IndIGO School – Lecture 1

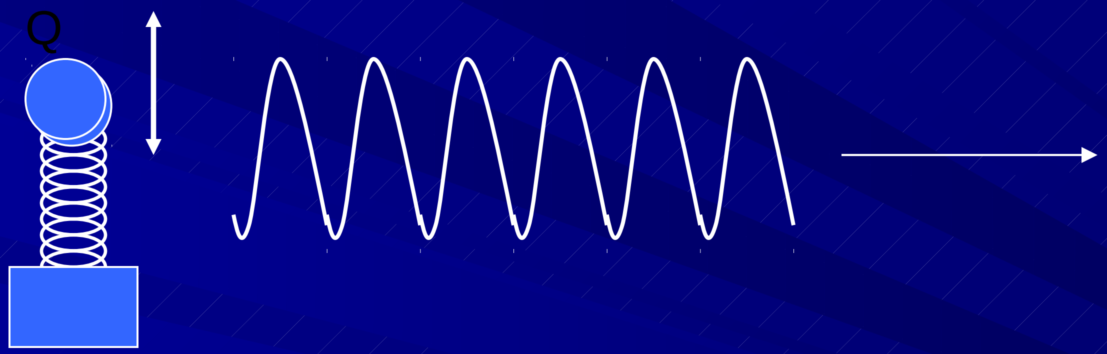
Another Introduction

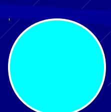
C. S. Unnikrishnan
DHEP/Gravitation Group
TIFR, Mumbai


3 km

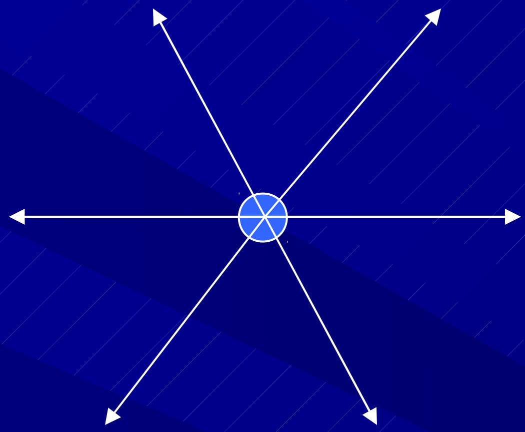
What is a gravitational wave?

What is an electromagnetic wave?

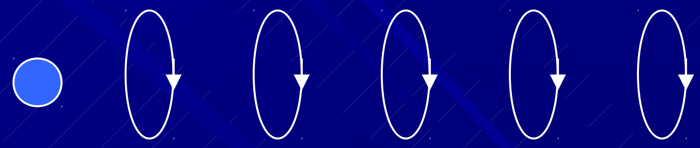




So, can we expect gravitational waves from oscillating (accelerated) masses?



Electric Field



Magnetic Field

Charges and Currents are fundamental, fields are 'theoretical'.

Current → Magnetic field

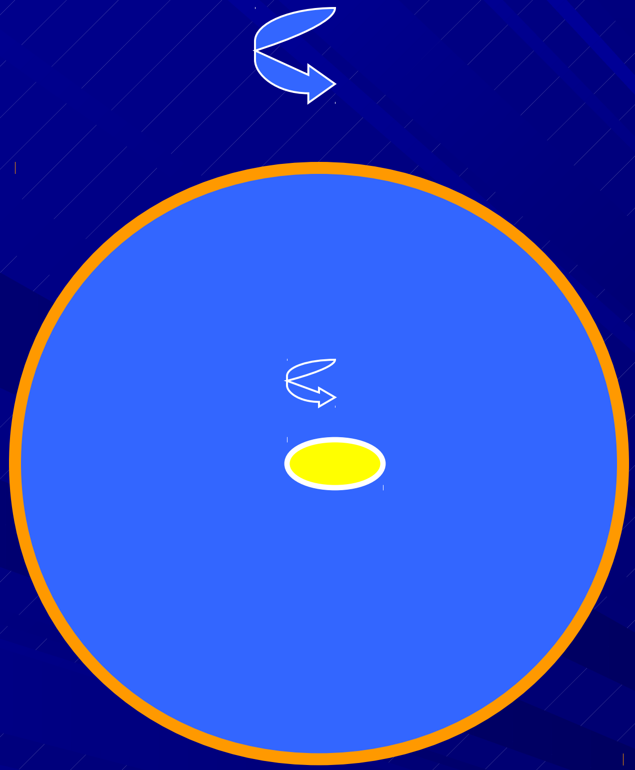
Electromagnetic waves need electric AND magnetic field for generation and propagation.

$$-\frac{\partial B}{\partial t} = \nabla \times E, \quad \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} = \nabla \times B \quad \ldots Maxwell$$

So, we seem to need a magnetic-like gravitational field, related to current of mass or matter, before we can think of gravitational waves…

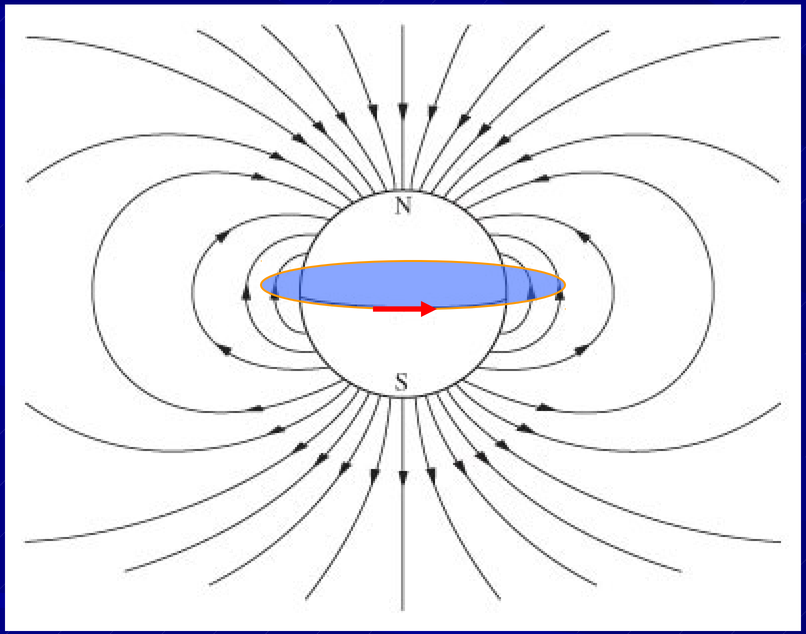
Does such a thing exist?!

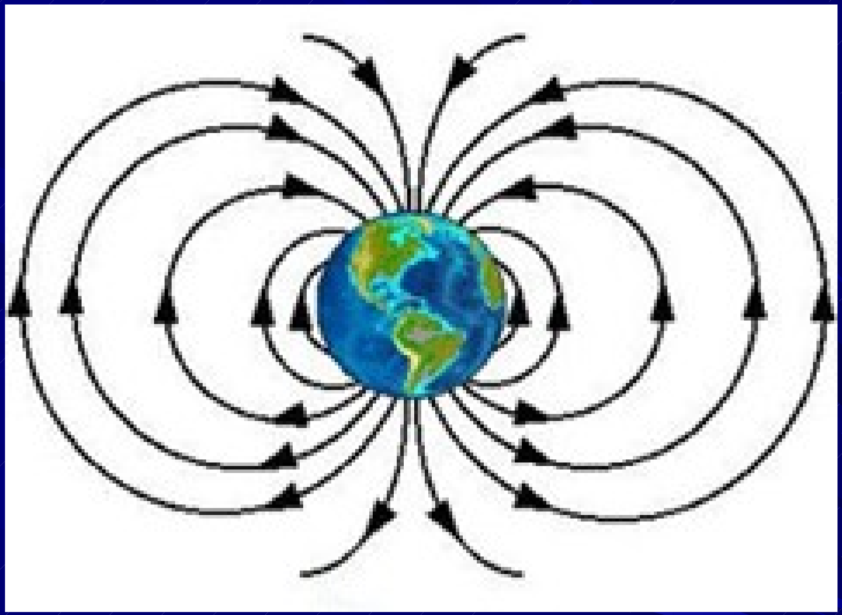
$$\omega = \Omega \frac{GM}{c^2 R}$$



Gravito-magnetism

A natural consequence of relativistic gravity, and yet, was not detected experimentally till recently.



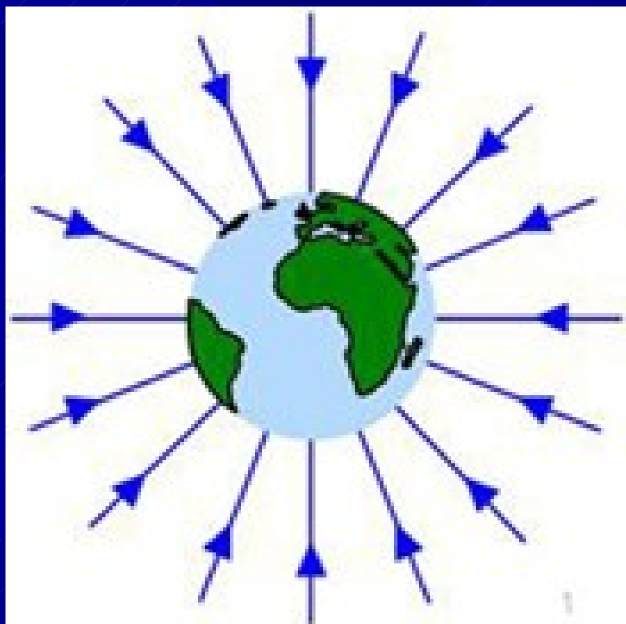


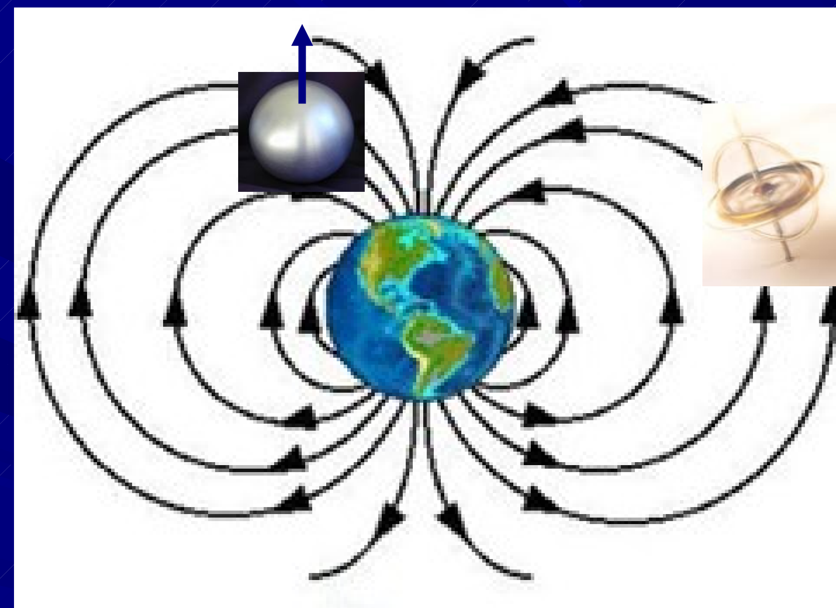
$$B \sim \frac{\mu_0 M}{r^3}$$

$$B_g \sim \frac{GJ}{c^2 r^3} = \frac{GI\omega}{c^2 r^3} \sim \frac{GM}{c^2 r}\omega$$

The real gravitational field near the earth

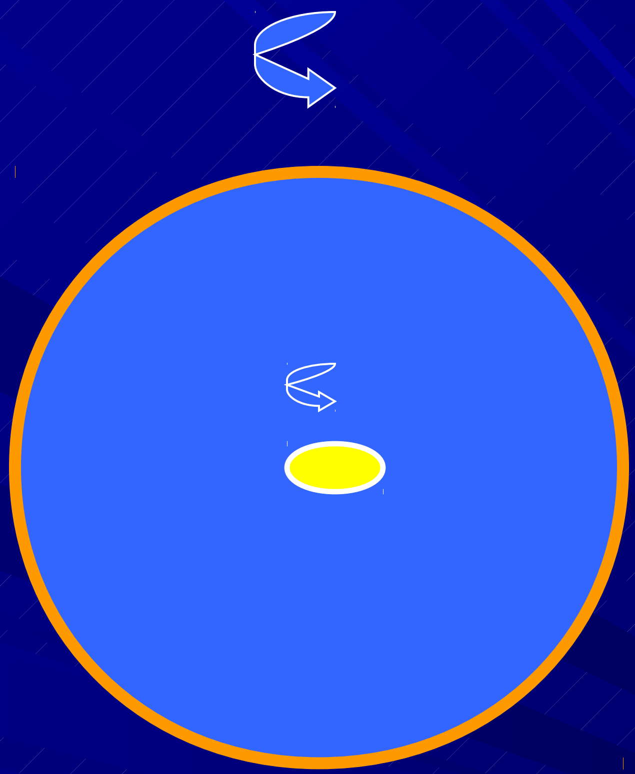
$$Torque = \vec{\mu}_g \times \vec{B}_g \quad and \quad B_g \sim \frac{GI\omega}{c^2 r^3} \sim \frac{GM}{c^2 r}\,\omega \sim 5 \times 10^{-14}\ rad/s$$





$$B_g \sim \frac{GI\omega}{c^2 r^3} \sim \frac{GM}{c^2 r}\,\omega \sim 5 \times 10^{-14}\ rad/s$$

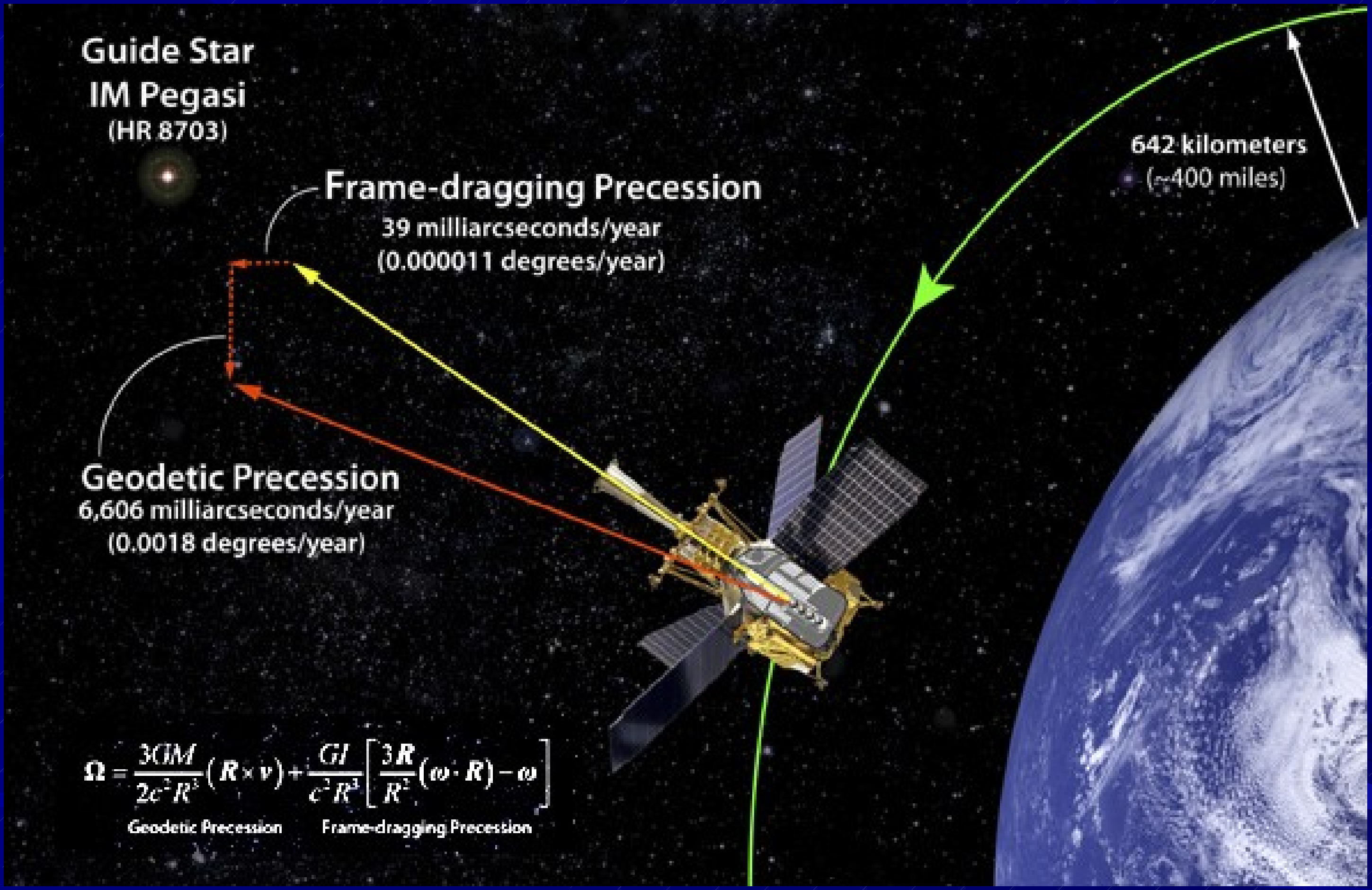
$$\omega = \Omega \frac{GM}{c^2 R}$$



Universe, Coriolis…

$$B_g = \frac{\Phi_U}{c^2} \nabla \quad \vec{V} = 2\Omega$$

Gravity Probe – B (Stanford U.)



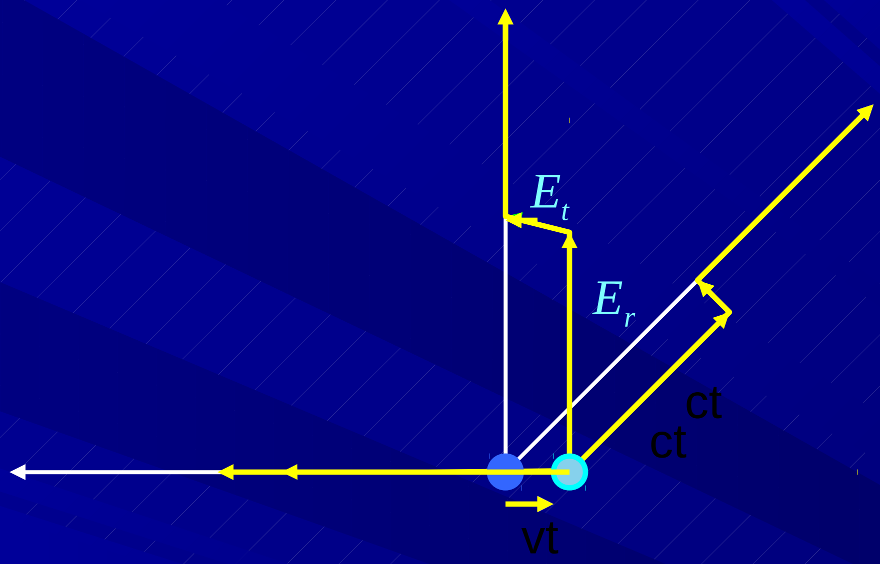
Gravity and Electromagnetism

Both have ‘electric’ and ‘magnetic’ parts → Charges and currents.

Mass is the ‘charge’ of gravity and Spin is its ‘gravito-magnetic moment’

One important different between the two is that while electric and magnetic fields have no electric charge, gravitational field has gravitational charge!

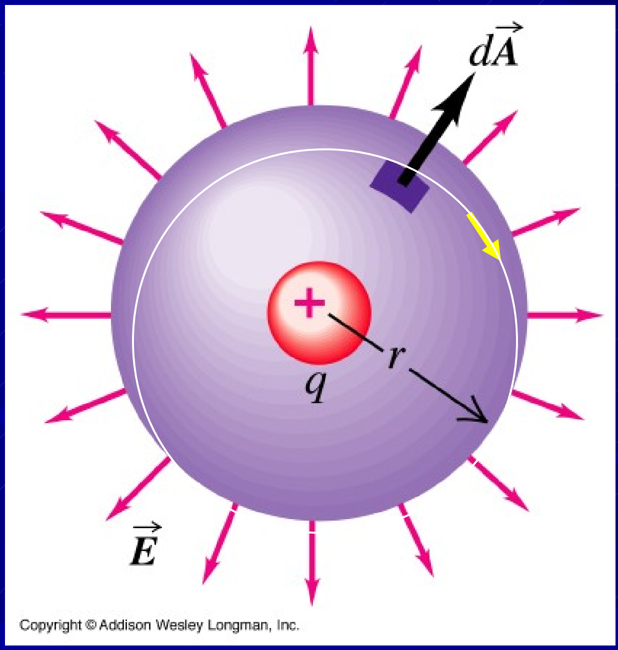
With $m=E/c^2$, all forms of energy is equivalent to mass, and hence generate gravity. Therefore, all fields including the gravitational field, which carry energy, also generate gravitational fields. This is one reason why the theory of gravity (The General Theory of Relativity) is complicated to work with.



$$\frac{E_t}{E_r} = \frac{vt}{ct} \square \frac{at}{c} = \frac{aR}{c^2} \text{ and } E_r = q / R^2$$

$$E_t = \frac{q}{R^2}\frac{aR\sin\theta}{c^2} = \frac{q\ddot{r}\sin\theta}{Rc^2}$$

$$qr = d \square \quad E_t = \frac{\ddot{d}\sin\theta}{Rc^2}$$



Copyright © Addison Wesley Longman, Inc.

Since area=R^2, number of flux lines/area → $1/R^2$

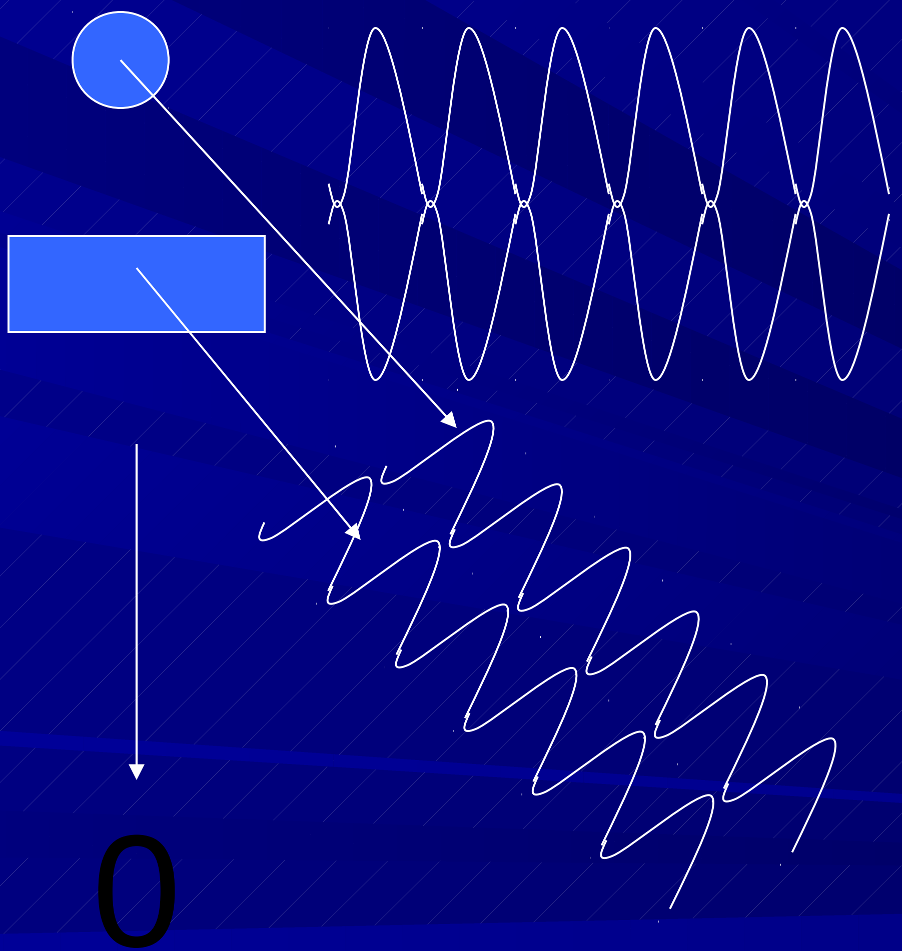
So, radial Electric field (flux/area) ~ $1/R^2$

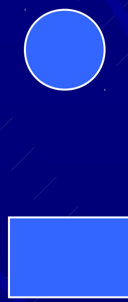
What about the Transverse Electric field (radiation)?

Since, circumference of a great circle on the sphere increases only as R, transverse radiation field decrease as 1/R.

What do we expect for gravitational waves, if the basic physics is similar?

$$\cancel{E_{gt} = \frac{GM\dot{r}\sin\theta}{Rc^{2}}}$$





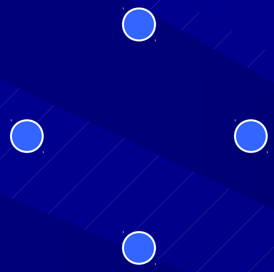
0

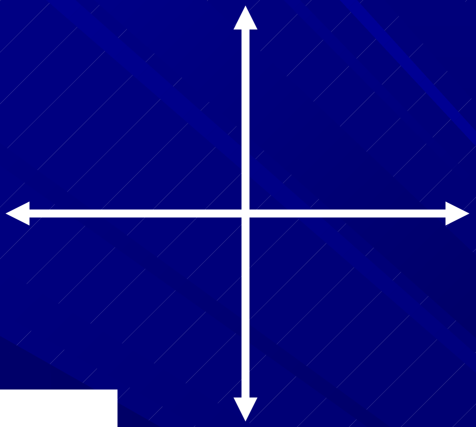
Phase diff.

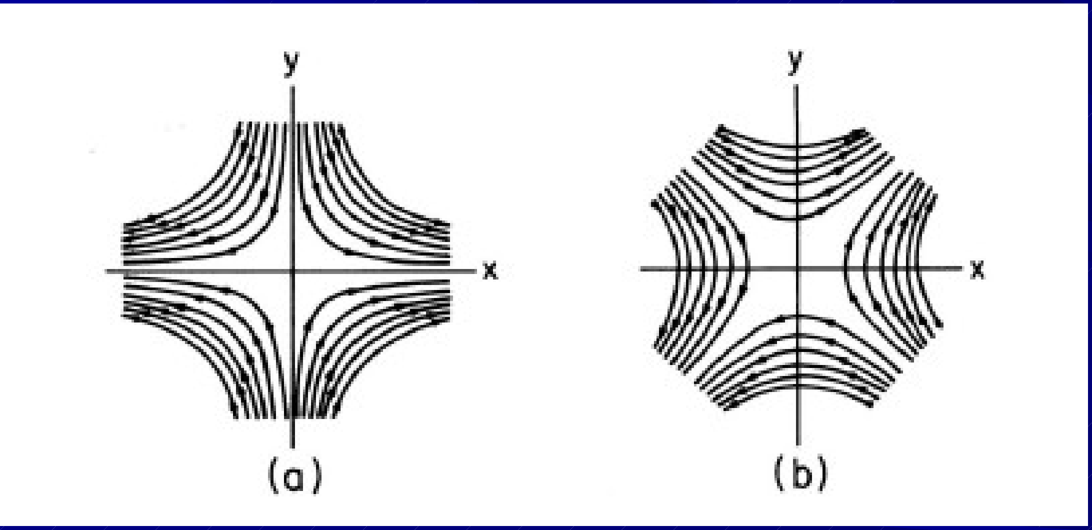
$$E_{gt} = \frac{GM\dot{r}\sin\theta}{Rc^2} \; \square \; \frac{\omega r\cos\theta}{c}$$

$$\square \;\; \square \;\; \frac{GMr^2\omega^3}{c^3 R}$$

What is the physical effect of a passing gravitational wave?







Quadrupole Radiation formula

$$\text{Strain}\ \ h = \frac{\Delta L}{L} \sim \frac{G}{c^4}\frac{Mr^2\omega^2}{R} \qquad \frac{G}{c^2R}\frac{Mv^2}{c^2}$$

Are we confident that Gravitational Waves exist, apart from the belief in the correctness of the theory?

ON GRAVITATIONAL WAVES.

Jl. Franklin Inst. 1937

BY

A. EINSTEIN and N. ROSEN.

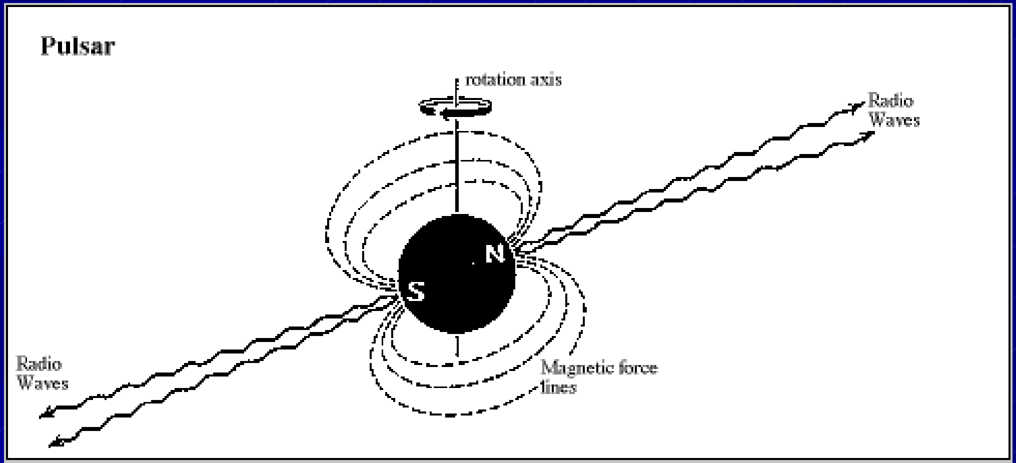
ABSTRACT.

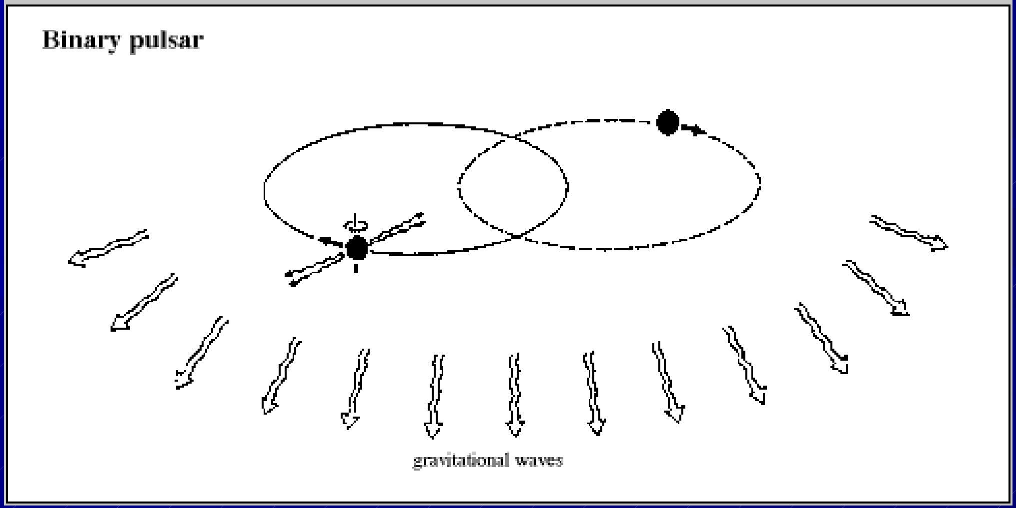
The rigorous solution for cylindrical gravitational waves is given. For the convenience of the reader the theory of gravitational waves and their production, already known in principle, is given in the first part of this paper. After encountering relationships which cast doubt on the existence of *rigorous* solutions for undulatory gravitational fields, we investigate rigorously the case of cylindrical gravitational waves. It turns out that rigorous solutions exist and that the problem reduces to the usual cylindrical waves in euclidean space.

Note.—The second part of this paper was considerably altered by me after the departure of Mr. Rosen for Russia since we had originally interpreted our formula results erroneously. I wish to thank my colleague Professor Robertson for his friendly assistance in the clarification of the original error. I thank also Mr. Hoffmann for kind assistance in translation.

A. EINSTEIN.

Binary Pulsar 1913+16 (Hulse-Taylor)

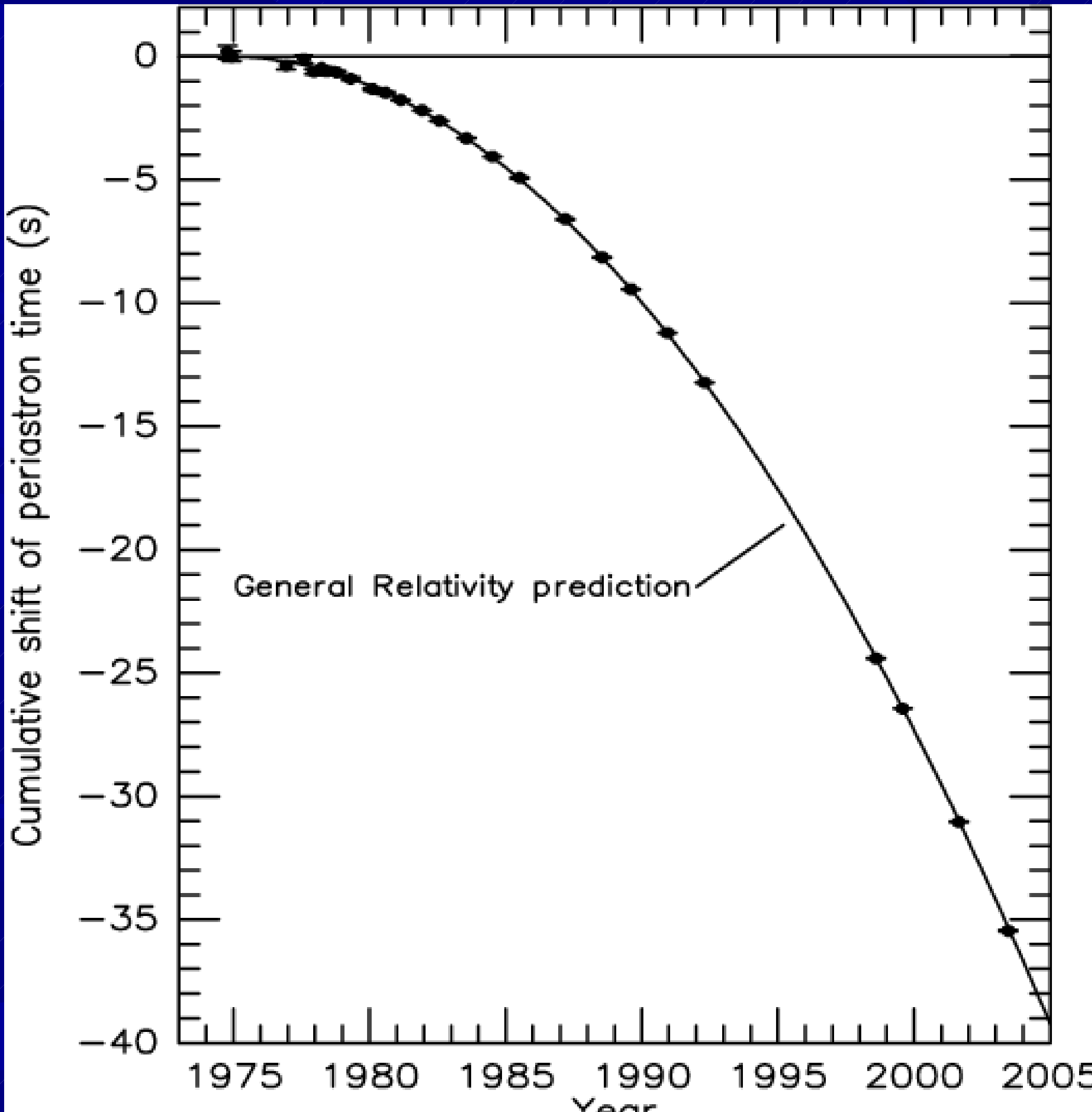




$$\dot{E}_G = \frac{dE_G}{dt} \approx \frac{32G}{c^5} M^2 \omega^6 r^4$$

$$t_c \approx 3 \times 10^8 \ yrs$$

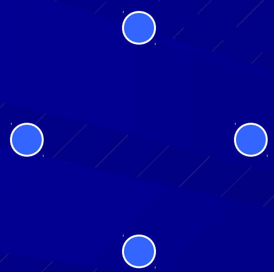
Orbital decay and speeding up of the binary pulsar:

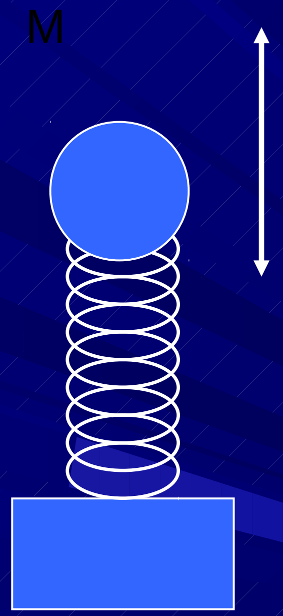


$$\dot{E}_G = \frac{dE_G}{dt} \approx \frac{32G}{c^5} M^2 \omega^6 r^4$$

Can we detect gravitational waves on the earth?

A gravitational Wave detector





Signal Strength at Earth for neutron star spiral in milky way:

Distance: 10 kpc ~ 10^{20} meters

$$\text{Strain } h \sim \frac{G}{c^4}\frac{Mv^2}{R} \sim 10^{-64} Mv^2$$

$$v \sim \sqrt{\frac{Gm}{2r}} = 0.1c \ (3 \times 10^7 \text{ m/s}) \text{ for neutron stars at } r \sim 100 \text{ km}$$

With M~10^{30} kg, v~3x10^7 m/s, $\text{Strain } h = \frac{\Delta l}{l} \sim 10^{-19}$

If the event happens in another galaxy, 100 Mpc (10^{24} m away),

$$\text{Strain } h \sim 10^{-23}$$

This small strain requires the measurement of <10^{-20} meters in a detector of size 1 km! (almost million times smaller than the atomic nucleus).

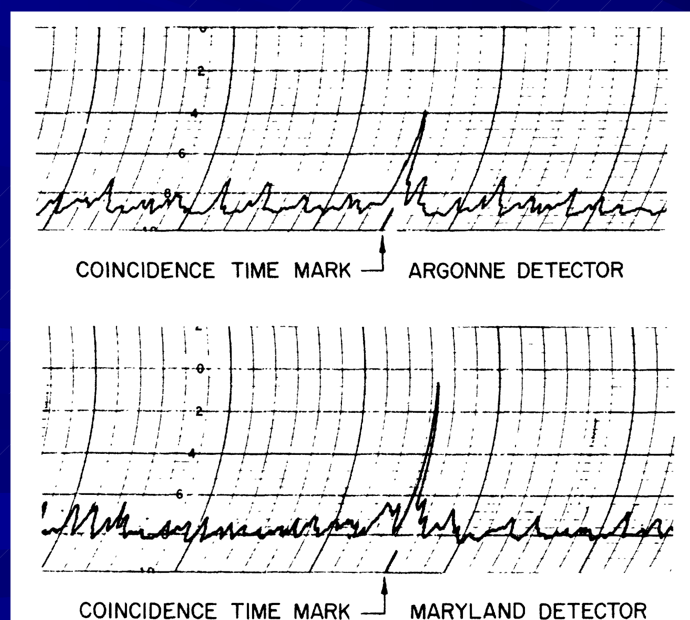
Is it a mad venture trying to make a 'detector'?

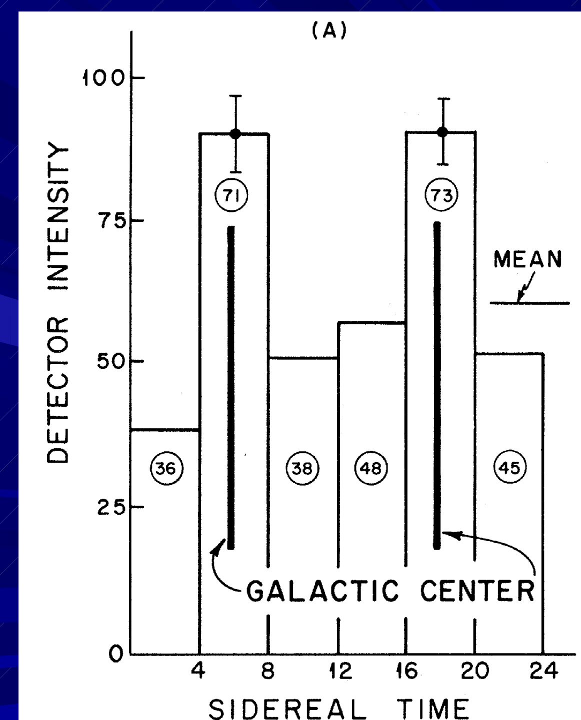
EVIDENCE FOR DISCOVERY OF GRAVITATIONAL RADIATION*

J. Weber
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 29 April 1969)

Coincidences have been observed on gravitational-radiation detectors over a base line of about 1000 km at Argonne National Laboratory and at the University of Maryland. The probability that all of these coincidences were accidental is incredibly small. Experiments imply that electromagnetic and seismic effects can be ruled out with a high level of confidence. These data are consistent with the conclusion that the detectors are being excited by gravitational radiation.

$$h \sim 10^{-17}$$





MiniGRAIL Cryogenic

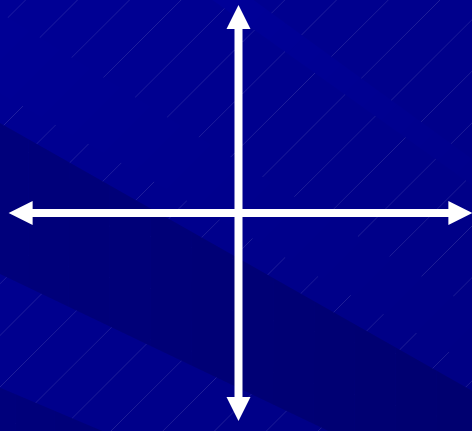
$$h \sim 10^{-20}$$

Later lecture on resonant detectors
+Laboratory on resonant transducer

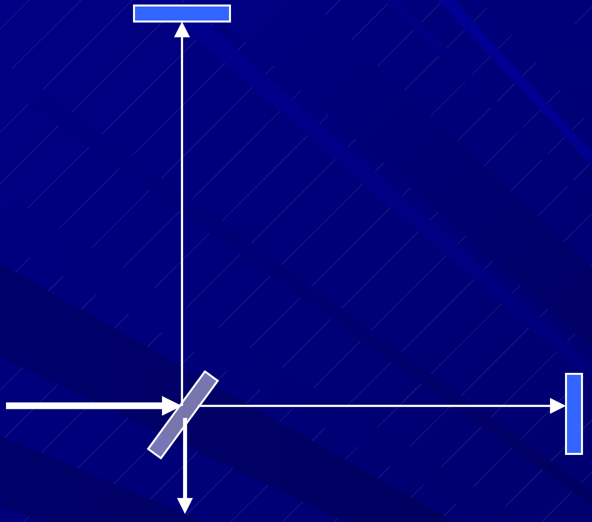
But, every bit counts… because waves strength is 1/R

If sensitivity is increased by factor X, then the distance reach increases by X, and the number of astrophysical sources increases as X^3!

So, a factor of 10 in sensitivity means a factor of 1000 in number of possible detections.

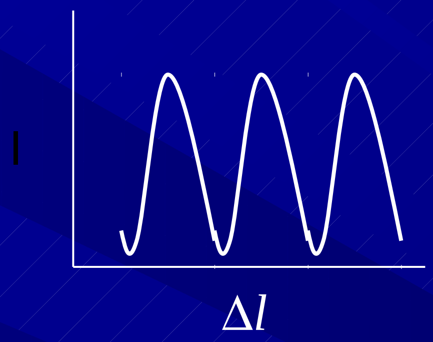


Michelson Interferometer



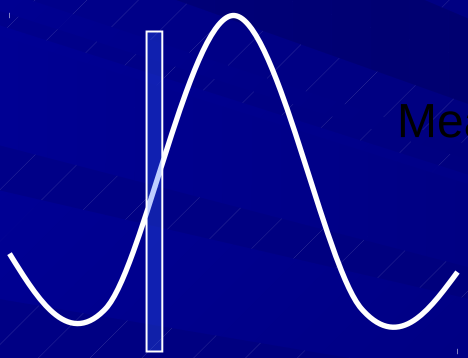
This is the primary device for gravitational wave detection

What is its sensitivity? And what are the noise sources and limitations?



Signal in the interferometer

The general problem of ‘fringe splitting’ (centroid, locking…)



Mean number of photons @ intensity I= N, $N = P / h\nu$

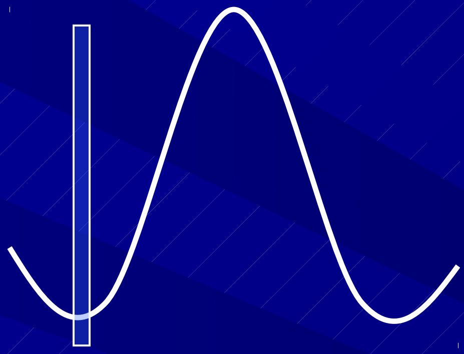
Change in the number of photons, $\Delta N \approx \dfrac{N \Delta\phi}{\pi}$

Noise $N_n = \sqrt{N}\ ;\ \Delta N_{\min}$

With 1 W of optical power, N=10^{19} / s,

$$\Delta\phi_{\min}\ ;\ \frac{\pi \Delta N_{\min}}{N} = \frac{\pi}{\sqrt{N}}\ ;\ 10^{-9}$$

However, the actual operation is on the dark fringe!,



Clearly, at the minimum (or maximum), the intensity at the detector does no change when phase changes, to first order.

We will learn in detail how a linear sensitivity is achieved while operating on a dark fringe

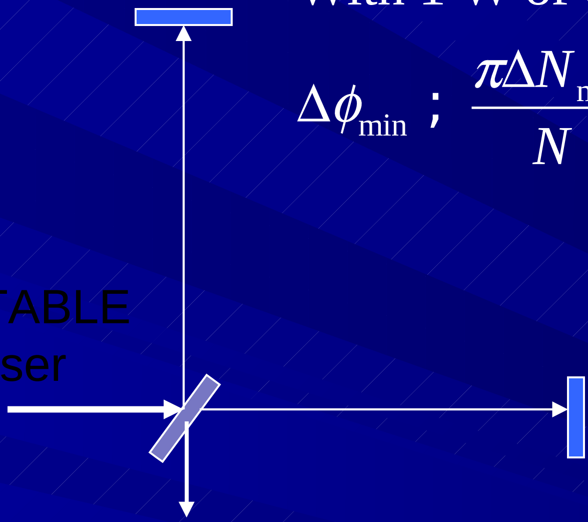
Pound-Drever-Hall technique, Phase modulation and side bands…Lock-in detection, negative feed back and control…

Detection of gravitational waves requires the measurement of movements 10^{-17} to 10^{-20} meters in a detector of size 1 km.

With 1 W of optical power, N=10^{19} / *s*,

$$\Delta\phi_{\min} \; ; \; \frac{\pi \Delta N_{\min}}{N} = \frac{\pi}{\sqrt{N}} \; ; \; 10^{-9} \; \square \quad \Delta L = \Delta\phi_{\min} \; \square \; \lambda \; ; \; 10^{-15} \; m \, (1\, s)$$

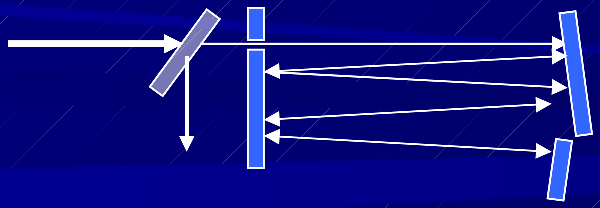
STABLE Laser



1) Increase Laser Power $100\, W \; \square \quad \Delta L \; \square \; 10^{-16} \; m$

2) Increase Length up to 4 km: Not much gain, though very important (1/R).

3) Fold optical path



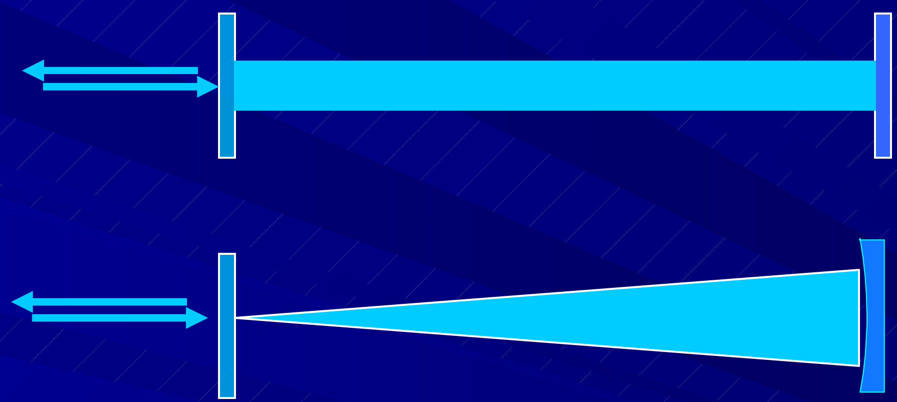
$n = 10 \; \square \quad 10^{-17} \; m$

Reaching there, but not comfortable yet!

Detection of gravitational waves requires the measurement of movements 10^{-17} to 10^{-19} meters in a detector of size several km.

Improvements:

1) Folding → Fabry-Perot Cavity



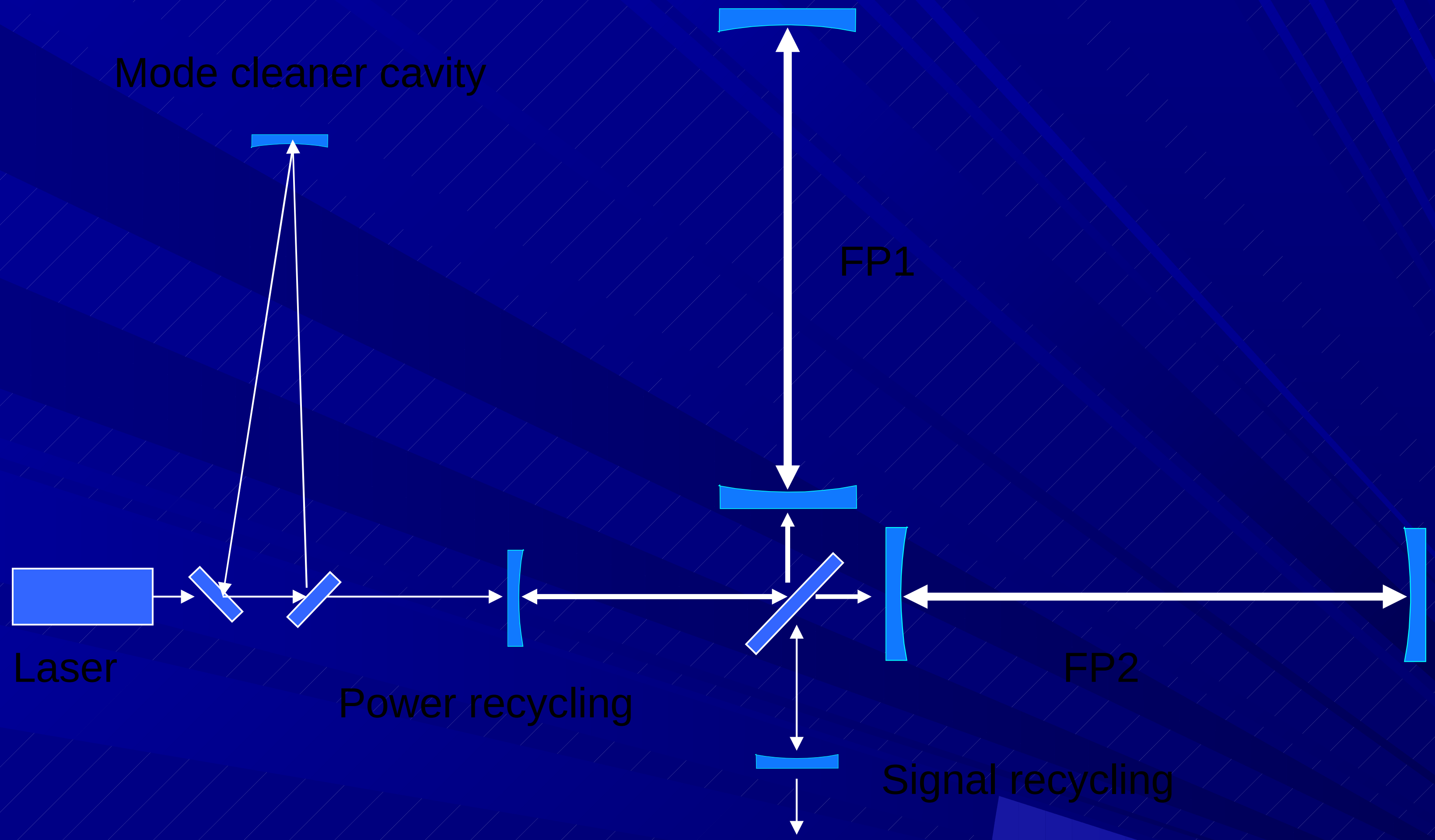
Finesse ~ n : 300 +

$$100\,W,\ 3\,km \Rightarrow 10^{-16}\,m \Rightarrow 3 \times 10^{-19}\,m \text{ with } F \approx 300$$

We will look at details of F-P operation, T and R, and do experiments on its use

Intra-cavity power > 10 -100 kW

Radiation Pressure Noise and Thermal Lensing are problems

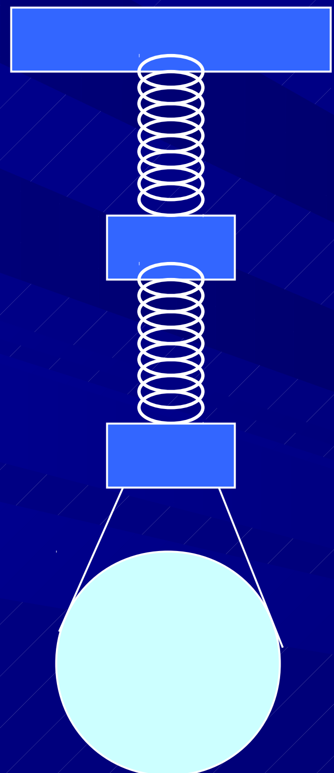


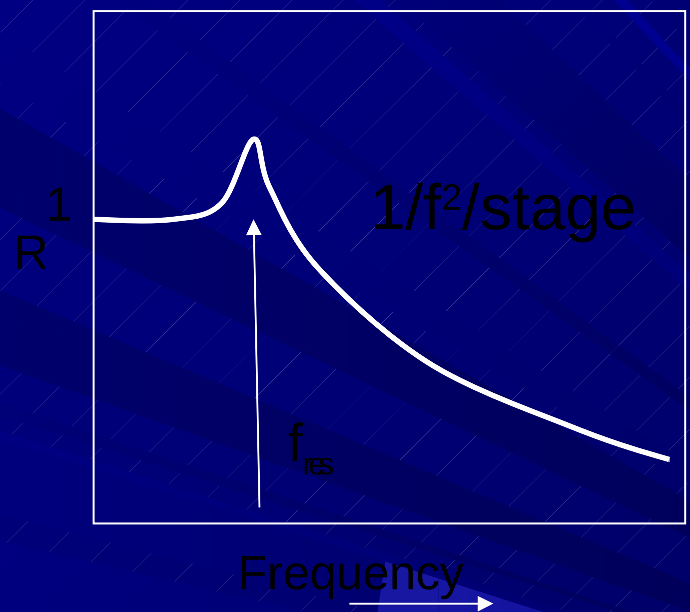
Scheme of the Interferometric detector

How can we hope to measure 10^{-19} m when the ground vibrations are like 1 micrometer?!

Immunity to vibrations needed by a factor of 10^{14} !

Solution:





Ground vibrations: 10^{-6} m at 1 Hz, 10^{-9} m at 30 Hz

3 stages of springs and pendulum with each resonance at around 1 Hz

→ Response down by a factor $(10^3)^3 = 10^9$ at 30 Hz!

Possible to isolate from vibrations at the 10^{-20} m level at 100 Hz with 3-4 stages.

Noise from Light:

$$\Delta l = \lambda \Delta \phi_{\min} \; ; \; \frac{\lambda \pi}{\sqrt{N}}$$

Photon Shot Noise: $h_{sn} = \alpha \sqrt{\frac{1}{P_i}}$

$$\sqrt{N} \; ; \; \sqrt{P_i / \left(hc / \lambda \right)}$$

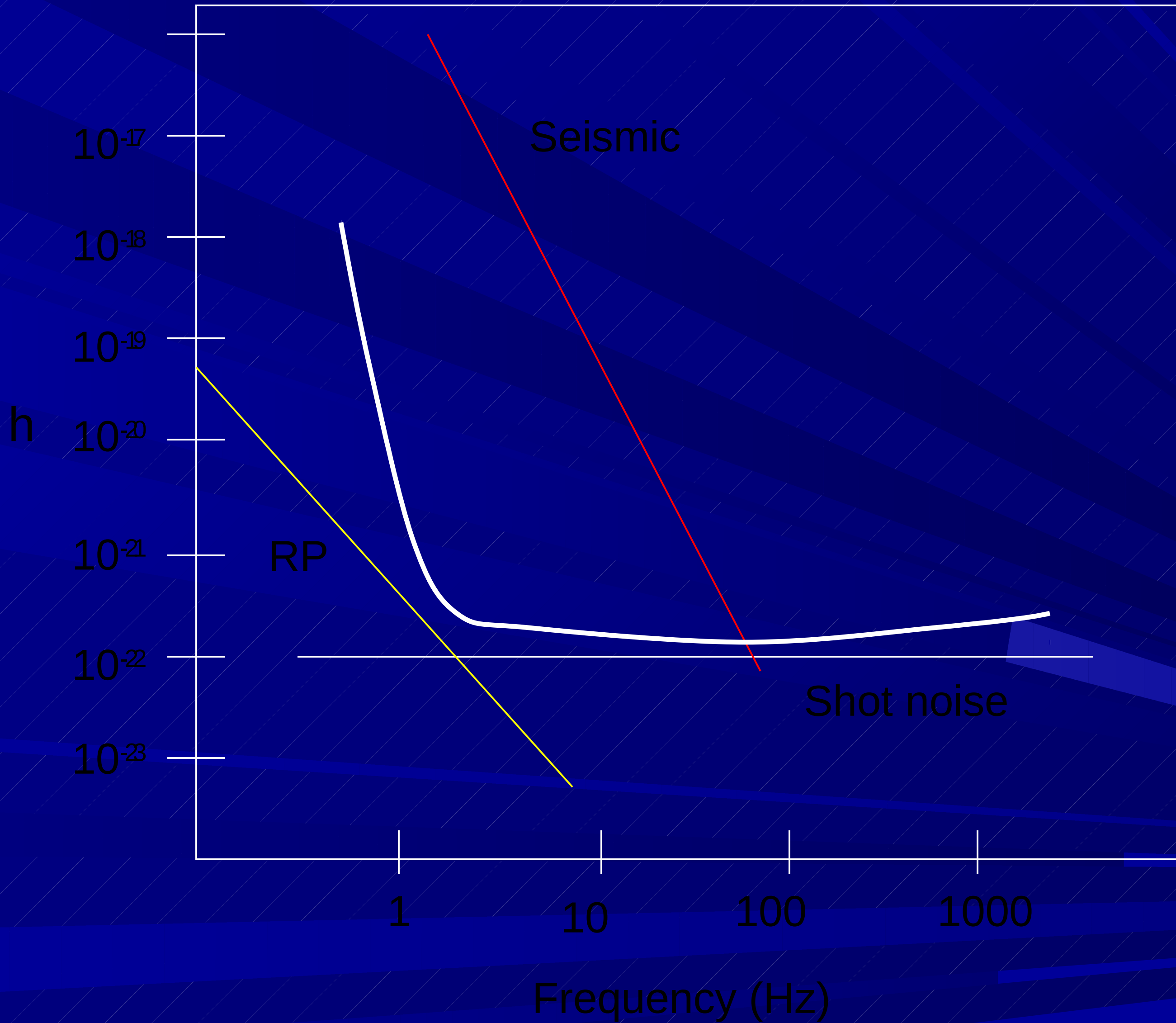
Radiation Pressure Noise $F_{rad} = \sqrt{N} \, \left(h / \lambda \right) = \sqrt{\frac{hP_i}{c\lambda}}$

Movement noise due to this force: $h_{rp} \, \square \, F_{rad} / m\omega^2 = \beta \sqrt{P_i}$

Standard Quantum Limit

$$h_{total} = \sqrt{h_{sn}^2 + h_{rp}^2} \; \square \; h_{\min} = h_{SQL} = \frac{1}{\pi L f} \sqrt{\frac{h}{m}}$$

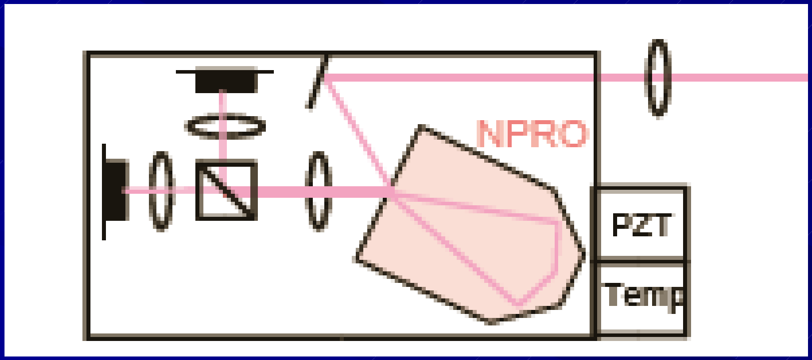
$$h_{total} = \sqrt{h_{sn}^2 + h_{rp}^2} \;\square\; h_{\min} = h_{SQL} = \frac{1}{\pi L f}\sqrt{\frac{\text{h}}{m}}$$

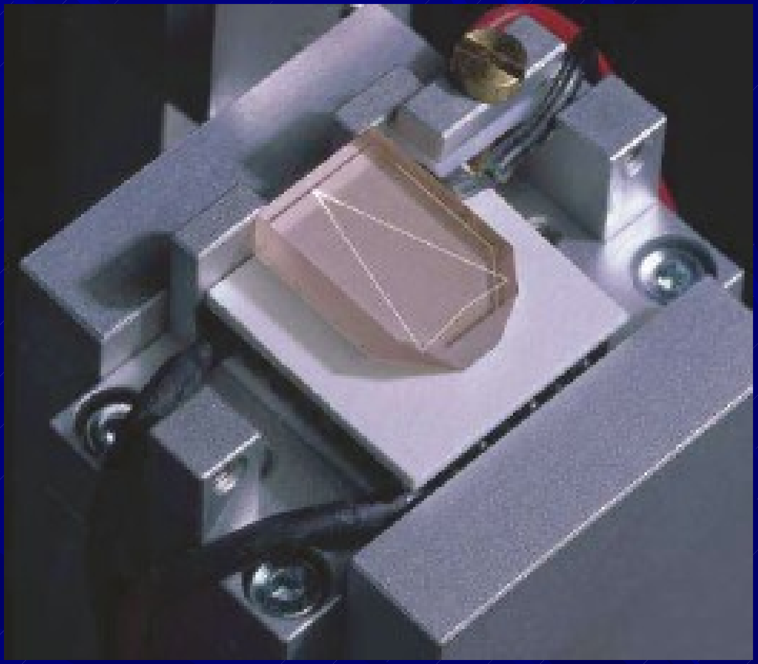


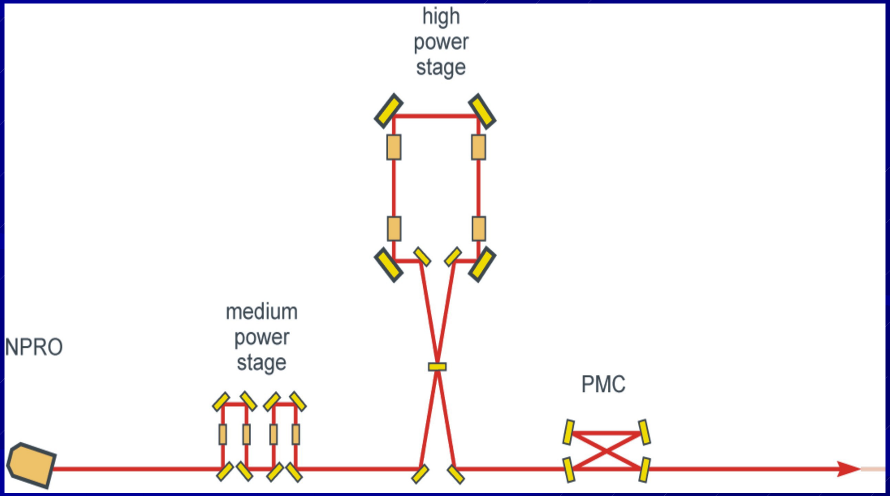
High Power Laser : 200 W

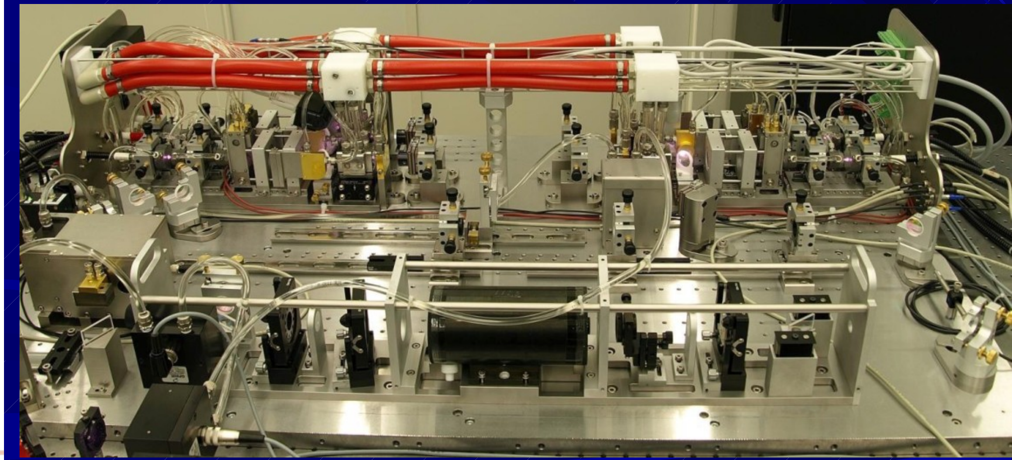
Intensity Noise < 10^{-9} (sub-micro-watts)

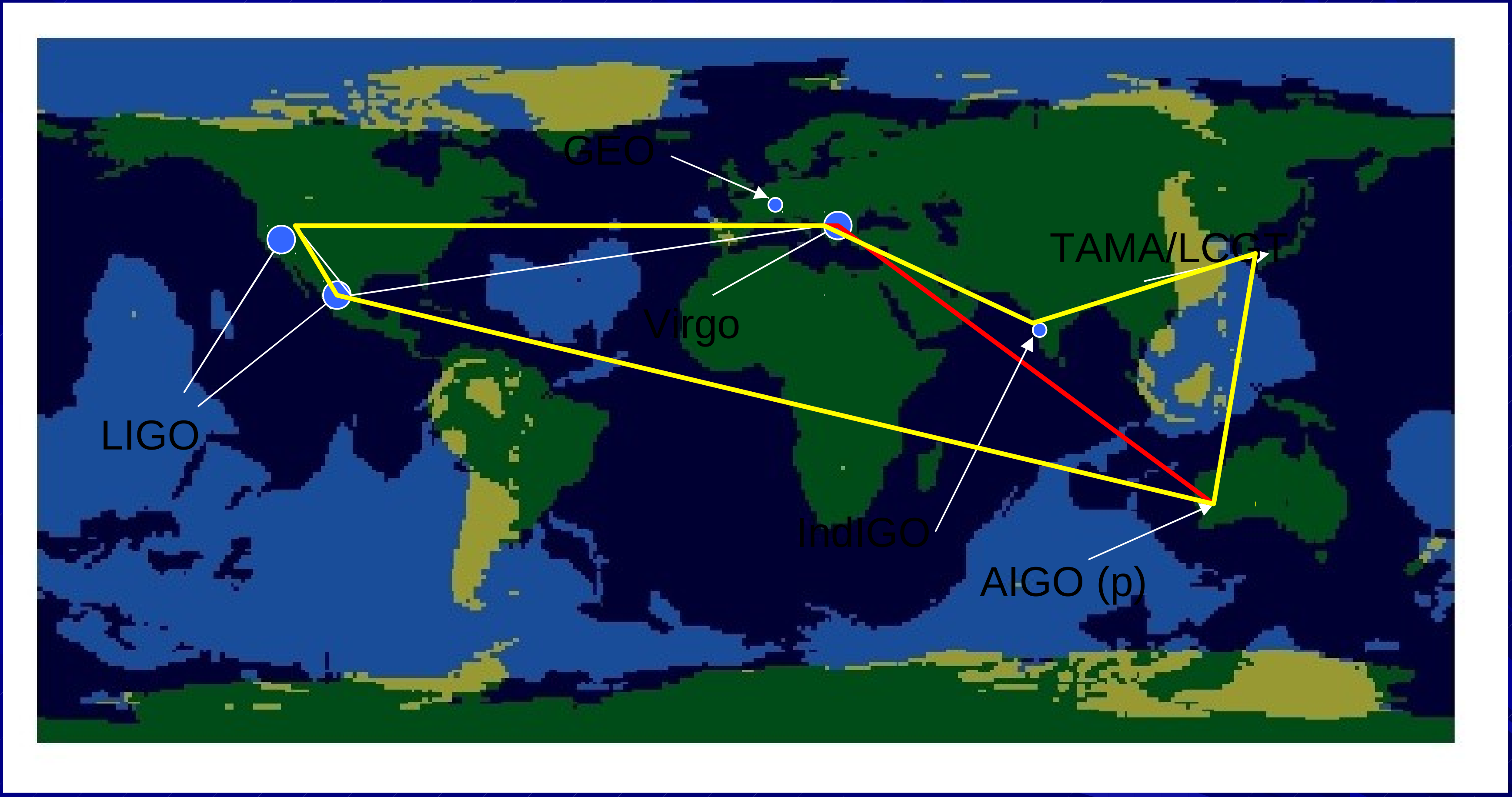
Frequency Noise < 1 Hz (in 10^{14} Hz!)







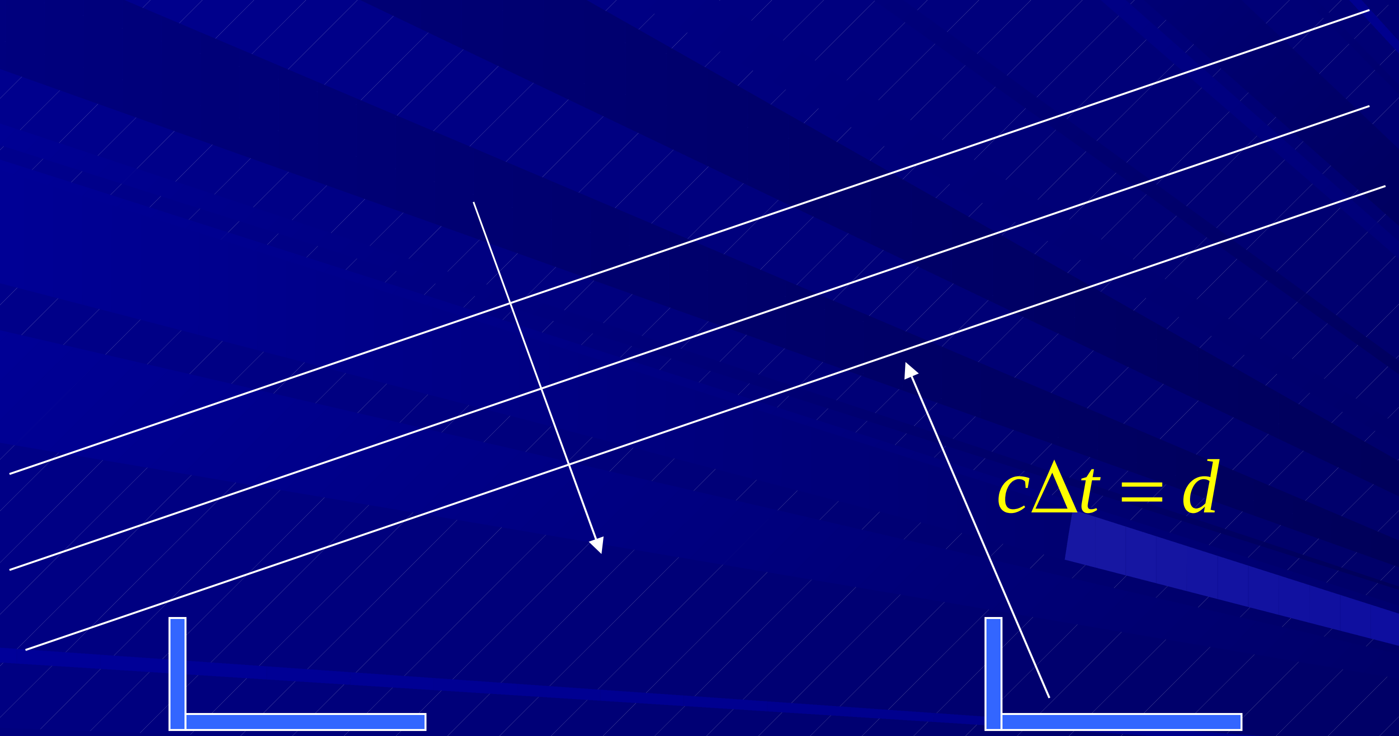




GEO
TAMA/LCGT
Virgo
LIGO
IndIGO
AIGO (p)

Why do we need more detectors?

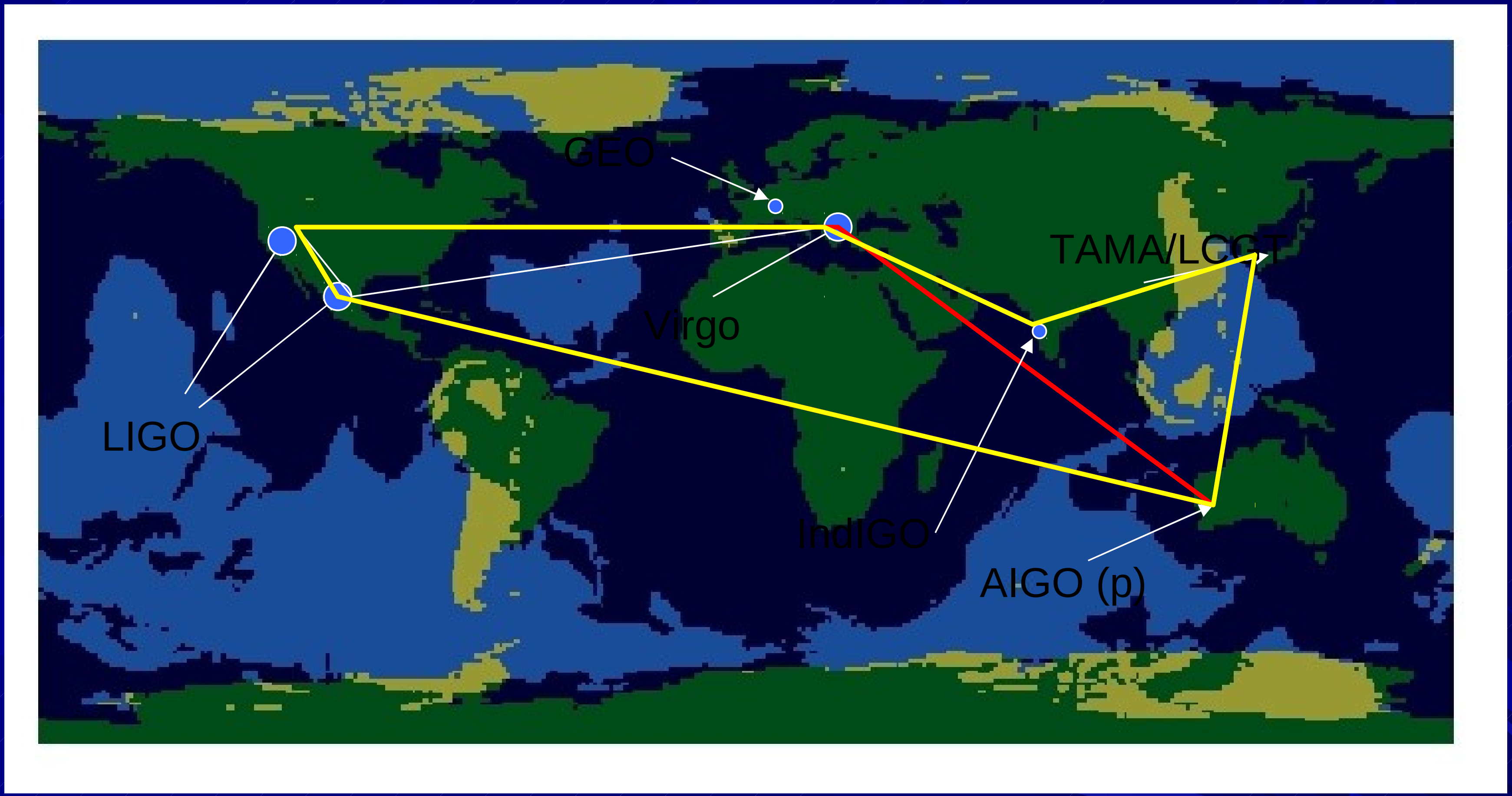
GW detectors are not telescopes. They cannot pin-point a source.



$$c\Delta t = d$$

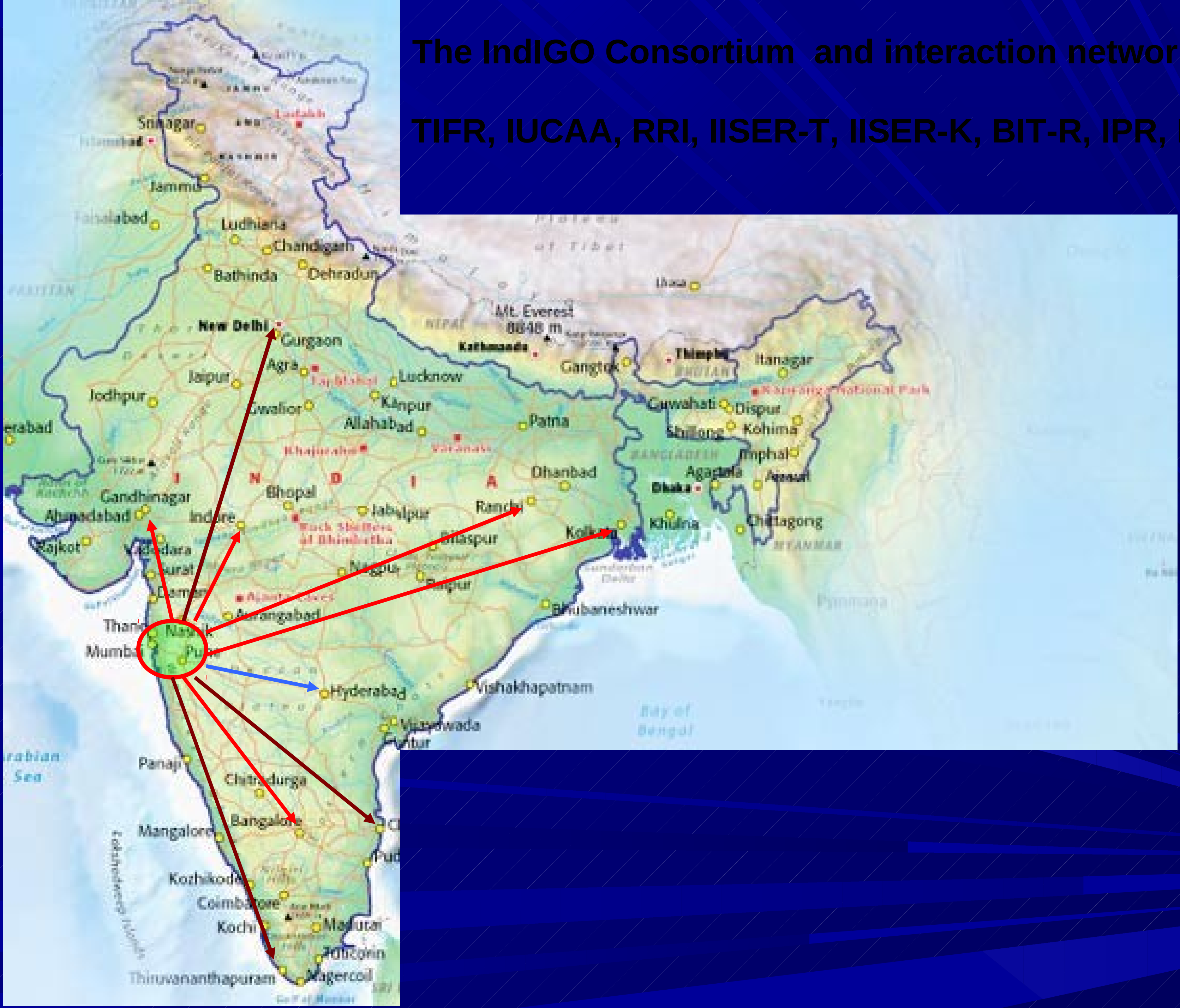
Timing (and only timing) can fix a direction

Need at least 3


GEO
TAMA/LCGT
Virgo
LIGO
IndIGO
AIGO (p)

The IndIGO Consortium and interaction network

TIFR, IUCAA, RRI, IISER-T, IISER-K, BIT-R, IPR, DU, CMI



The roadmap prepared by the IndIGO Consortium (www.gw-indigo.org)

(about 25 Indian scientists from TIFR, IUCAA, RRI, RRCAT, DU, IISERs, IITs, IPR, CalTech, MIT, and Cardiff)

(a) the construction and operation of an advanced 3 meter scale prototype within the next three years (TIFR)

(b) collaborative participation in the 4 km class advanced detector. Australia has shown keen interest in exploring close collaboration with India for participation in its AIGO detector.

(c) Simultaneously with (a) and (b), feasibility study and construction of an advanced prototype in the 30 meter class in a carefully chosen place with full implementation of all the advanced detector technologies (5-6 years).

(d) the construction of a 4 km class detector, Indian Interferometric Gravitational wave Observatory (IndIGO), as the final step. With the accumulated experience from the earlier steps, one may expect that such a detector to come up during 2020-22.

There are great possibilities and bright future for gravitational wave-based astronomy… if we manage to detect gravitational waves with these detectors