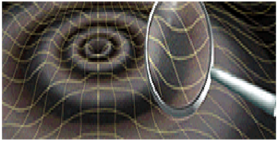

Multi-detector GWaves networking

Archana Pai
Indian Institute of Science Education and Research
Trivandrum

ISGWA2010, 22 December 2010

Interferometer antenna pattern

Geodesic deviation equation in linearised gravity

$$\frac{\partial^2 \delta L^i}{\partial t^2} = -\frac{1}{2}\omega^2 L^k h^i_k \Longrightarrow \delta L^i = \frac{1}{2} L^k h^i_k$$

$L^x = L \cos \delta \quad L^y = L \sin \delta \quad h^i_k = h_+ \rightarrow \delta L^x, \delta L^y$?

$h^i_k = h_\times \rightarrow \delta L^x, \delta L^y$?

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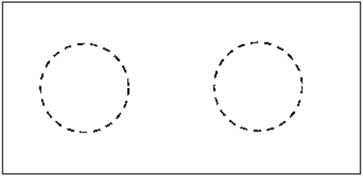
GWaves incident on the ring of particles

GWave Polarisation: h_+ $\quad h_\times \quad \delta L \sim hL/2$



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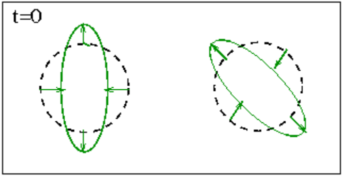

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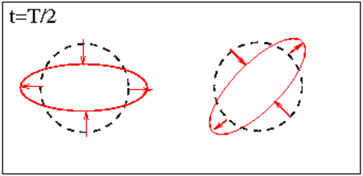

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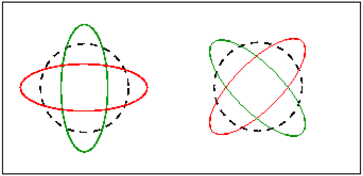
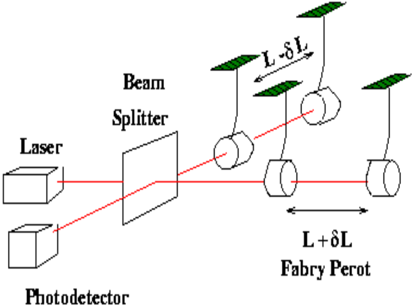


Interferometer antenna pattern



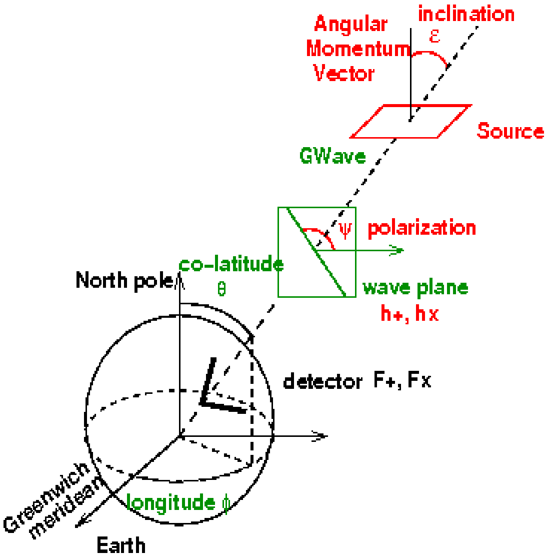
Detector – (X', Y', Z')

$$L'_x = \{L, 0, 0\}$$
$$L'_y = \{0, L, 0\}$$

Detector response

$$h \equiv \frac{\delta L'_x - \delta L'_y}{L} = \frac{h'_{xx} - h'_{yy}}{2}$$

Interferometer antenna pattern



Detector response

$$h \equiv \frac{\delta L'_x - \delta L'_y}{L} = \frac{h'_{xx} - h'_{yy}}{2}$$

GWave frame:

$(X, Y, Z) \quad h_+, h_\times$

Rotation transformation:

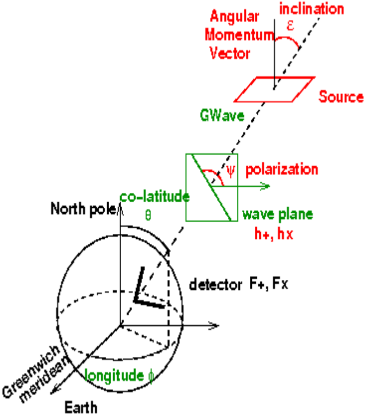
$(X, Y, Z) \rightarrow (X', Y', Z')$

$$\mathcal{T} = \mathcal{R}(\phi)\mathcal{R}(\theta)\mathcal{R}(\psi)$$

GWave in detector frame:

$$h'_{ij} = \mathcal{T}_{ik}\mathcal{T}_{jl}h_{kl}$$

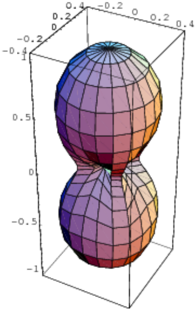
Interferometer antenna pattern



Detector response

$$h \equiv \frac{\delta L'_x - \delta L'_y}{L} = \frac{h'_{xx} - h'_{yy}}{2}$$

$$h = F_+(\phi,\theta,\psi)\; h_+ + F_\times(\phi,\theta,\psi)\; h_\times$$



Antenna Pattern $\psi = 0$

$$F_+^2 + F_\times^2$$

$$F_+ = \frac{1+\cos^2\theta}{2}\cos 2\phi$$

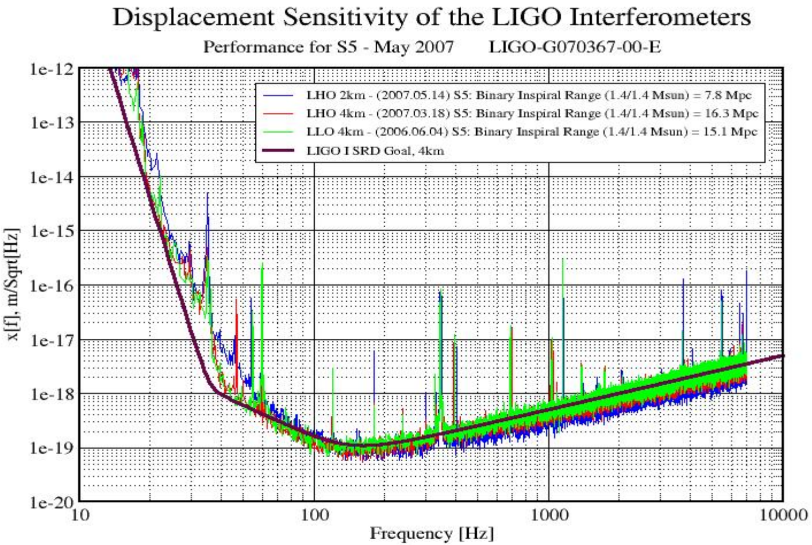
$$F_\times = cos\theta\; sin2\phi$$

Terrestrial Interferometric Detectors (10 Hz – few kHz)

Long baseline LIGO-Virgo interferometers have taken the first joint data at the designed sensitivity of $h \sim 10^{-22}$ in 2007

Back

Terrestrial Interferometric Detectors (10 Hz – few kHz)



Astrophysical Sources of GWaves

Dynamical frequency: $f_{\rm dyn} \sim \sqrt{G\rho}$

$$f_{\rm dyn} = 1\,{\rm mHz} \left[\frac{M}{2.8M_\odot}\right]^{1/2} \left[\frac{R}{2\times10^8 m}\right]^{-3/2}$$

Black hole:– $R = 2GM/c^2$

$$\rho \sim 2\times10^6 \left[\frac{f}{3\,{\rm mHz}}\right]^2 {\rm kg/m^3}$$

Terrestrial detectors

Frequency: $10Hz - few\ kHz$

Sources with high ρ

Compact, dense systems

Space based mission

$0.1mHz - few\ Hz$

Sources with wide range of ρ

Astrophysical Sources of GWaves

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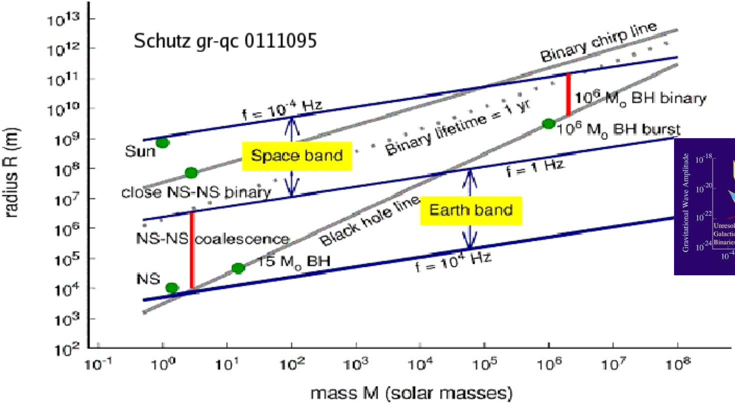
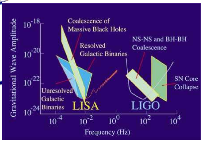
Sources with high ρ Sources with wide range of ρ





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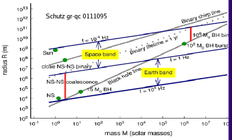
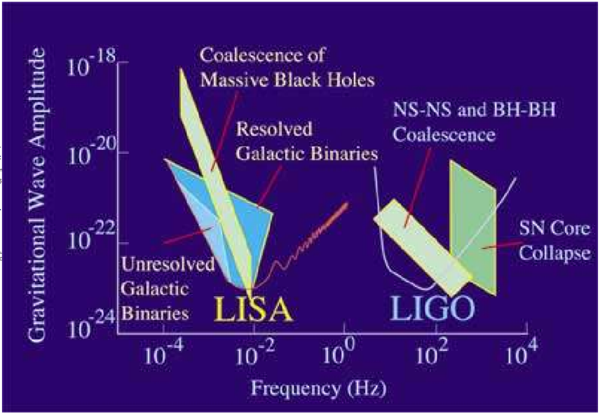
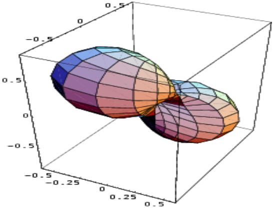


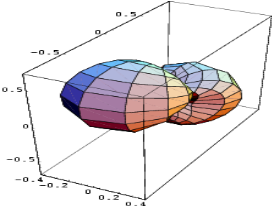


Advantages of Multi-detector over Single detector
1. Better Sky Coverage

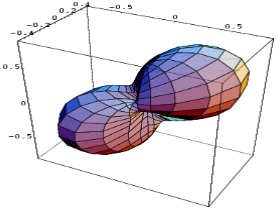
LIGO-H



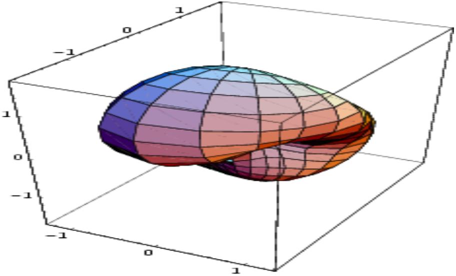
LIGO-L



VIRGO

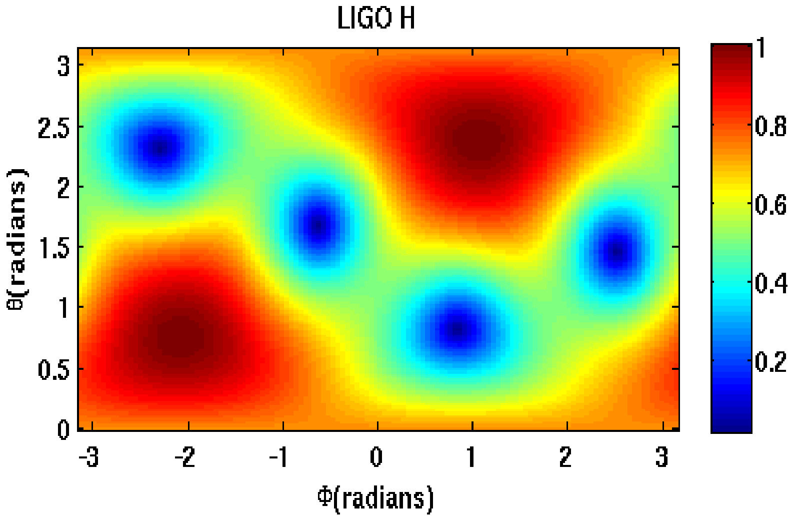


LIGO-VIRGO network has better sky-coverage



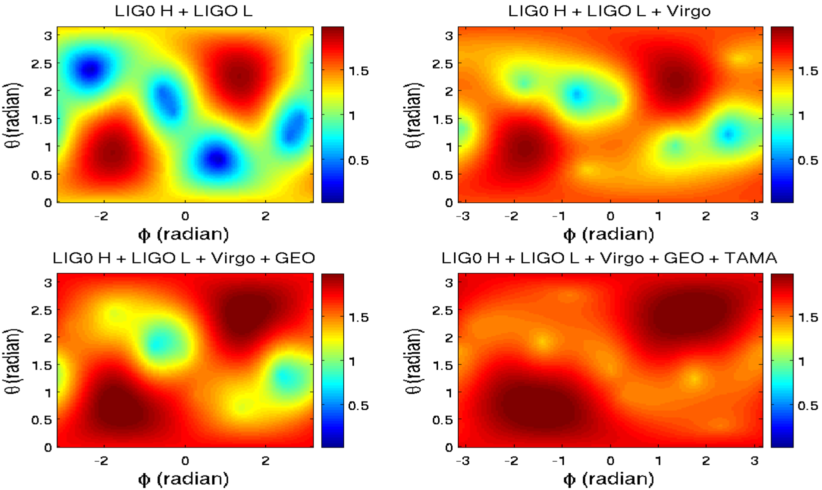
Advantages of Multi-detector over Single detector
1. Better Sky Coverage

Single Detector Antenna Pattern Sky Map — $F_+^2 + F_\times^2$



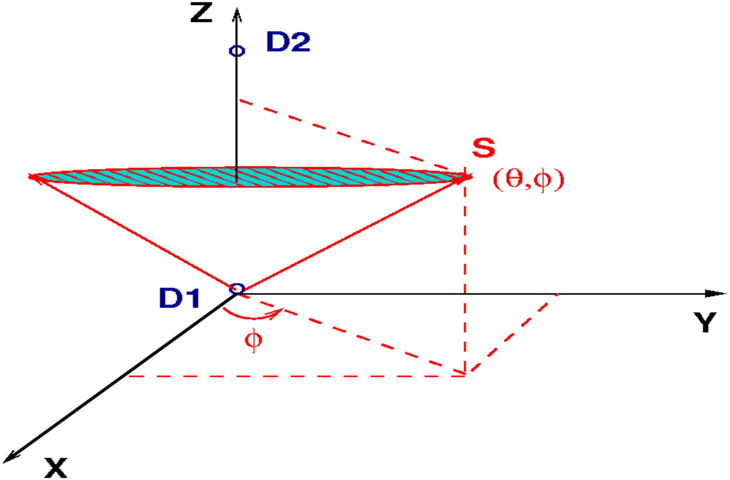
Advantages of Multi-detector over Single detector
1. Better Sky Coverage

Multi-detector Antenna Pattern Map: $\sum_{Det}(F_{+}^{2} + F_{\times}^{2})$



Advantages of Multi-detector over Single detector
2. Localisation of Source in the Sky

Two Detector Network



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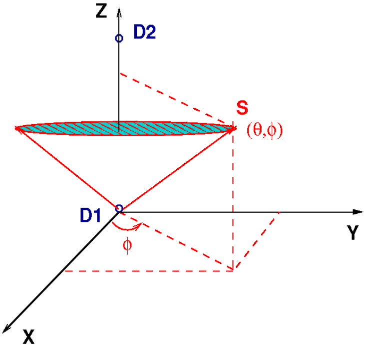


Light travel time between D1-D2 ;

$$T2 = L(D1 - D2)/c$$

GWaves arrival time delay between D1,D2

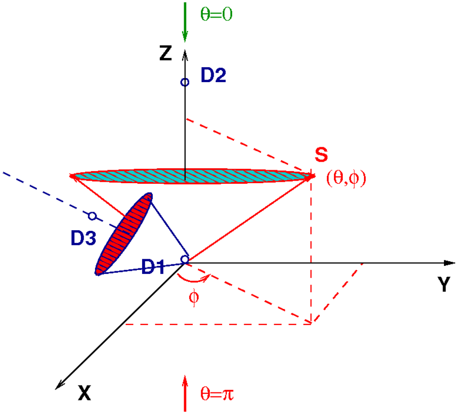
$$\tau_2 = T2\ cos(\theta)$$

$$-T2 \leq \tau_2 \leq T2$$

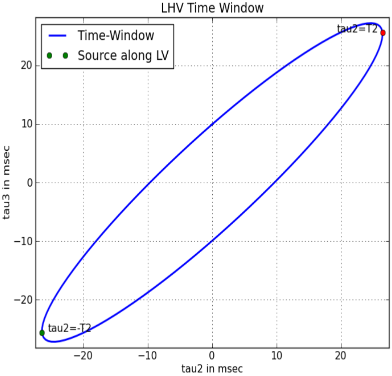
Determine θ, Degeneracy in ϕ.

Advantages of Multi-detector over Single detector
2. Localisation of Source in the Sky

Three Detector Network



Time window == Ellipse in (τ_2, τ_3)



Advantages of Multi-detector over Single detector
2. Localisation of Source in the Sky

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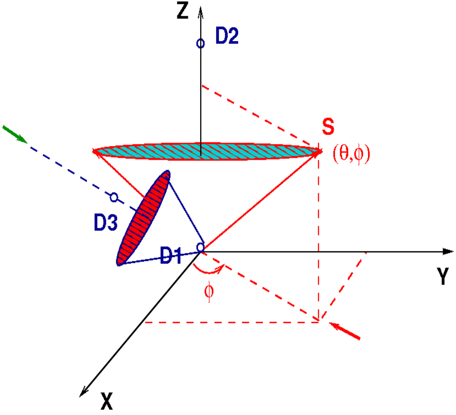

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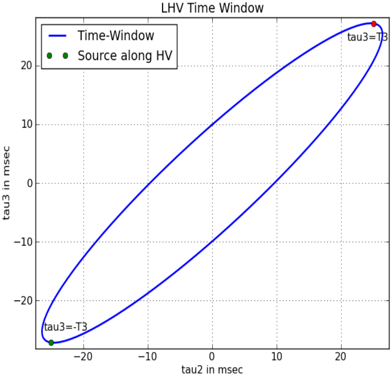

Advantages of Multi-detector over Single detector
2. Localisation of Source in the Sky

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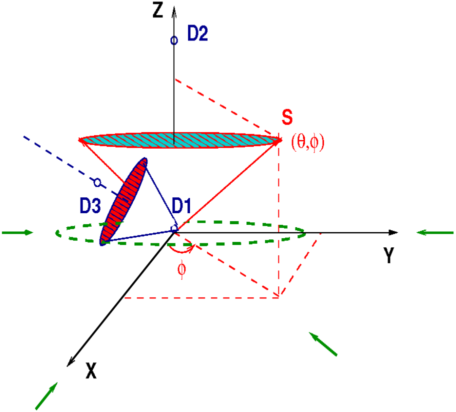

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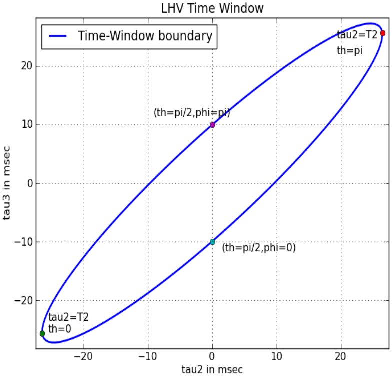

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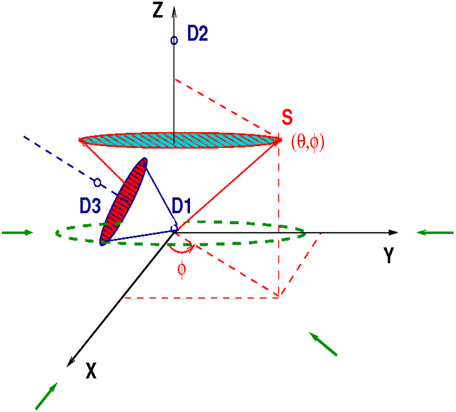

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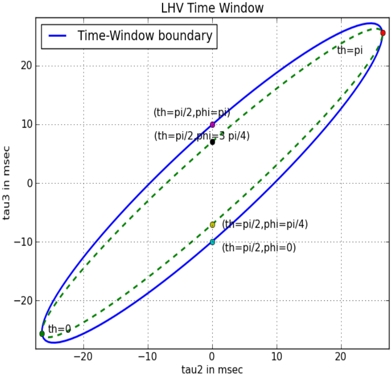

Advantages of Multi-detector over Single detector
2. Localisation of Source in the Sky

Three Detector Network

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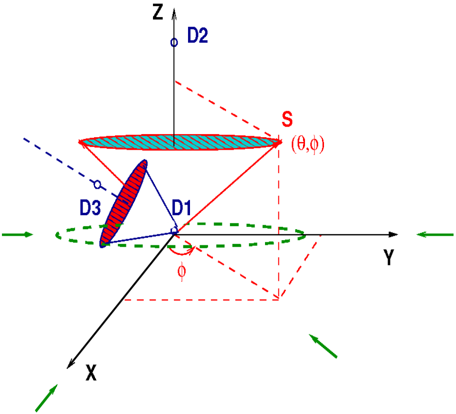
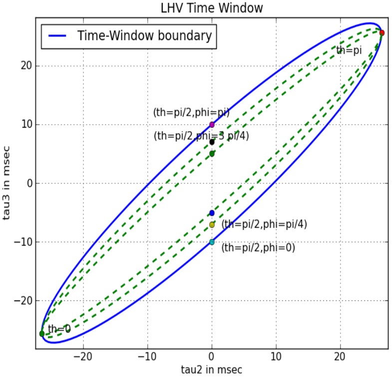
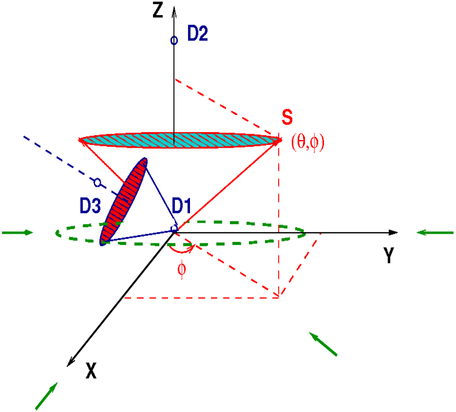




Advantages of Multi-detector over Single detector
2. Localisation of Source in the Sky

Three Detector Network



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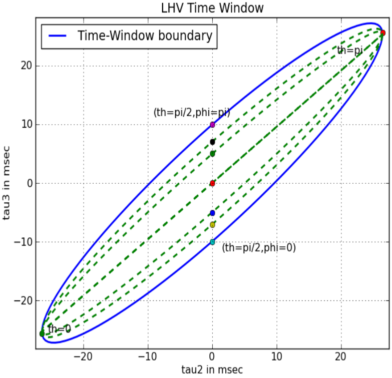

Advantages of Multi-detector over Single detector
2. Localisation of Source in the Sky

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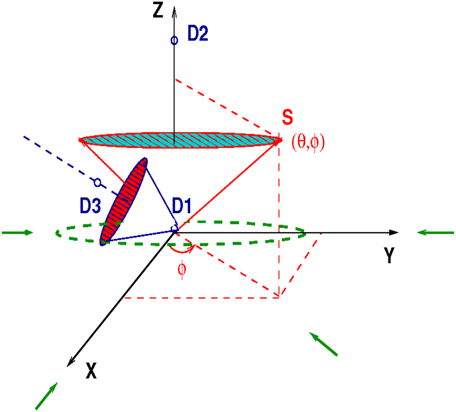
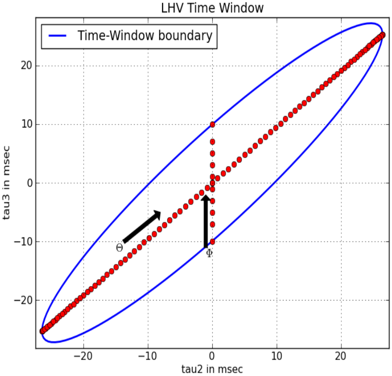


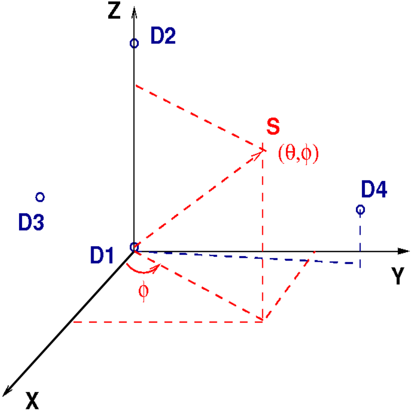
Time window == Ellipse in (τ_2, τ_3)



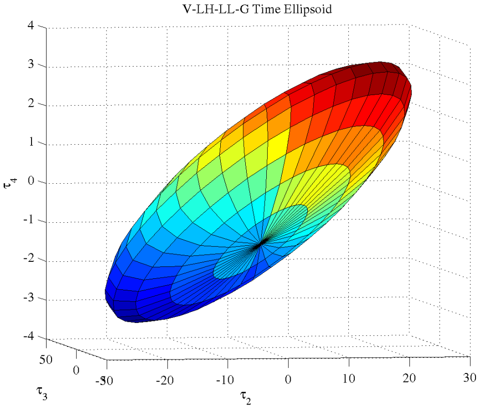
$(\theta, \pm\phi) \Rightarrow (\theta, \phi)$ – Degeneracy in ϕ
Source above and below D1-D2-D3

Advantages of Multi-detector over Single detector
2. Localisation of Source in the Sky

Four Detector Network



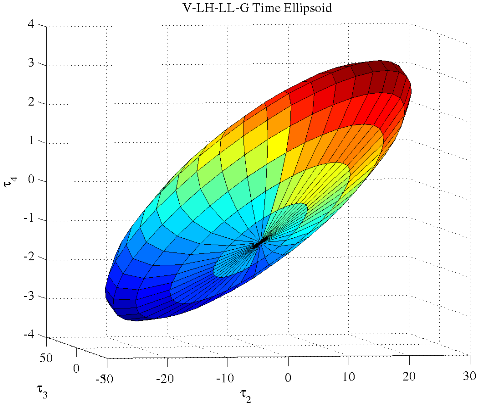
Time window: Ellipsoid in (τ_2, τ_3, τ_4)



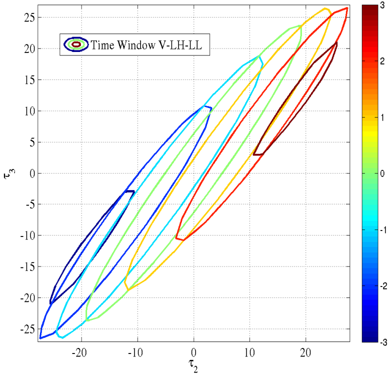
Advantages of Multi-detector over Single detector
2. Localisation of Source in the Sky

Four Detector Network

Time window: Ellipsoid in (τ_2, τ_3, τ_4)



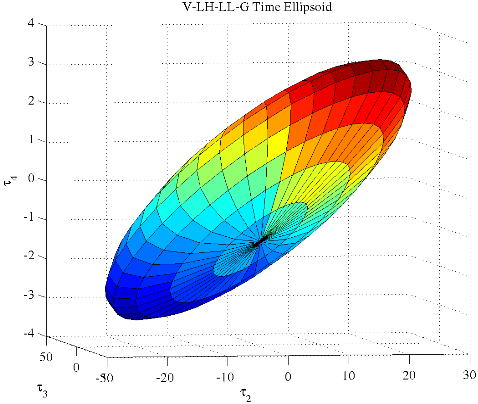
Projection on $\tau_2 - \tau_3$ plane:



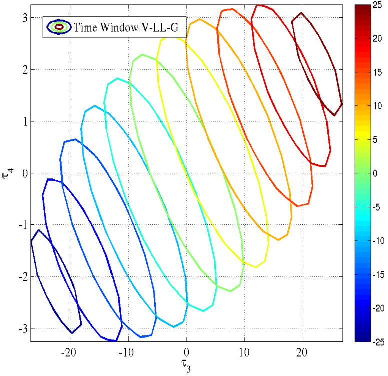
Advantages of Multi-detector over Single detector
2. Localisation of Source in the Sky

Four Detector Network

Time window: Ellipsoid in (τ_2, τ_3, τ_4)



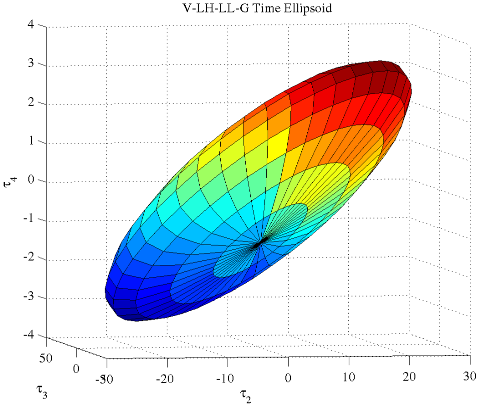
Projection on $\tau_3 - \tau_4$ plane:



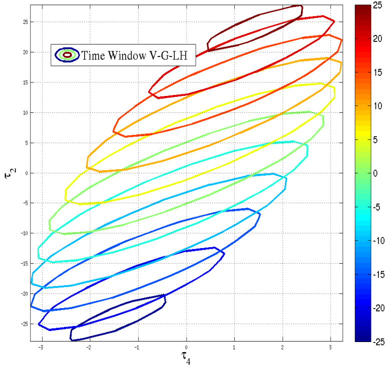
Advantages of Multi-detector over Single detector
2. Localisation of Source in the Sky

Four Detector Network

Time window: Ellipsoid in (τ_2, τ_3, τ_4)



Projection on $\tau_4 - \tau_2$ plane:



Advantages of Multi-detector over Single detector
2. Localisation of Source in the Sky

Four Detector Network

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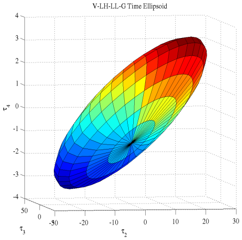
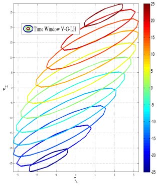


Projection on $\tau_4 - \tau_2$ plane:



Four detectors determine source location (θ, ϕ)

D detector network $(D \geq 4)$

Number of ellipsoids: ${}^{D-1}C_3$

Each time-delay triplet gives independent estimate of (θ, ϕ).

Advantages of Multi-detector over Single detector
3. Improved Signal-To-Noise Ratio

Matched Filter SNR

$$SNR^2 = \int \frac{|\tilde{h}(f)|^2}{S(f)}\, df$$

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$$SNR^2 = \int \frac{|\tilde{h}(f)|^2}{S(f)}\, df$$

Network of D GWaves Detectors

Det 1 : – $h_1(t) = F_{1+}(\theta, \phi, \psi)\ h_+(t) + F_{1\times}(\theta, \phi, \psi)\ h_\times(t)$
Det 2 : – $h_2(t) = F_{2+}(\theta, \phi, \psi)\ h_+(t) + F_{2\times}(\theta, \phi, \psi)\ h_\times(t)$
Det 3 : – $h_3(t) = F_{3+}(\theta, \phi, \psi)\ h_+(t) + F_{3\times}(\theta, \phi, \psi)\ h_\times(t)$
.
.
Det D : – $h_D(t) = F_{D+}(\theta, \phi, \psi)\ h_+(t) + F_{D\times}(\theta, \phi, \psi)\ h_\times(t)$

Advantages of Multi-detector over Single detector
3. Improved Signal-To-Noise Ratio

Matched Filter SNR

$$SNR^2 = \int \frac{|\tilde{h}(f)|^2}{S(f)} \, df$$

Network of D GWaves Detectors

$$\text{Det 1 :} - h_1(t) = F_{1+}(\theta,\phi,\psi)\; h_+(t) + F_{1\times}(\theta,\phi,\psi)\; h_\times(t)$$
$$\text{Det 2 :} - h_2(t) = F_{2+}(\theta,\phi,\psi)\; h_+(t) + F_{2\times}(\theta,\phi,\psi)\; h_\times(t)$$
$$\text{Det 3 :} - h_3(t) = F_{3+}(\theta,\phi,\psi)\; h_+(t) + F_{3\times}(\theta,\phi,\psi)\; h_\times(t)$$

.

.

$$\text{Det D :} - h_D(t) = F_{D+}(\theta,\phi,\psi)\; h_+(t) + F_{D\times}(\theta,\phi,\psi)\; h_\times(t)$$

Network Matched Filter SNR

$$SNR^2_{Net} = \sum_{i=1}^{D} SNR_i^2 = \sum_{i=1}^{D} \int \frac{|\tilde{h}_i(f)|^2}{S_i(f)} \, df$$

Advantages of Multi-detector over Single detector
3. Improved Signal-To-Noise Ratio

Network Matched Filter SNR

$$SNR^2_{Net} = \sum_{i=1}^{D} SNR_i^2 = \sum_{i=1}^{D} \int \frac{|\tilde{h}_i(f)|^2}{S_i(f)} \, df$$

- D Colocated detectors with identical noise: $SNR_{Net} = \sqrt{D} SNR$.
- D arbitrarily oriented detectors with identical noise: $SNR_{Net}(\theta, \phi, \psi) \propto \sum_i (F_{i+})^2 + (F_{i\times})^2$
- GWaves network probes deeper in the sky
  $\Rightarrow$ Probe deeper by $\sqrt{D}$ factor
  $\Rightarrow$ Increase events by $D^{3/2}$

Advantages of Multi-detector over Single detector
4. More knowledge about the source

Non-spinning Compact binaries with NS, BH :

$$r, \mathcal{M}, \epsilon, \psi, \theta, \phi, t_a, \delta$$

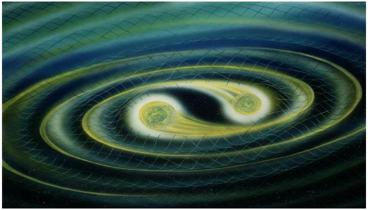
$h_+(t) = A(t)\,(1 + \cos^2 \epsilon)\,\cos(\phi(t))$

$h_\times(t) = 2\,A(t) \cos(\epsilon)\,\sin(\phi(t))$

Amplitude $A(t) = \mathcal{M}^{5/3} f^{2/3}/r$

Chirp mass $\mathcal{M} = [\mu^3 M^2]^{1/5}$

Frequency $f \propto \mathcal{M}^{-5/8}(t_{coal} - t)^{-3/8}$



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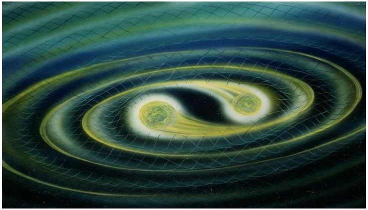

Single Detector:

$s(t) = F_+\ h_+(t) + F_\times\ h_\times(t) = \mathcal{A}(t) \cos(\phi(t) + \chi)$

Can not separate $(\epsilon, \theta, \phi, \psi, r)$

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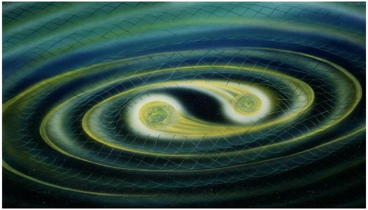

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EM: Info of projected semi-major axis, no info of ϵ

Advantages of Multi-detector over Single detector
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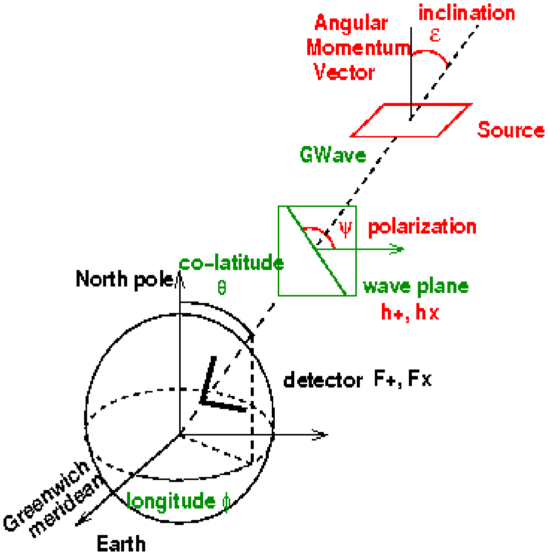

Network response to GWave $\equiv$ Project GWaves on the detector network

$$\mathbf{s} = \mathbf{F}_+ \otimes \mathbf{h}_+ + \mathbf{F}_\times \otimes \mathbf{h}_\times$$

GWave Polarisation:
$\mathbf{h}_+(\mathbf{t}), \mathbf{h}_\times(\mathbf{t}) \sim f(\epsilon, A(t), \varphi(t))$

Detector antenna pattern :
$\mathbf{F}_+, \mathbf{F}_\times \sim f(\theta, \phi, \psi, \alpha, \beta, \gamma)$

Advantages of Multi-detector over Single detector
5. Indepedent Veto Technique

D detector network

- Directional Null Veto:

  $$\textit{Signal} \quad \mathbf{h} = h_+ \mathbf{F}_+ + h_\times \mathbf{F}_\times$$

  Signal lies on the 2-dim plane of $\mathbf{F}_+$ and $\mathbf{F}_\times$.
  $\mathbf{h}$ projected along $\mathbf{F}_+ \times \mathbf{F}_\times$ contains no signal.
  Null data stream $(\mathbf{F}_+ \times \mathbf{F}_\times) \cdot \mathbf{h}$
- D detectors have $D - 2$ directional null streams
- Cross-correlation between the detectors to look for common signal
- False Alarm rate decreases. Low chance to find chance coincidences between the detectors.

Advantages of Multi-detector over Single detector
6. Improves the parameter estimation

- Noisy data introduces bias in the parameter estimation.

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γ^{ii}: Inverse of Fisher information

Advantages of Multi-detector over Single detector 6. Improves the parameter estimation

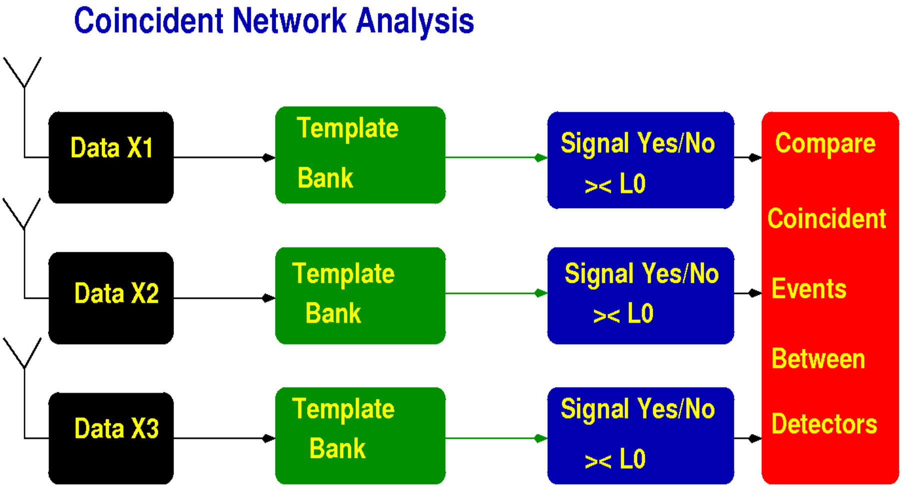
- Noisy data introduces bias in the parameter estimation.
- Error in parameter estimation is inversely proportional to the SNR.

$$\sigma_i = \frac{\sqrt{\gamma^{ii}}}{SNR}$$

γ^{ii}: Inverse of Fisher information

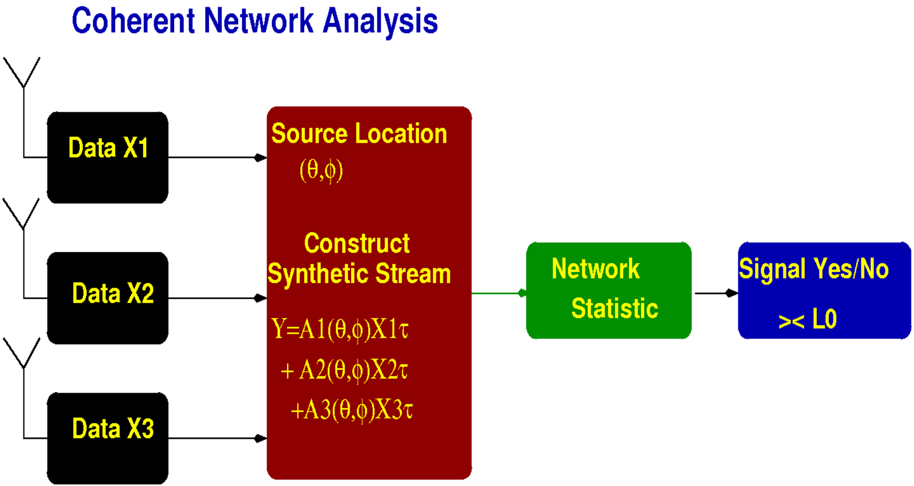
- Multi-detector analysis improves the SNR. This improves the parameter estimation.

Network Schemes



Signal phase is not accounted

Network Schemes



Signal phase is accounted

Network Schemes

Coherent detection is expensive :

Example: Newtonian chirp with multi-detectors

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Example: Newtonian chirp with multi-detectors

Signal detector : $\{t_a, \mathcal{M}, \delta, A\}$
Numerical maximisation : $\mathcal{M}$
Matched filtering technique, scan the $\mathcal{M}$ space
Look for the maximum in the filtered output
Templates: $M = 5000$, $m_1 = m_2 = 0.5M_\odot$, $N = 10^6$
Comp Cost: $\sim 6 * M * N * log_2 N \rightarrow 1.5 GFlops$

Network Schemes

Coherent detection is expensive :

Example: Newtonian chirp with multi-detectors

Multi-detectors: $\{t_a, \mathcal{M}, \delta, A, \epsilon, \psi, \theta, \phi\}$

Numerical maximisation : $\mathcal{M}, \theta, \phi$

Matched filtering technique, scan the $\mathcal{M}, \theta, \phi$ space

Look for the maximum in the filtered output

Templates: $\mathcal{M} \sim 7500$, $\Omega \sim 25000 \rightarrow$ *Tens of Tflops*

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Look for the maximum in the filtered output
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Open questions:
How to speed up the coherent search?
Set up hierarchy in parameter space?
Coherent detections with spins? higher harmonics?