Gravitational waves: Sources and the Astrophysics

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Life of a star!

- Stellar equilibrium: Balance between inward pull of gravity and outward radiation pressure.
- Nuclear burn out:As the star burns out all its nuclear fuel, it collapses under gravity as there is nothing to balance.
- Ultimate fate:The final 'fate' of the star will depend on what is *the mass of the core* of the collapsing star

Compact objects: White dwarfs, Neutron Stars and Black holes

White dwarf

White dwarf is the state where gravity of the collapsing core is balanced by **electron degeneracy pressure**. The maximum mass which can be balanced by electron degeneracy pressure is $\sim 1 M_\odot \Rightarrow$ **Chandrasekhar limit**.
Mass $\sim 1.4 M_\odot$, radius $\sim$ Earth's radius.

Neutron Stars

If the mass is more, the collapse ensues and at some stage the density becomes so much that the neutron degeneracy pressure balances the gravity $\Rightarrow$ Neutron stars.
Max mass of the core $\sim 3 M_\odot$ (depends on various factors).
Mass $\sim 1.5 M_\odot$, Radius$\sim$ 10kms, High magnetic field 10^{12} Gauss, Rapidly rotating ($\sim$1-100s rotation per second!!)

Black holes

If the mass of the core is so high that even neutron degeneracy pressure cannot balance gravity, the collapse continues and a Black hole is born!

Distance scales in astronomy

Distances

- Light Year = The distance light travels in a year= $365 \times 24 \times 3600 \times 3 \times 10^8$ meters$\sim 10^{16}$ meteres
- 1 parsec (pc)= 3.26 light years ($\sim 10^{16}$ meters).
- 1 kilo parsec (kpc)=10^3 pc
- 1 mega parsec (Mpc) = 10^6 pc
- 1 Gpc = 10^9 pc

To get a feel

- Distance to moon $\sim$ 1.3 light seconds.
- Distance to sun $\sim$ 6 light minutes.
- Distance to the centre of galaxy $\sim$ 10 kpc.
- Distance to our neighbouring galaxy (Andromeda) $\sim$ 750 kpc.
- Max distance up to which initial GW detectors can see: $\simeq$ 100 Mpc
- Max distance up to which second generation GW detectors can see: $\sim$ Gpc

'Strength' of the GW signal

Strength of the GW can be measured in terms of a dimensionless strain $h = \frac{\Delta L}{L}$, where L is the original separation between two test masses and ΔL its change due to the passing of the GW.

Waveform

$$h \sim \frac{1}{r}\frac{d^2Q}{dt^2} \Rightarrow \frac{M}{r}v^2_{\rm nonspherical}$$

This implies that the strength of the GWs would depend on the time dependence of the deviation from spherical symmetry, of the mass distribution of the source.

The more is the asymmetry, the stronger the GW emission.

Hence an astrophysical phenomenon which emits enormous amount of electro-magnetic energy can be relatively silent in the GW band, if the Quadrupole moment of the source is small.

Outline: An overview of GW sources and astrophysics

- Classification of GW signals, frequency ranges.
- Astrophysical origins of various types of sources.
- What do we hope to learn by observing GW signals
- What are the typical strengths of the GW signals for each type of sources.
- How many events of this sort do we hope to detect?

Differentiating the sources based on the signal duration

Duration of the signals is a good way to characterize different types of GW sources. Here I have used the expected duration of GW signals in the *ground based detector bandwidth* as a way to characterize them.

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2. Inspiral and merger of compact binaries: a few seconds.
3. Monochromatic GW signals: Long duration ($\sim$ years), typically as long as the detector can take data.
4. Stochastic background of GWs.

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Another way to characterize the sources would be based on the frequencies of the GWs: Low frequency GWs (< 1 Hz) which can be observed with space-based GW missions as opposed to high frequency ($\geq$ 1Hz) sources detectable by the ground-based detectors.

We will stick to the classification scheme based on signal duration and will discuss the low and high frequency sources in each classes.

GW burst sources

GW Burst sources: Collapse of the cores of massive stars

Core collapse of Massive stars

- Stars are held in equilibrium by the subtle balance of the gravitational pull of the core with the radiation pressure of due to thermo-nuclear reactions.
- When the star exhausts its fuel, there is nothing to balance gravity and the core of the star collapses under gravity.
- The sudden release of gravitational potential energy will heat up the outer layers and throw them out.
- During this short interval (a few weeks), the energy thats emitted can be as high as that emitted by sun in its entire life time!

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Observed as

Believed to be the mechanism responsible for supernova(SN) explosions & (long duration) Gamma Ray Bursts (GRBs), which are the most energetic explosions since big bang!

How strong is GW emission from core collapse of massive stars?

- If the collapse is **asymmetrical (non-spherical)** , there can be significant GW emission associated with them.
- Recent numerical simulations have shown that in a typical supernova, GWs could extract $10^{-5} - 10^{-7}$ of the the total mass-energy.
- Typical frequencies might be in the range $200 - 1000\text{Hz}$.

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$$h \sim 6 \times 10^{-21} \left(\frac{E}{10^{-7} M_\odot}\right)^{1/2} \left(\frac{1 \text{ ms}}{T}\right)^{1/2} \left(\frac{1 \text{ kHz}}{f}\right) \left(\frac{10 \text{ kpc}}{r}\right),$$

- Typical distance up to which such events can be observed is roughly of order of a few 10s of kilo parsecs (thats roughly the distance to the centre of our galaxy).
- Event rates could be as low as 1 per century!

What can we learn from GW observations?

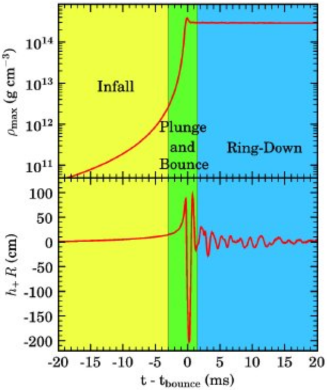

Figure: Typical Gravitational Waveform from Supernova explosion

[Dimmenmeir et al, 2007]

- EM observations cannot probe deep into the core which GWs can.

* Gravitational Waveforms are functions of mass of the core, angular momentum of the core& nuclear equation of state (EOS).

- GW observations can hence constrain rotation of the core, details of the nuclear physics (EOS) at the core.
- Possibility of detecting electromagnetic, neutrino and gravitational wave emissions from the same source!!!

Summary

- There can be GW emissions associated with the collapse of the massive stellar cores, the strength of GW signals will depend on the asymmetry of the collapse.
- These are observed electromagnetically (and in the neutrino) as supernova explosions and long duration Gamma Ray Bursts.
- GW observations can provide us useful insights on the various aspects of the core collapse like the rotation of the core, nuclear equation of state etc.
- We may be able to observe this type of events in and around our galaxy.

Continuous GW signals from Pulsars

Monochromatic Waves from Pulsars

Pulsars

* Neutron stars (NS) (which are observed as pulsars) are stellar objects which are the outcome of core collapse of massive stars.
* In a NS, the gravity of the core is balanced by neutron degeneracy pressure (arising from Pauli's exclusion principle).
* They emit highly regular pulses (hence the name Pulsars).
* They are highly spherically symmetric and magnetized objects.

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In order to emit GWs, there should be some mechanisms which induce a quadrupole moment.

Modelled as Bumps on NS surface

Let a NS have a mass M and radius R. Assuming uniform density, its moment of inertia is $2/5 M R^2$. Let there be a small bump of mass m on its surface. Its moment of inertia is $\sim mR^2 \Rightarrow$ fractional asymmetry $\epsilon = \frac{m}{M} \Rightarrow$ Time varying quadrupole moment as the NS rotates.

Typical signal strengths

[Sathyaprakash & Schutz, 2009.]

Assuming NS has a rotational frequency of f, the non-spherical velocity $v_{\rm nonspherical} = 2\pi\, f\, R$. Hence typical order of magnitude of signal strength can be estimated to be

$$h \sim Mass \times \frac{1}{r} v^2_{\rm nonspherical} = \frac{4}{5}(2\pi\, m\, f\, R)^2 \frac{\epsilon\, M}{r}$$

And the gravitational wave luminosity is given by

$$L_{\rm GW} \sim \frac{16}{125}(2\pi f)^6\, \epsilon^2 M^2\, R^4$$

This energy should come from the rotational energy of the NS, given by $M\, v^2/5$. Hence due to the emission of GWs, the NS would *spin down* in a time scale

$$t_{\rm spin-down} \sim \frac{1/5 M v^2}{L_{\rm GW}} \sim \epsilon^{-2} f^{-1} \left(\frac{M}{R}\right)^{-1} v^{-3}$$

Estimating spin down of Pulsars due to GW emission

If from EM observations we know M/R, v & f, spin down rate and from GW observations we know $L_{\rm GW}$, one can estimate the asymmetry parameter ϵ.(Assuming the spin down is solely due to GW emission).

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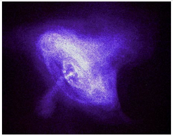
Example: The Crab Pulsar 

- Crab Pulsar @ 2 kpc, $(M/R) \sim 0.2$, $v/c \sim 6.2 \times 10^{-3}$, $t_{\rm spin-down} = 2500 {\rm years}$ [All from EM observations] $\Rightarrow \epsilon \sim 7 \times 10^{-4}$.
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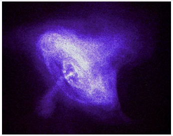

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GW upper limits

GW observations can set limits on the asymmetry parameter ϵ of Crab Pulsar. $\epsilon \leq 1.8 \times 10^{-4}$ to be consistent with non detection. [Abbott et al, Astrophys.J.683:L45]

Astrophysical Processes of GW emission from pulsars

Instabilities in NSs

Instabilities in NSs: various modes of deformations (r-mode, f-mode etc) $\Rightarrow$ helps one to undertand the internal composition of the NS.

Low mass X ray binaries

Narrow range of rotational periods of LMXBs may be due to the accretion torque being balanced by angular momentum loss from GW emission [Bildsten, 1998]

Pulsar Glitches

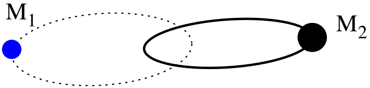
Star quakes in NSs can give rise to GW emission

Summary

- There can be GW emission from pulsars due to asymmetries in the mass distribution. They are monochromatic waves at twice the rotational frequency of the pulsar.
- Combining EM and GW observations, one can derive interesting upper limits, even in the absence of direct detection.

Inspiralling compact binaries

Inspiralling Compact binaries



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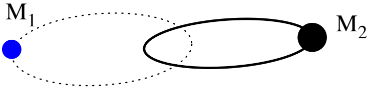

- Binary system consisting of Neutron stars (NS) and/or Black holes (BHs) have a time varying quadrupole moment which leads to gravitational radiation.
- GWs take away energy and angular momentum from the source, which results in the decay of the binary's orbit and gives rise to observable signatures even electromagnetically (See pic). Finally they merge to form a single BH.
- Especially towards the late stages of this process (called inspiral), these binaries emit GWs which are strong enough to be detected on earth.

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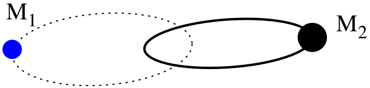

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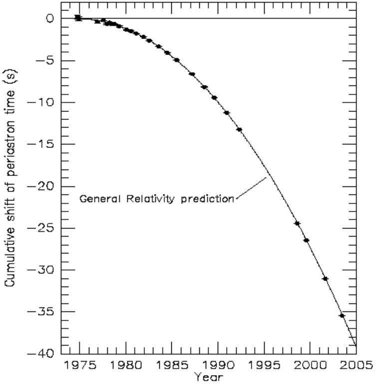


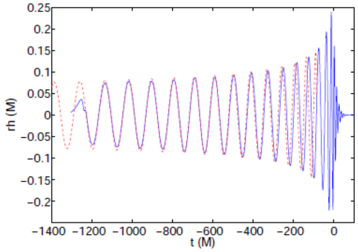
Indirect evidence for the existence of GWs: Orbital decay of Hulse-Taylor pulsar matches fantastically with GR predictions

GWs from inspiralling compact binaries

- Inspiralling compact binaries are the most promising sources of GWs for ground-based as well as space based detectors.
- The are three phases of the binary evolution: inspiral, merger and ringdown.
- All three of these phases can be very accurately modelled using analytical or numerical approaches to GR.
- This prior knowledge of the waveform enables the data analyst to employ 'matched filtering' technique to detect the signal.

Challenge to the theorist: Accurately model the source to improve chances of detection (and parameter estimation).

Baker et al, 2006



Three phases

- Inspiral: Post-Newtonian approximation to GR.
- Merger: Numerical Relativity simulations.
- Ringdown: BH perturbation theory.

Typical frequencies

Last Stable Orbit

Formally inspiral phase of the binary's evolution is assumed to stop when the binary reaches the last stable orbit (LSO) when the orbital frequency is given by $f = \frac{1}{2\pi m 6^{3/2}}$.

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BH mergers and strength of the signal (initial interferometers)

$$h_{\rm eff} = 3\times 10^{-21}\left(\frac{30\ \mathrm{Mpc}}{r}\right)\left(\frac{\eta}{0.25}\right)^{1/2}\left(\frac{M}{2.8 M_{\odot}}\right)^{5/6}\left(\frac{1\ \mathrm{kHz}}{f}\right)^{1/6},$$

$$h_{\rm eff} = 4\times 10^{-17}\left(\frac{1\ \mathrm{Gpc}}{r}\right)\left(\frac{\eta}{0.25}\right)^{1/2}\left(\frac{M}{10^6 M_{\odot}}\right)^{5/6}\left(\frac{1\ \mathrm{mHz}}{f}\right)^{1/6}.$$

Astrophysics, Cosmology & Fundamental physics from Inspiralling compact binaries: A demo

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Electromagnetic Observations

- Suppose *S*WIFT observes a ShGRB and follows it up with an optical counterpart.
- It gives information about the location of the source in the sky, and afterglow (or in the absence of it a possible coincidence with a known galaxy) $\Rightarrow$ redshift information.
- Difficult to unambiguously pin point the progenitor.

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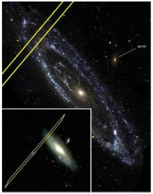
AdvLIGO-Virgo-AIGO observes it

- Estimation of the masses of the compact binary components $\Rightarrow$ Progenitor.
- Equation of state of the NSs may have imprints on the binary waveform $\Rightarrow$ insights about the core of NS.
- Measurement of Luminosity distance (but no redshift) $\Rightarrow$ Determination of Hubble's constant.
- Enables tests of General Relativity & Alternative theories of gravity

LIGO upper limit on GRB070201



[Abbott et al, 2007]

- GRB070201 occurred during the 5th Science run of LIGO
- EM observations showed that the location coincided with the spiral arms of our neighbouring galaxy Andromeda.
- LIGO search excluded this event to be due to a coalescing binary in Andromeda with components $1M_\odot \leq M_1 \leq 3M_\odot$ & $1M_\odot \leq M_2 \leq 40M_\odot$ at 99% confidence.
- Also binary NS scenario is ruled out up to a distance of 3.5Mpc, which is farther than distance to Andromeda (which is roughly 0.8Mpc.)

[Abbott et al, ApJ, 2007.]

Gravitational Wave Stochastic background

Gravitational wave stochastic background

Cosmological scenarios suggest we are bathed in a stochastic background of GWs similar to Cosmic Microwave Background in the EM spectrum. These are random superpositions of GW signals which are either not resolved and/or arise from some of the fundamental processes in the early universe (such as inflation).

GW stochastic background=Astrophysical background + Primordial background.

Astrophysical

- Astrophysical stochastic background contribution is due to the enormous number of sources whose signals reach the detectors, but they are so frequent and signal to noise ratio is low that detector cannot resolve the individual signals
- Carries the imprints of distribution of various sources in the universe.

Primordial

- Primordial stochastic background signal arise from some of the fundamental processes which occured in the very early phase of the universe soon after the big bang: e.g. Inflation.
- Unique way to probe the universe when it was 10^{-30} seconds old!!!

Concluding Remarks

- We did a quick overview of various types of GW sources. and the variety of astrophysical, cosmological and fundamental physics insights the GW observations would provide.

References

- *Physics, Astrophysics & Cosmology with Gravitational Waves*, B S Sathyaprakash& Bernard Schutz, Living Reviews in Relativity, 12 (2009), 2.
- For more references see "Resources" page of `gw-indigo.org` website.