\
&+\frac{76055}{1728}\eta^2 - \frac{127825}{1296}\eta^3 - \frac{6848}{21}\log(4\,v) \text{ (BFIJ, 2002, BDEI, 2004)}, \\
\alpha_7 &= \pi\left(\frac{77096675}{254016} + \frac{1014115}{3024}\eta - \frac{36865}{378}\eta^2\right) \text{ (BFIJ, 2002).}
\end{aligned}$$

Amplitudes in the PN waveform

Not just the phasing, amplitude also should be modelled as accurately as possible [Sintes Vecchio, 2000, Van den Broeck & Sengupta, 2006]

- Computation of higher order polarizations done up to 3PN [Arun, Blanchet, Faye, Iyer, Qusailah, Sinha, Will, Wiseman... (1996-2010)]

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- Design studies show that they are very important in data analysis, especially for *parameter estimation*. It can bring dramatic improvements to the source localization capability & distance measurement accuracy $\Rightarrow$ improved cosmography. [Van den Broeck& Sengupta, 2006;Arun, Iyer, Mishra, Sathyaprakash, Sinha& Van Den Broeck 2007, 2008; Trias & Sintes, 2007; Porter & Cornish, 2008]

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Incorporating spins in the inspiral waveforms

A considerable chunk of sources Advanced detectors might see can have non-negligible spins. Its very important to account for the spin and spin-induced effects such as precession!

- More difficult than modelling nonspinning systems
- Spin effects are higher PN effects, spin-orbit coupling starts to appear from 1.5PN onwards and spin-spin effects from 2PN.
- Phasing is at present available to 2.5PN order & Amplitude is computed to 2PN [Cutler, Apostalatos, Kidder, Will, Wiseman, Blanchet, Faye, Buonanno, Arun, Ochsner (1993-2008)]

Waveforms for elliptical orbit binaries

There may be astrophysical effects which may prevent the circularization of the binary via GW radiation reaction!!

- There may be significant lose in SNR if effect of eccentricity is not accounted for in the waveform [Martel&Poisson, 1999, Tessmer & Gopakumar 2009, Brown & Zimmerman 2009.]
- Phasing for elliptical orbit binaries are available to 3PN order [Peters, Mathews, Blanchet, Schaefer, Gopakumar, Iyer, Damour, Arun, Qusailah, Sinha.(1963-2008)]
- PN terms in amplitude are computed up to 2PN [Gopakumar & Iyer, 2002]
- 3PN order polarizations [Mishra, Arun, Iyer, In progress]

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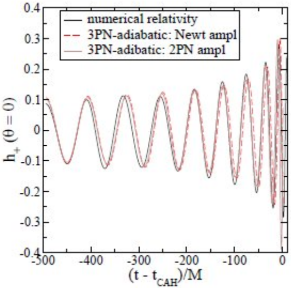
Merger waveforms & Numerical Relativity

Numerical Relativity

- Numerically solve Einstein's equations for a 2 body problem
- Very complicated problem, took many years' effort different groups around the globe to come up with a solution
- First result came from Pretorius in 2005. [Pretorius 2005]
- This was soon independently confirmed by many other groups [Baker et al, 2006, Campanelli etal 2006, Gonzalez et al 2007]
- Many papers on various aspects of the problem and results for various types of sources (spinning, eccentric..) followed

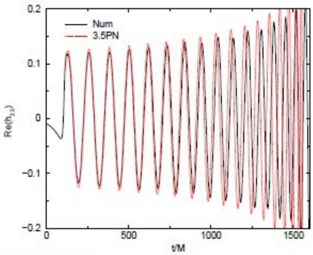
Numerical relativity confronts post-Newtonian theory

Circular Nonspinning binaries



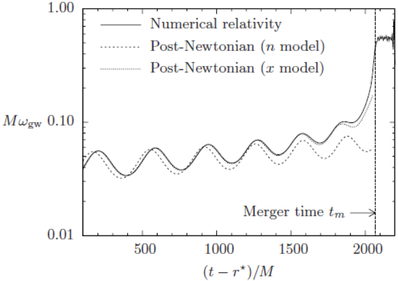
[Buonanno et al, 2005]

Circular orbit, spinning binaries:



[Campanelli et al, 2008]

Elliptic orbit binaries: Hinder et al 2007:



"Kick" of the binary from Numerical Relativity

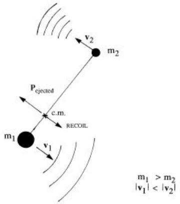


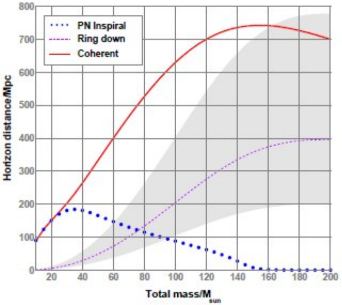
FIG. 3. Momentum ejection and subsequent recoil of a binary system.

[Wiseman, 1993]

- GWs also carry linear momentum with them ⇒ Recoil of the newly formed BH.
- The kick velocity can be of the order of 1000 km/sec and may have astrophysical implications.
- Many NR groups have computed the kick velocity from binary BH merger for various configurations and they tend to agree well with the expected contribution from PN regime of their evolution [Blanchet et al, Damour & Gopakumar· · · .]

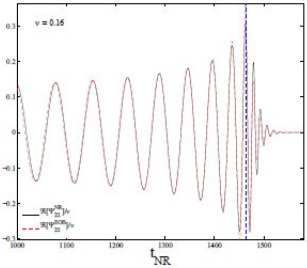
Analytical Modelling of NR waveforms

Phenomenological Modelling of NR waveforms [Ajith et al, 2007, 2008, 2009, 2010]



Effective One body approach

[Buonanno, Damour, Nagar et al]



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- Field theory approach to PN approximation.

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  - **Rajesh Nayak (IISER, Kolkata)**
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- Experiments
  - **P Unnikrishnan, TIFR Mumbai**

Interested students are welcome to apply for summer projects, depending on their areas of interest.