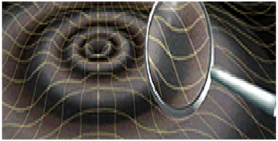
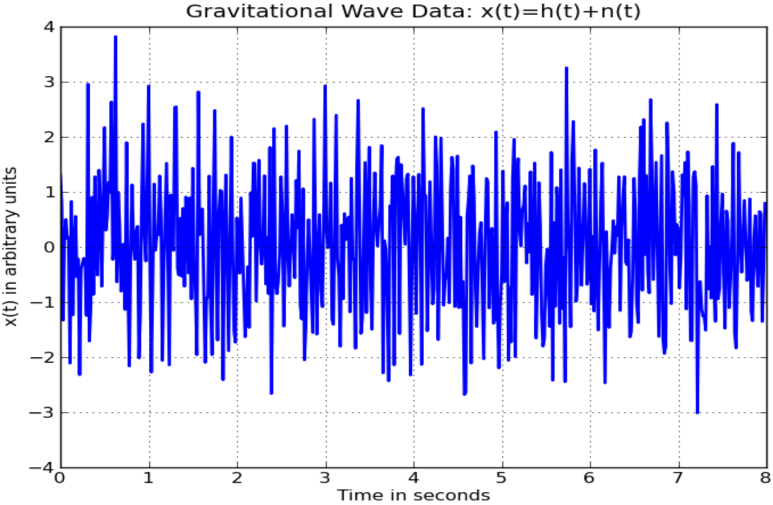


Matched Filtering Technique

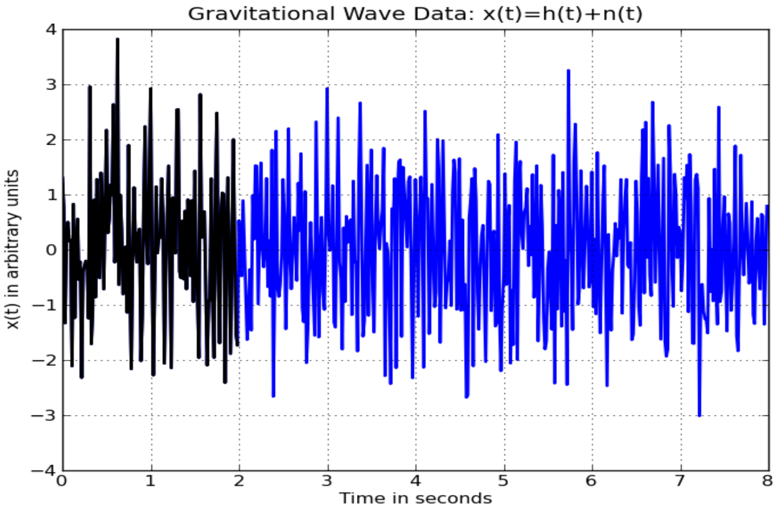
Archana Pai
Indian Institute of Science Education and Research
Trivandrum

ISGWA2010, 16 December 2010

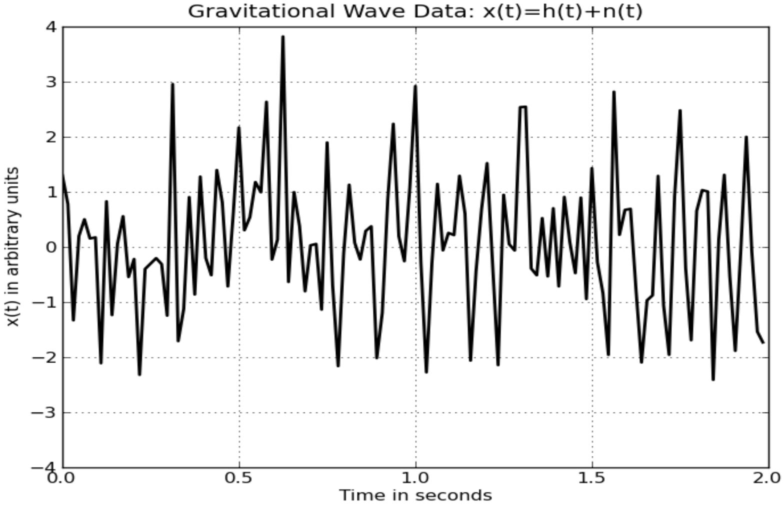
Linear Filtering of Data



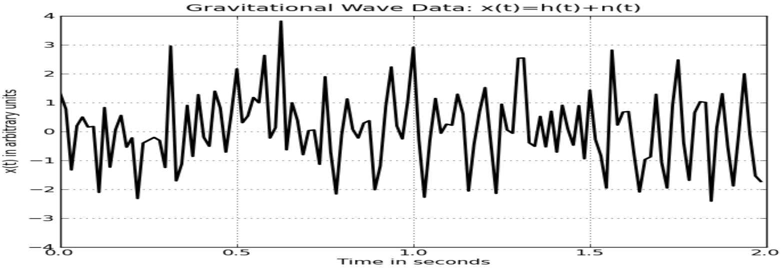
Linear Filtering of Data



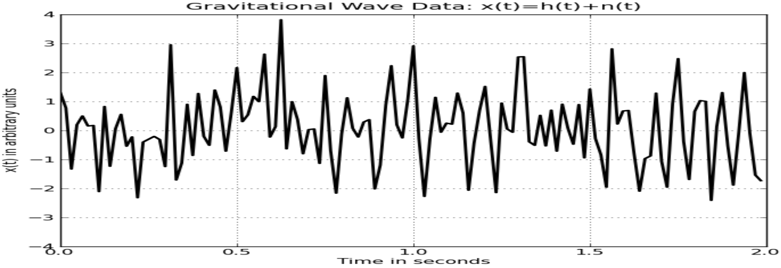
Linear Filtering of Data

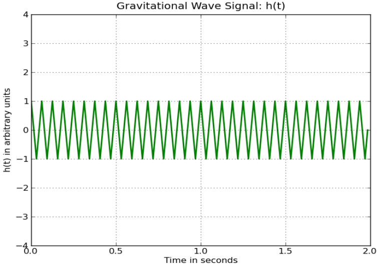


Linear Filtering of Data



Linear Filtering of Data





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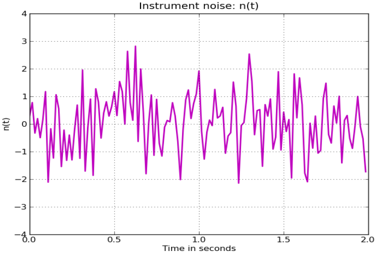
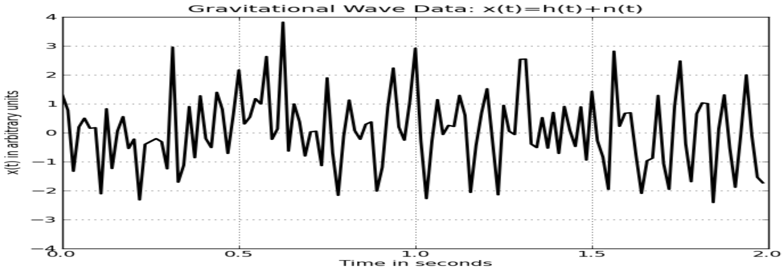
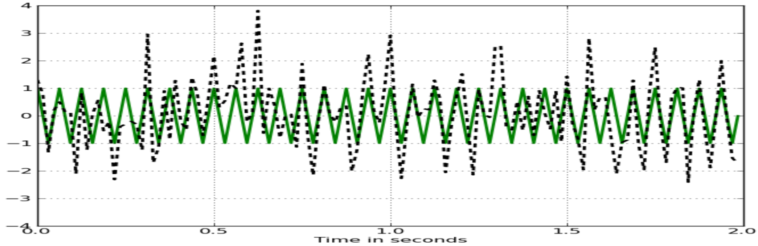
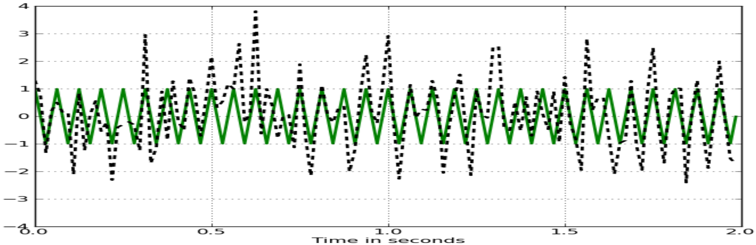


Linear Filtering of Data



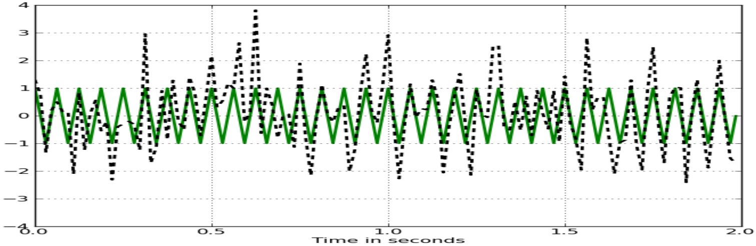


Linear Filtering of Data



Questions to address:
Is there any Gravity-Wave signal in the data?

Linear Filtering of Data

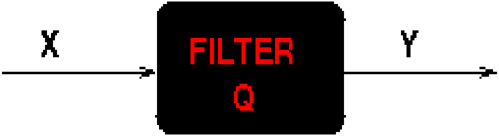


Questions to address:
Is there any Gravity-Wave signal in the data?
What kind of source and its astrophysical parameters?

Linear Filtering of Data

Conv of $x(t)$ and $q(t)$: $X \otimes Q$

$$y(T) = \int x(t)\, q(T - t)\, dt$$



Linear Filtering of Data

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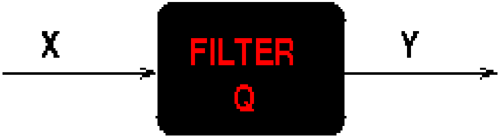
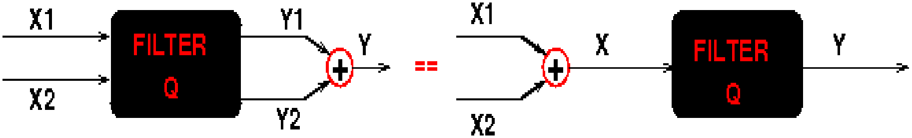


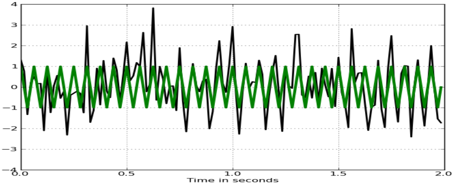
Linearity of a filter:



Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\, {\color{red} q(T - t)}\, dt$$



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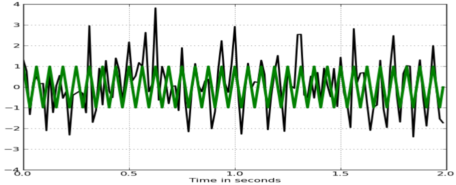
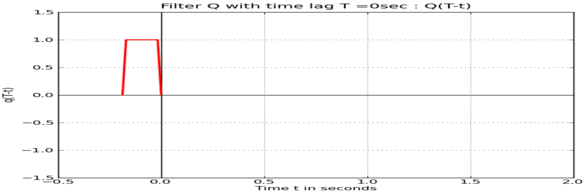


Filter function
${\color{red} q(T-t)}$



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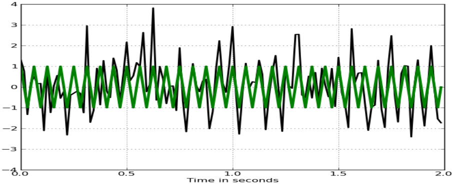
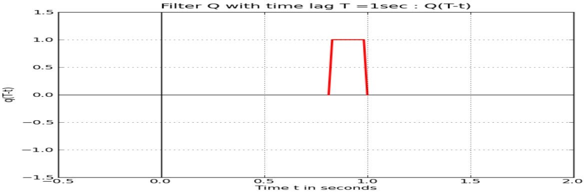


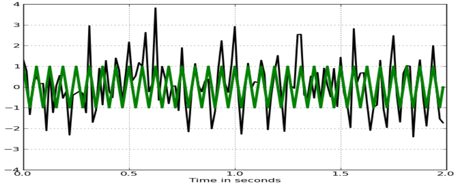
Filter function
$q(T-t)$



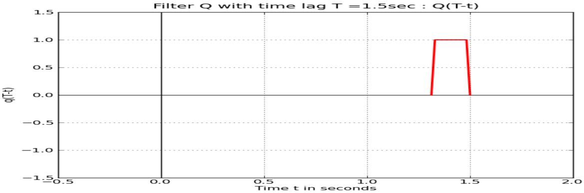
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\ q(T-t)\ dt$$



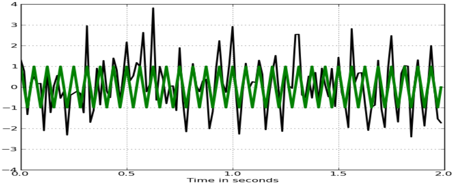
Filter function
$q(T-t)$



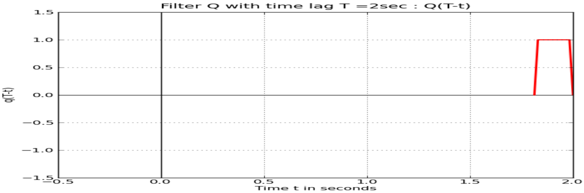
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\, q(T-t)\, dt$$



Filter function
$q(T-t)$



Linear Filtering of Data

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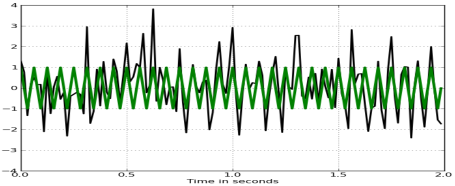
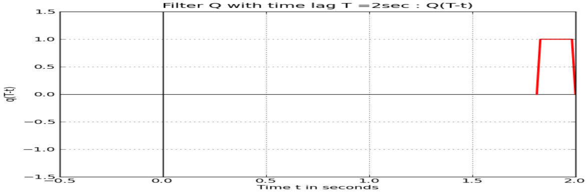
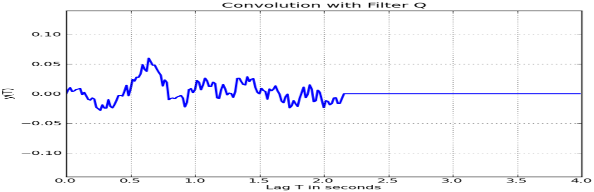
$$y(\tau) = \int x(t)\, q(T - t)\, dt$$



Filter function
$q(T - t)$



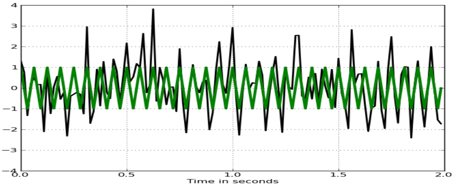
Filter Output y(T)



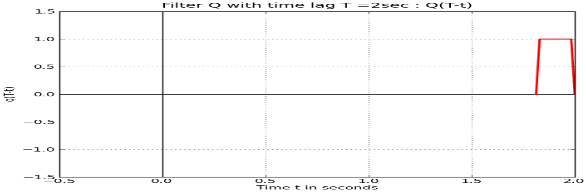
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

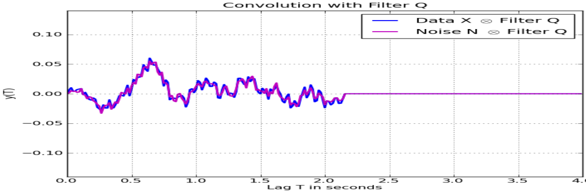
$$y(\tau) = \int x(t)\, {\color{red} q(T-t)}\, dt$$



Filter function
$\color{red} q(T-t)$



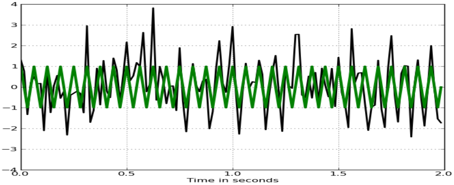
Filter Output y(T)



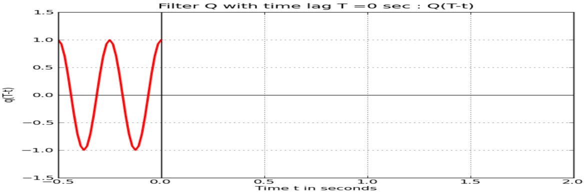
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\, {\color{red} q(T - t)}\, dt$$



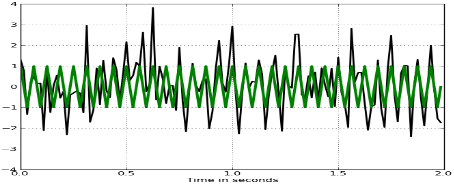
Cos Filter ${\color{red} q(T - t)}$
Freq = 4Hz



Linear Filtering of Data

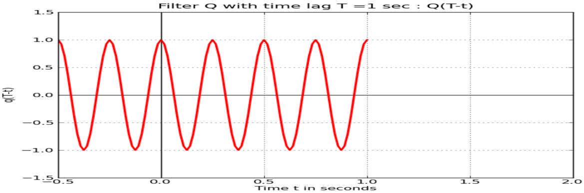
Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\, q(T - t)\, dt$$



Cos Filter $q(T - t)$

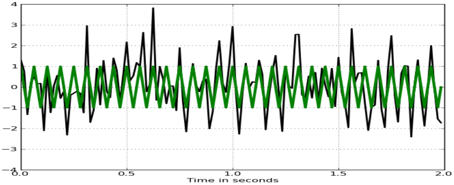
Freq = 4Hz



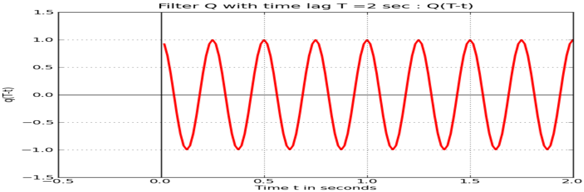
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\, q(T-t)\, dt$$



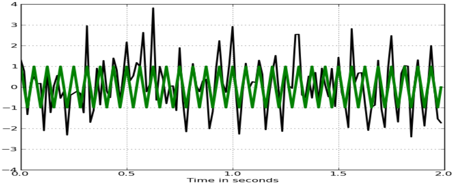
Cos Filter $q(T-t)$
Freq = 4Hz



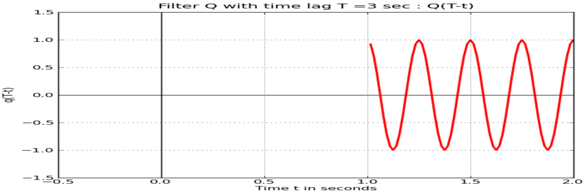
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\; q(T-t)\; dt$$



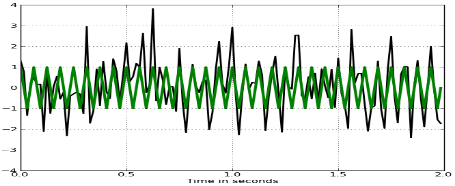
Cos Filter $q(T-t)$
Freq = 4Hz



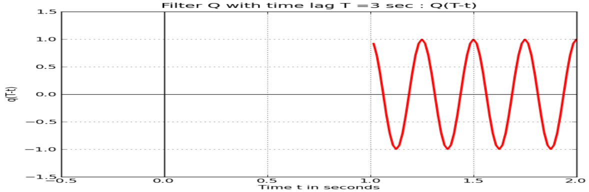
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

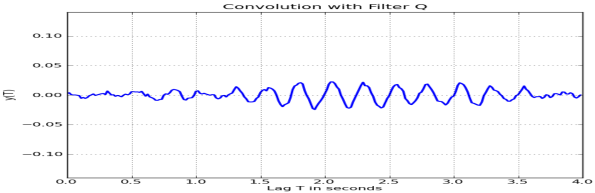
$$y(\tau) = \int x(t)\, {\color{red} q(T-t)}\, dt$$



Cos Filter ${\color{red} q(T-t)}$
Freq = 4Hz



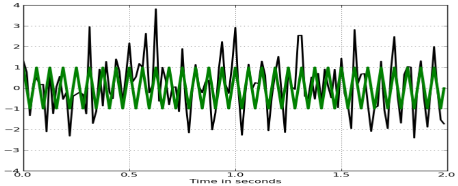
Filtered Output
$y(T)$



Linear Filtering of Data

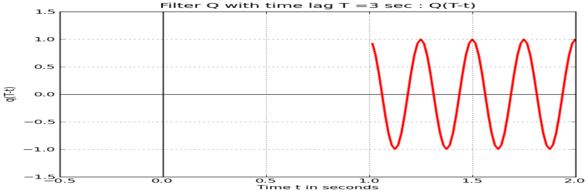
Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\, q(T - t)\, dt$$



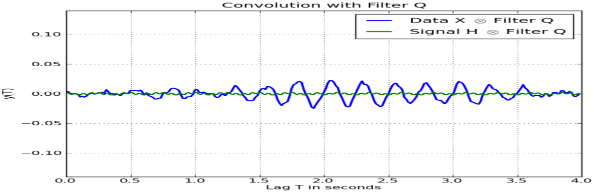
Cos Filter $q(T - t)$

Freq = 4Hz



Filtered Output

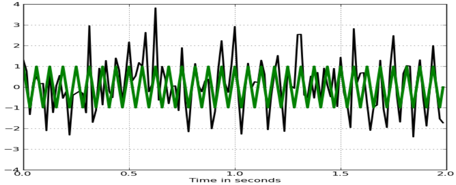
$y(T)$



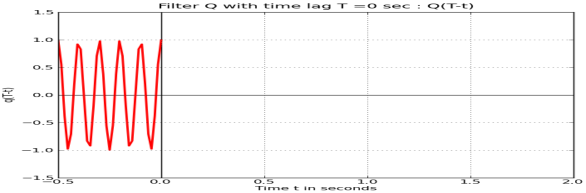
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\, q(T - t)\, dt$$



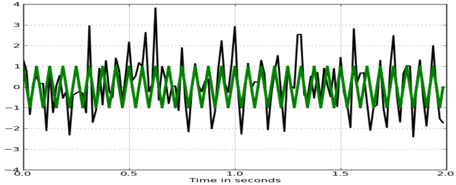
Cos Filter $q(T - t)$
Freq = 10Hz



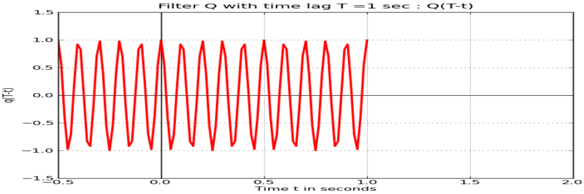
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\, {\color{red} q(T - t)}\, dt$$



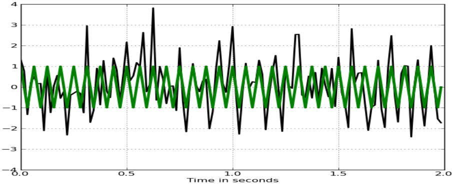
Cos Filter ${\color{red} q(T - t)}$
Freq = 10Hz



Linear Filtering of Data

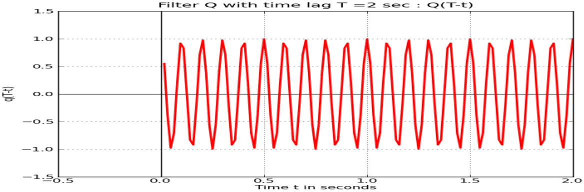
Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\, q(T-t)\, dt$$



Cos Filter $q(T-t)$

Freq = 10Hz



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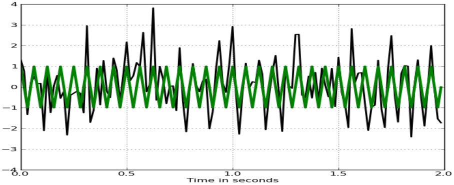

Cos Filter $q(T-t)$
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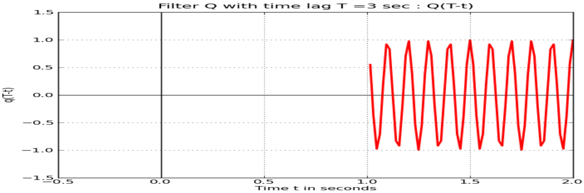

Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

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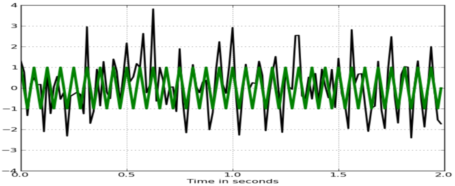
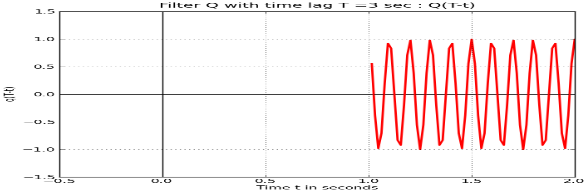
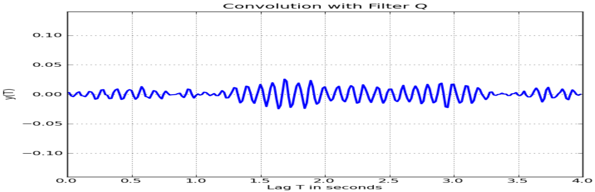


Cos Filter $q(T-t)$
Freq = 10Hz



Filtered Output
$y(T)$



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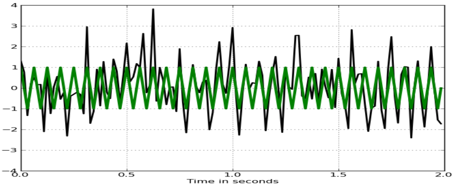
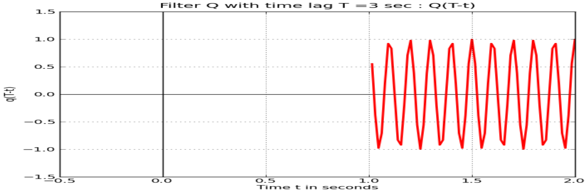
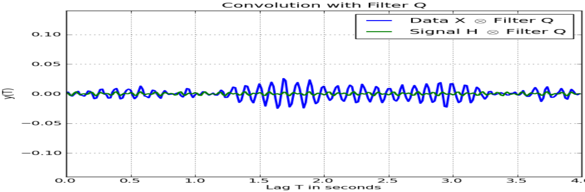


Cos Filter $q(T-t)$
Freq = 10Hz



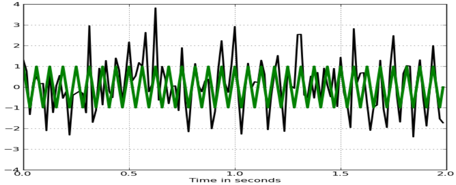
Filtered Output
$y(T)$



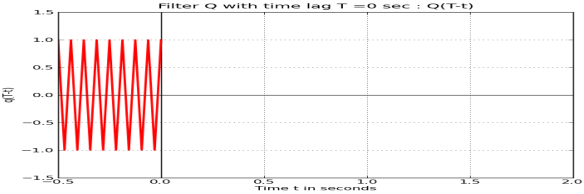
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\, {\color{red} q(T-t)}\, dt$$



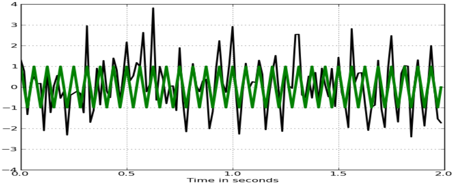
Cos Filter ${\color{red} q(T-t)}$
Freq = 16Hz



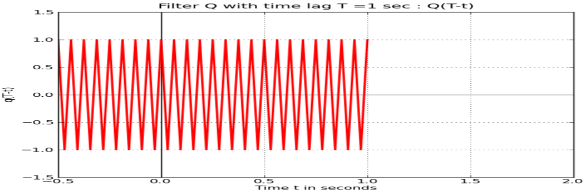
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\, q(T - t)\, dt$$



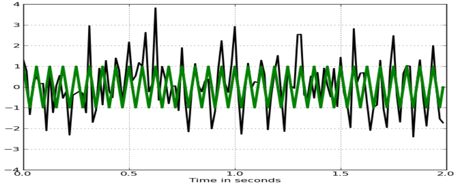
Cos Filter $q(T - t)$
Freq = 16Hz



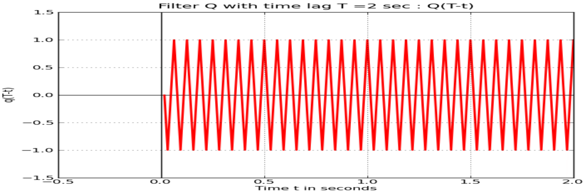
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

$$y(\tau) = \int x(t)\, q(T-t)\, dt$$



Cos Filter $q(T-t)$
Freq = 16Hz



Linear Filtering of Data

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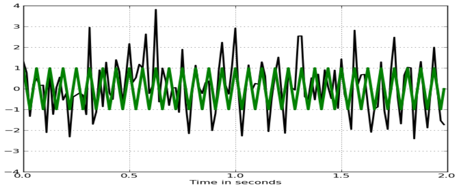

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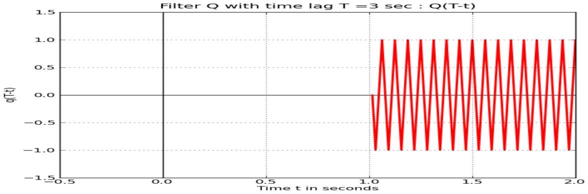

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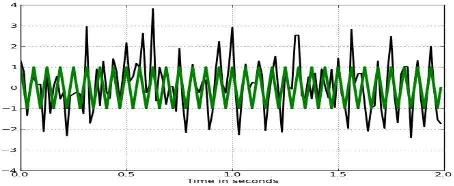

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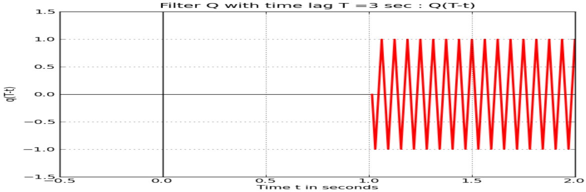
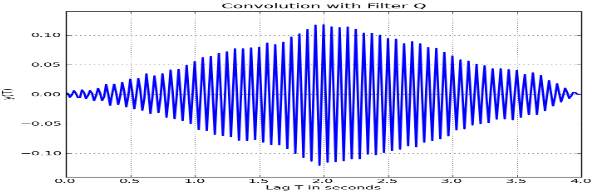


Filtered Output

$y(T)$



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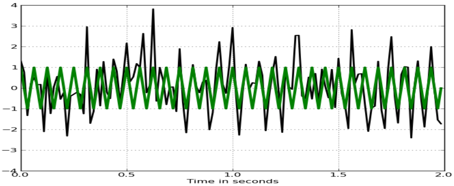

Cos Filter $q(T - t)$

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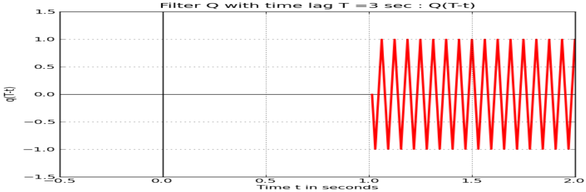
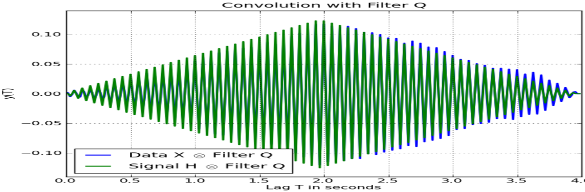


Filtered Output

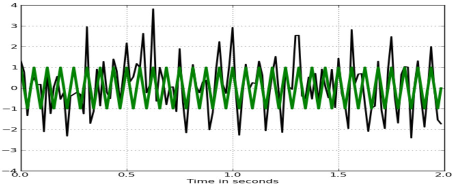
$y(T)$



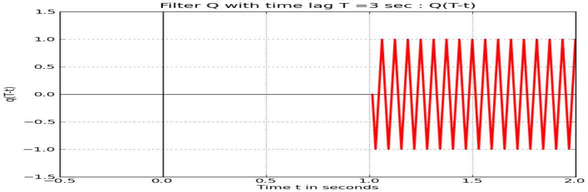
Linear Filtering of Data

Conv of $x(t)$ and $q(t)$:

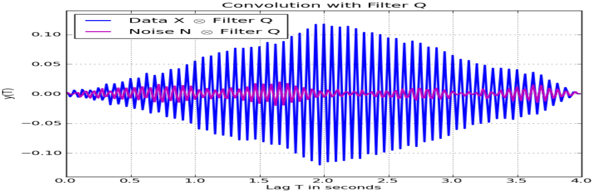
$$y(\tau) = \int x(t)\, {\color{red} q(T - t)}\, dt$$



Cos Filter ${\color{red} q(T - t)}$
Freq = 16Hz



Filtered Output
$y(T)$



Important questions in GW data analysis

Assume: GW signal is hidden in the noise

Important questions in GW data analysis

<u>Assume:</u> GW signal is hidden in the noise

Take home lesson –

Important questions in GW data analysis

Assume: GW signal is hidden in the noise

Take home lesson –

Best filter depends on signal shape

Important questions in GW data analysis

<u>Assume:</u> GW signal is hidden in the noise

Take home lesson –

Best filter depends on signal shape

Phase matching is crucial for signal detection

Important questions in GW data analysis

Assume: GW signal is hidden in the noise

Several Questions

Important questions in GW data analysis

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Several Questions

GW signal – Known/Unknown shape?

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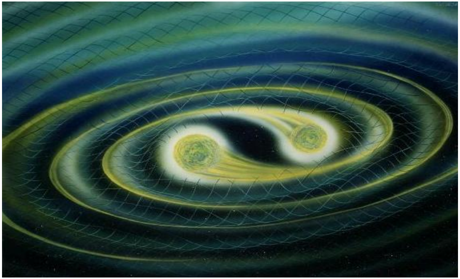


Known signal

Compact Binary Stars

Neutron Star – Black Hole

Neutron Star – Neutron Star

Black hole – Black hole

Important questions in GW data analysis

Assume: GW signal is hidden in the noise

Several Questions

GW signal – Known/Unknown shape?

Known signal
Pulsars – Pulsating Neutron Stars

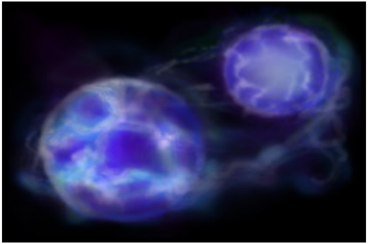
Hester et al, 2003; CXC, HST, NASA

Important questions in GW data analysis

Assume: GW signal is hidden in the noise

Several Questions

GW signal – Known/Unknown shape?



Unknown/Unmodeled signal

Supernova event *[DA IV]*

Accreting systems *[DA IV]*

Important questions in GW data analysis

Assume: GW signal is hidden in the noise

Several Questions

GW signal – Known/Unknown shape?

Signal of known shape

Important questions in GW data analysis

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Signal of known shape

What is the best filter in presence of noise? *[DA III]*

Important questions in GW data analysis

Assume: GW signal is hidden in the noise

Several Questions

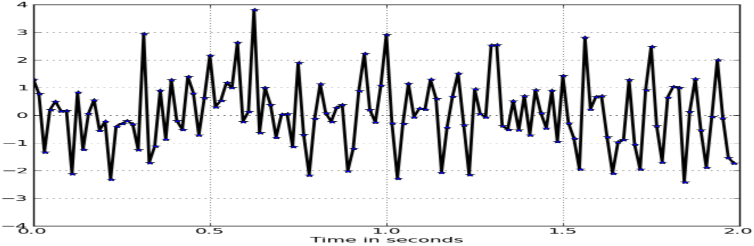
GW signal – Known/Unknown shape?

Signal of known shape

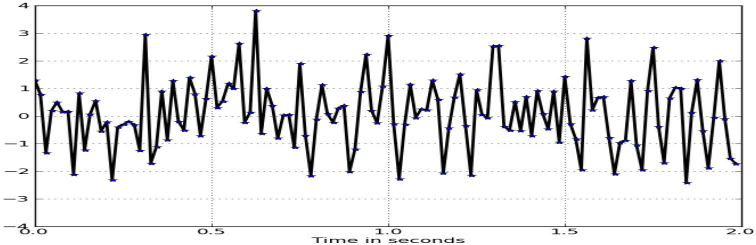
What is the best filter in presence of noise? *[DA III]*

Parameters affect the signal phase? One/Many ?
How filters should be spaced?
Phase mismatch between filter and signal? — *[DA V]*

Revise: Continuous and Discrete Fourier Transform



Revise: Continuous and Discrete Fourier Transform



<u>Continuous Domain</u>

Time series:

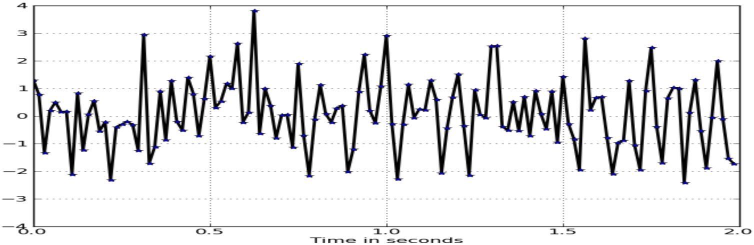
$$x(t) \qquad 0 < t < T$$

<u>Discrete Domain</u>

Sampled Data at $f_{samp} = \Delta^{-1}$,

$$x_j = x(t_j) \quad t_j = 0, \Delta, \ldots, (N-1)\Delta$$

Revise: Continuous and Discrete Fourier Transform



<u>Continuous Domain</u>

Time series:

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FT: $\tilde{x}(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi i f t} dt$

<u>Discrete Domain</u>

Sampled Data at $f_{samp} = \Delta^{-1}$,

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DFT: $\tilde{x}_k = \sum_0^{N-1} x_j e^{-2\pi i j k / N}$

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$$\tilde{x}(f) = \tilde{x}(f_k) \Rightarrow \Delta \times \tilde{x}_k \qquad\qquad f_k = k/T$$

Revise: Continuous and Discrete Fourier Transform

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IFT: $x(t) = \int_{-\infty}^{\infty} \tilde{x}(f) e^{2\pi i f t} df$

DIFT: $x_j = \frac{1}{N} \sum_0^{N-1} \tilde{x}_j e^{2\pi i j k/N}$

Revise: Continuous and Discrete Fourier Transform

<u>Continuous Domain</u>

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FT: $\tilde{x}(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi i f t} dt$

$$\tilde{x}(f) = \tilde{x}(f_k) \Rightarrow \Delta \times \tilde{x}_k$$

IFT: $x(t) = \int_{-\infty}^{\infty} \tilde{x}(f) e^{2\pi i f t} df$

Parseval Theorem:

$$\int x^2(t) dt = \int |\tilde{x}(f)|^2 df$$

<u>Discrete Domain</u>

Sampled Data at $f_{samp} = \Delta^{-1}$,

$$x_j = x(t_j) \quad t_j = 0, \Delta, \ldots, (N-1)\Delta$$

DFT: $\tilde{x}_k = \sum_0^{N-1} x_j e^{-2\pi i jk/N}$

$$f_k = k/T$$

DIFT: $x_j = \frac{1}{N} \sum_0^{N-1} \tilde{x}_j e^{2\pi i jk/N}$

$$N \sum_0^{N-1} y_j \, z_j = \sum_0^{N-1} \tilde{y}_k \, \tilde{z}_k^*$$

Revise: Continuous and Discrete Fourier Transform

<u>Continuous Domain</u>

Time series:

$$x(t) \qquad 0 < t < T$$

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Convolution $Y = X \otimes Q$:

$$y(T) = \int x(t)\, q(T-t) dt \qquad y_j = \sum_{l=0}^{N-1} x_l\, q_{j-l}$$

Noise: White and Colored

What is the color of the noise?

Noise: White and Colored

What is the color of the noise?

White noise $==$ White light

Noise: White and Colored

What is the color of the noise?

White noise $==$ White light

White light $==$ All colors in equal proportions

White noise $==$ All frequencies in equal proportions

Frequency spectrum is flat

Noise: White and Colored

What is the color of the noise?

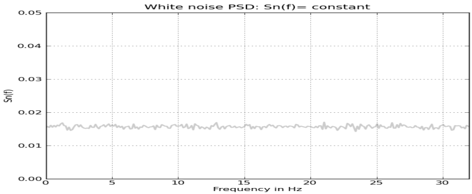
White light $==$ All colors in equal proportions

White noise $==$ All frequencies in equal proportions

Frequency spectrum is flat

Noise Power Spectral Density:

$$Sn(f) = \frac{< |\tilde{n}(f)|^2 >}{T}$$



Application : Elctronic music, Audio testing etc

Noise: White and Colored

What is the color of the noise?

What is the colored noise?

Noise: White and Colored

What is the color of the noise?

What is the colored noise?

How to obtain blue color from white color??

Noise: White and Colored

What is the color of the noise?

What is the colored noise?

Colored noise - Pass white noise through a band-pass filter

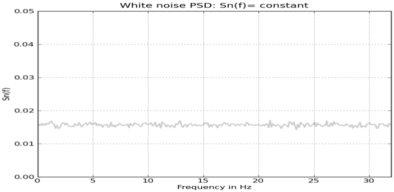
Noise: White and Colored

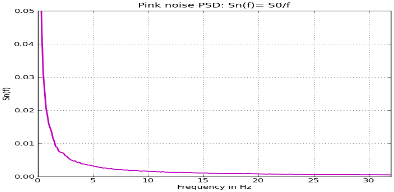
What is the color of the noise?

Colored noise - Pass white noise through a band-pass filter

Example: Power law noise   Noise power spectrum $1/f^{\alpha}$

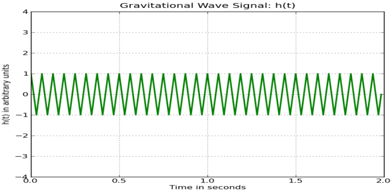
$\alpha = 0, 1, 2, \ldots \Rightarrow$ white, pink, brown, $\ldots$ noise





What is the best filter?

Signal – Known shape



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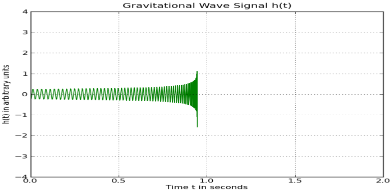

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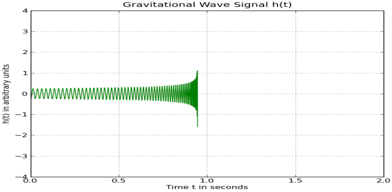
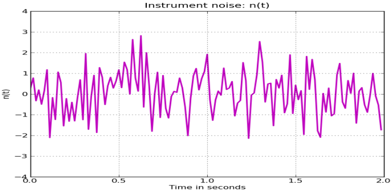
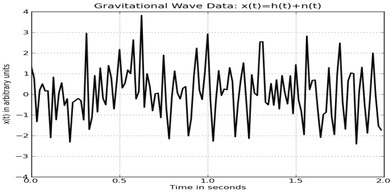


Noise – White



What is the best filter?

Data $x(t) = h(t) + n(t)$
Known signal in Stationary noise



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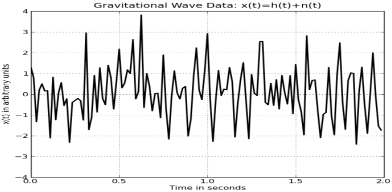

$$Y \equiv X \otimes Q = H \otimes Q + N \otimes Q$$

Signal Noise

What is the best filter?

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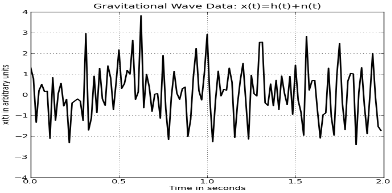

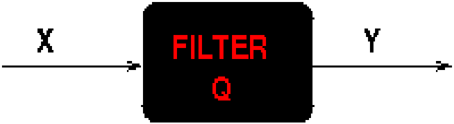

$$Y \equiv X \otimes Q = H \otimes Q + N \otimes Q$$

Signal Noise

Signal-To-Noise Ratio:

$$SNR = \frac{< X \otimes Q >}{\sigma(X \otimes Q)} = \frac{H \otimes Q}{\sigma(N \otimes Q)}$$

What is the best filter?

$\boxed{\text{Signal-To-Noise Ratio:}}$

$$SNR = \frac{< X \otimes Q >}{\sigma(X \otimes Q)} = \frac{H \otimes Q}{\sigma(N \otimes Q)}$$

Which filter function Q optimises the SNR?

What is the best filter?

Signal-To-Noise Ratio:

$$SNR = \frac{< X \otimes Q >}{\sigma(X \otimes Q)} = \frac{H \otimes Q}{\sigma(N \otimes Q)}$$

Recall — $y_j = \sum_{l=0}^{M-1} x_l \, q_{-l+j}$

What is the best filter?

Signal-To-Noise Ratio:

$$SNR = \frac{< X \otimes Q >}{\sigma(X \otimes Q)} = \frac{H \otimes Q}{\sigma(N \otimes Q)}$$

Recall — $y_j = \sum_{l=0}^{M-1} x_l \; q_{-l+j}$
$= \sum_{l=0}^{M-1} x_l \; q^r_{l+j}$

What is the best filter?

Signal-To-Noise Ratio:

$$SNR = \frac{< X \otimes Q >}{\sigma(X \otimes Q)} = \frac{H \otimes Q}{\sigma(N \otimes Q)}$$

$$\text{Recall — } y_j = \sum_{l=0}^{M-1} x_l \; q_{-l+j} = \sum_{l=0}^{M-1} x_l \; q^r_{l+j} = \frac{1}{M} \sum_{k=0}^{M-1} \tilde{x}_k \; \tilde{q}^{r*}_k e^{2\pi j \; k/M}$$

What is the best filter?

Signal-To-Noise Ratio:

$$SNR = \frac{< X \otimes Q >}{\sigma(X \otimes Q)} = \frac{H \otimes Q}{\sigma(N \otimes Q)}$$

$$\text{Recall — } y_j = \sum_{l=0}^{M-1} x_l \; q_{-l+j}$$

$$= \sum_{l=0}^{M-1} x_l \; q^r_{l+j}$$

$$= \frac{1}{M} \sum_{k=0}^{M-1} \tilde{x}_k \; \tilde{q}^{r*}_{k} e^{2\pi j \; k/M}$$

$$Y = X \otimes Q = \frac{1}{M}(\tilde{X} \cdot \tilde{Q}^s)$$

What is the best filter?

$\boxed{\text{Signal-To-Noise Ratio:}}$

$$SNR = \frac{< X \otimes Q >}{\sigma(X \otimes Q)} = \frac{H \otimes Q}{\sigma(N \otimes Q)}$$

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$$SNR^2 = \frac{(\tilde{H} \cdot \tilde{Q}^s)^2}{< (\tilde{N} \cdot \tilde{Q}^s)^2 >}$$

What is the best filter?

Signal-To-Noise Ratio:

$$SNR = \frac{< X \otimes Q >}{\sigma(X \otimes Q)} = \frac{H \otimes Q}{\sigma(N \otimes Q)}$$

$$Y = X \otimes Q = \frac{1}{M}(\tilde{X} \cdot \tilde{Q}^s)$$

$$SNR^2 = \frac{(\tilde{H} \cdot \tilde{Q}^s)^2}{< (\tilde{N} \cdot \tilde{Q}^s)^2 >} = \frac{(\sum_{k=0}^{M-1} \tilde{h}_k \; \tilde{q}_k^*)^2}{\sum_{l=0}^{M-1} S_k \; |\tilde{q}_k|^2}$$

Use: Stationary $< \tilde{n}_k \tilde{n}_k'^* >= S_k \delta(k, k')$

What is the best filter?

Signal-To-Noise Ratio:

$$SNR = \frac{< X \otimes Q >}{\sigma(X \otimes Q)} = \frac{H \otimes Q}{\sigma(N \otimes Q)}$$

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Use: Stationary $< \tilde{n}_k \tilde{n}_k'^* >= S_k \delta(k, k')$

$$SNR^2 = \left(\sum_{k=0}^{M-1} \frac{\tilde{h}_k}{\sqrt{S_k}} \;\; \underline{\tilde{q}_k^*} \right)^2 = (A \cdot \underline{B})^2$$

What is the best filter?

Signal-To-Noise Ratio:

$$SNR = \frac{< X \otimes Q >}{\sigma(X \otimes Q)} = \frac{H \otimes Q}{\sigma(N \otimes Q)}$$

$$SNR^2 = \frac{(\tilde{H} \cdot \tilde{Q}^s)^2}{< (\tilde{N} \cdot \tilde{Q}^s)^2 >} = \frac{(\sum_{k=0}^{M-1} \tilde{h}_k \; \tilde{q}_k^*)^2}{\sum_{l=0}^{M-1} S_k \; |\tilde{q}_k|^2}$$

Use: Stationary $< \tilde{n}_k \tilde{n}_k'^* >= S_k \delta(k, k')$

$$SNR^2_{max} = \sum_{k=0}^{M-1} \frac{|\tilde{h}_k|^2}{S_k} \qquad \textit{Matched filter} : -\tilde{q}_k \propto \frac{\tilde{h}_k}{S_k}$$

What is the best filter?

$$SNR^2_{max} = \sum_{k=0}^{M-1} \frac{|\tilde{h}_k|^2}{S_k} \qquad Matched \;\; filter: \;\; \tilde{q}_k = M\frac{\tilde{h}_k}{S_k}$$

$$Y = X \otimes Q == y_j = \sum_{k=0}^{M-1} \tilde{x}_k \, \frac{\tilde{h}_k^*}{S_k} e^{2\pi jk/M}$$

Matched filter SNR

$$SNR_{MF}^2 = \sum_{k=0}^{M-1} \frac{|\tilde{h}_k|^2}{S_k} \qquad \textit{Matched filter}: \quad \tilde{q}_k = M \frac{\tilde{h}_k}{S_k}$$

- Noise PSD $S_k = E(|\tilde{x}_k|^2)$ — Measure of spectral noise variance
- White: All frequencies are present $(S_k = S_0)$; $q_k \propto h_k$.
- Colored: Noise variance is a function of frequency.

  $$\text{Small } S_k \Rightarrow \text{small variance.}$$

  More SNR contribution with small S_k frequencies.
  Demonstrates how noise distribution limits the sensitivity.
- Band-pass operation: Allows data to pass through a frequency band given by the instrument (S_k).

Matched filter SNR and Signal duration

$$SNR_{MF}^2 = \sum_{k=0}^{M-1} \frac{|\tilde{h}_k|^2}{S_k} = \int_{-\infty}^{\infty} \frac{|\tilde{h}(f)|^2}{S(f)} df \qquad S(f) = \frac{T}{M^2} S_k$$

Matched filter SNR and Signal duration

$$SNR^2_{MF} = \sum_{k=0}^{M-1} \frac{|\tilde{h}_k|^2}{S_k} = \int_{-\infty}^{\infty} \frac{|\tilde{h}(f)|^2}{S(f)} df \qquad S(f) = \frac{T}{M^2} S_k$$

Sinusiod signal $h(t) = A \cos(2\,\pi\, fo\; t) \qquad 0 < t < T$

Matched filter SNR and Signal duration

$$SNR^2_{MF} = \sum_{k=0}^{M-1} \frac{|\tilde{h}_k|^2}{S_k} = \int_{-\infty}^{\infty} \frac{|\tilde{h}(f)|^2}{S(f)} df \qquad S(f) = \frac{T}{M^2} S_k$$

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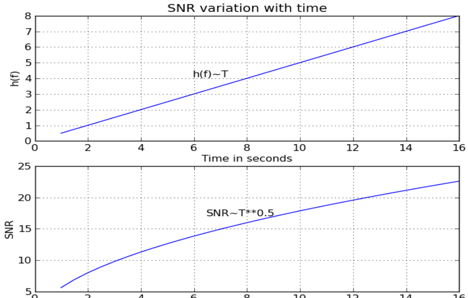

$$|h(f)|^2 \sim A^2\ T^2 \quad df = T^{-1}$$

$$|h(f)|^2 df \sim A^2\ T$$

$$SNR_{MF} \propto \sqrt{T}$$
$$\propto \sqrt{\#\ cycles}$$

$$A\sqrt{T}\ \ vs\ \ \sqrt{S(f)}$$

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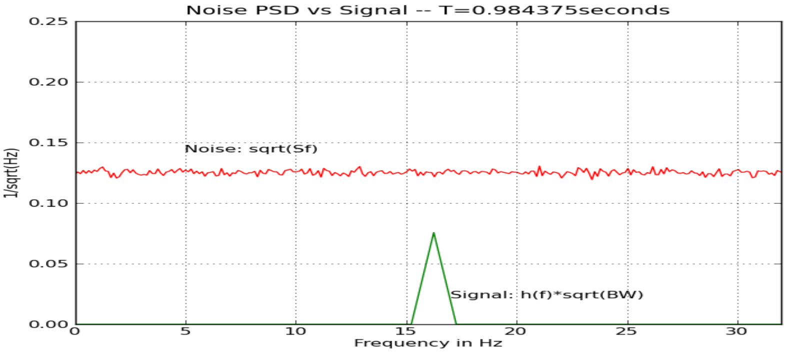

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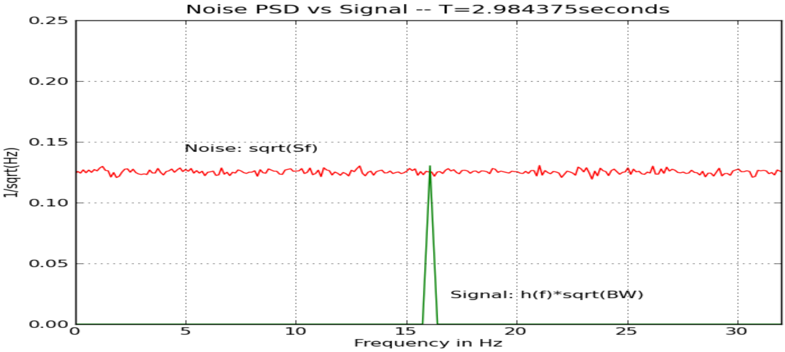

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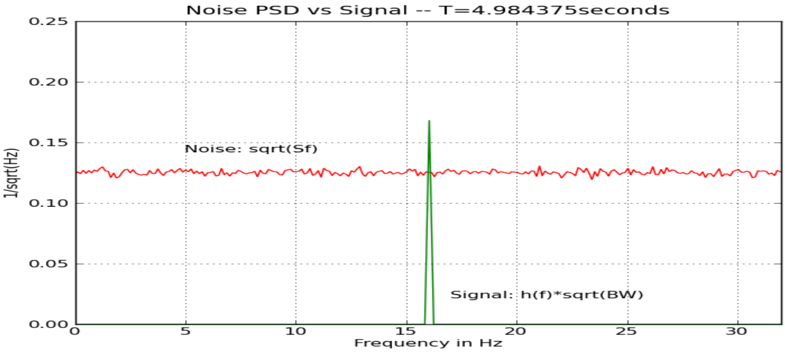

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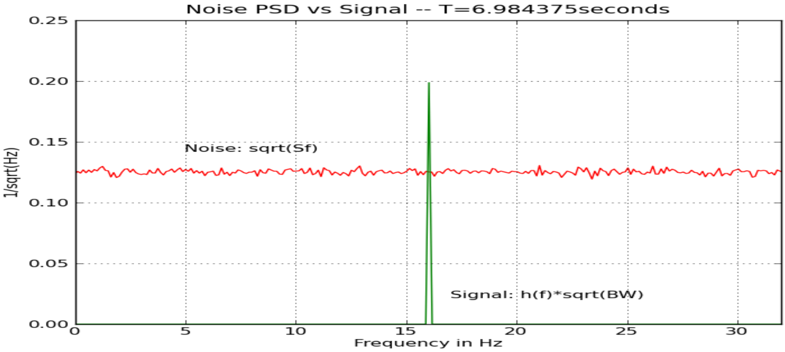

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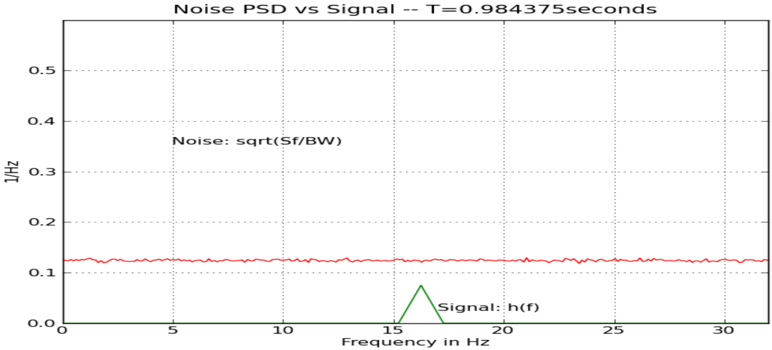

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Noise is incoherent (random walk) – $\tilde{n}(f) \propto \sqrt{T}$

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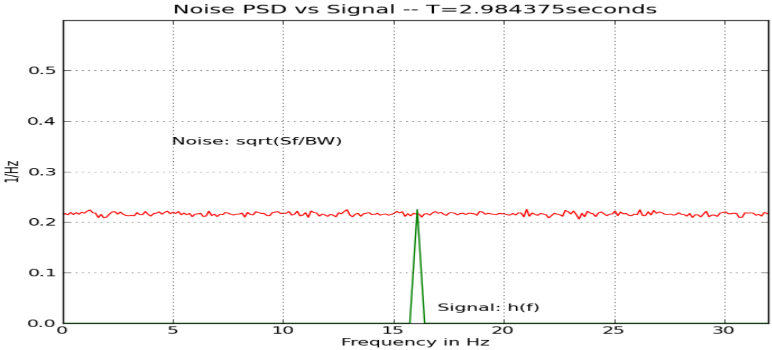

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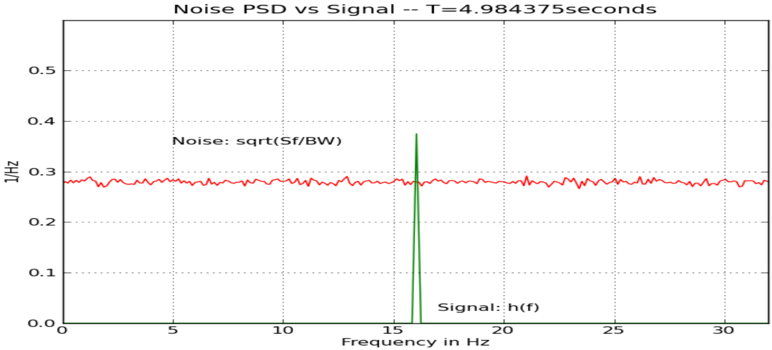

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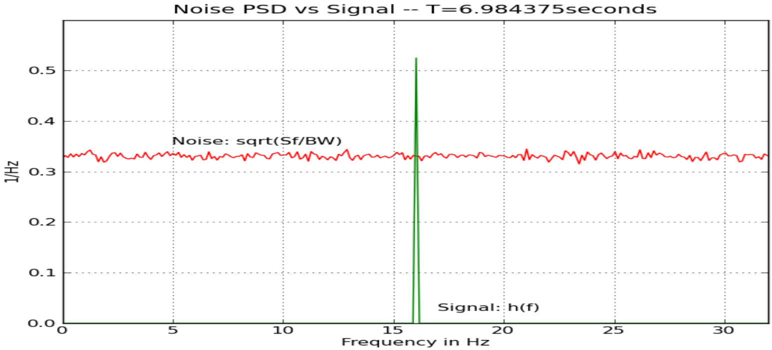

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Matched filtering: Inspiral waveforms

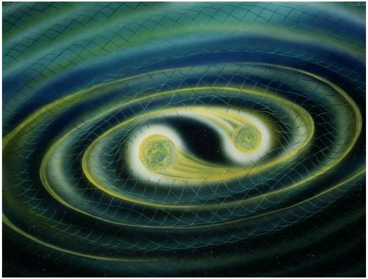
Compact binaries with NS, BH

$h_+ \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \cos(2\pi \int f(t)dt)$

$h_\times \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \sin(2\pi \int f(t)dt)$

Chirp mass $\mathcal{M} = [\mu^3 M^2]^{1/5}$

Freq. $f \propto \mathcal{M}^{-5/8}(t_{coal} - t)^{-3/8}$



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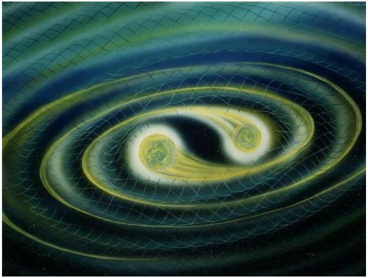

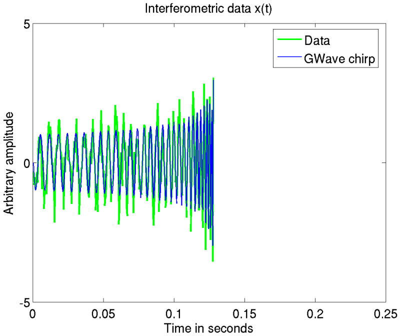

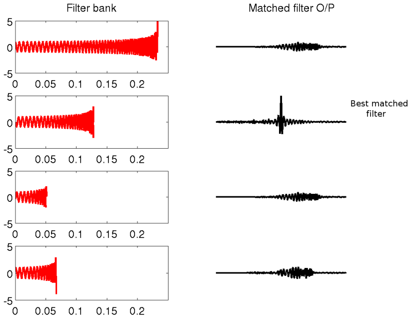

Signal identification issues

Illustration:

Pick 10 random digits between 0-9

S1 – 3 4 6 2 1 0 9 8 2 6

S2 – 1 4 5 8 2 1 0 3 7 2

S3 – 1 2 3 4 0 2 4 7 8 3 .

.

S98 – 3 2 6 2 1 0 7 8 3 6

S99 – 1 3 5 8 1 1 0 5 7 8

S100 – 0 3 5 2 1 2 3 4 0 2

Illustration:

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Prob (1 2 3 4)? == 7 *in* 10 *thousand*

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2 in 100 instead of 7 in 10000 – ??

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Prob (1 0)? == 0.09

Signal identification issues

Strategy –

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Searching multi-parameter signal increases the false alarm rate

Time-Frequency analysis