Gravitational Wave Astronomy With

LISA

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IISER-Kolkata

Over View

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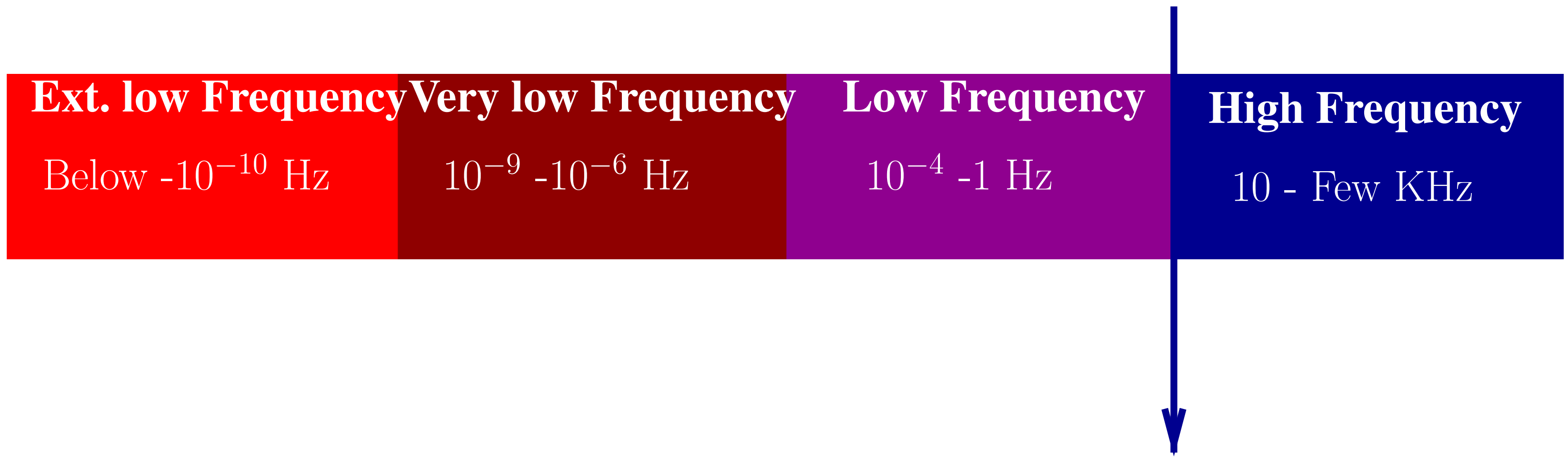
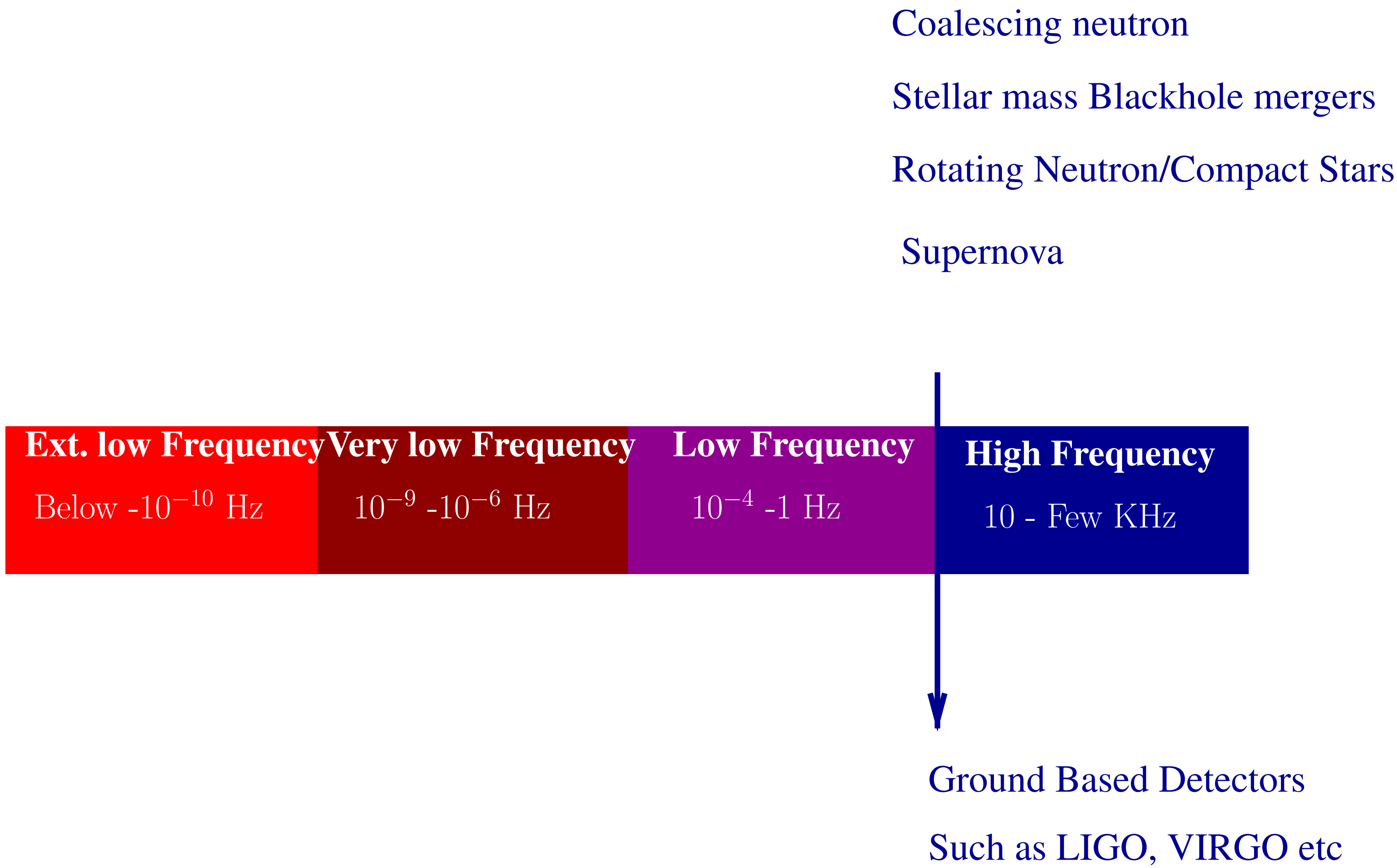
- ☞ Time-Delay interferometer.



Ext. low Frequency
Below -10^{-10} Hz
Very low Frequency
10^{-9} -10^{-6} Hz
Low Frequency
10^{-4} -1 Hz
High Frequency
10 - Few KHz


Coalescing neutron
Stellar mass Blackhole mergers
Rotating Neutron/Compact Stars
Supernova
Ext. low Frequency
Below -10^{-10} Hz
Very low Frequency
10^{-9} -10^{-6} Hz
Low Frequency
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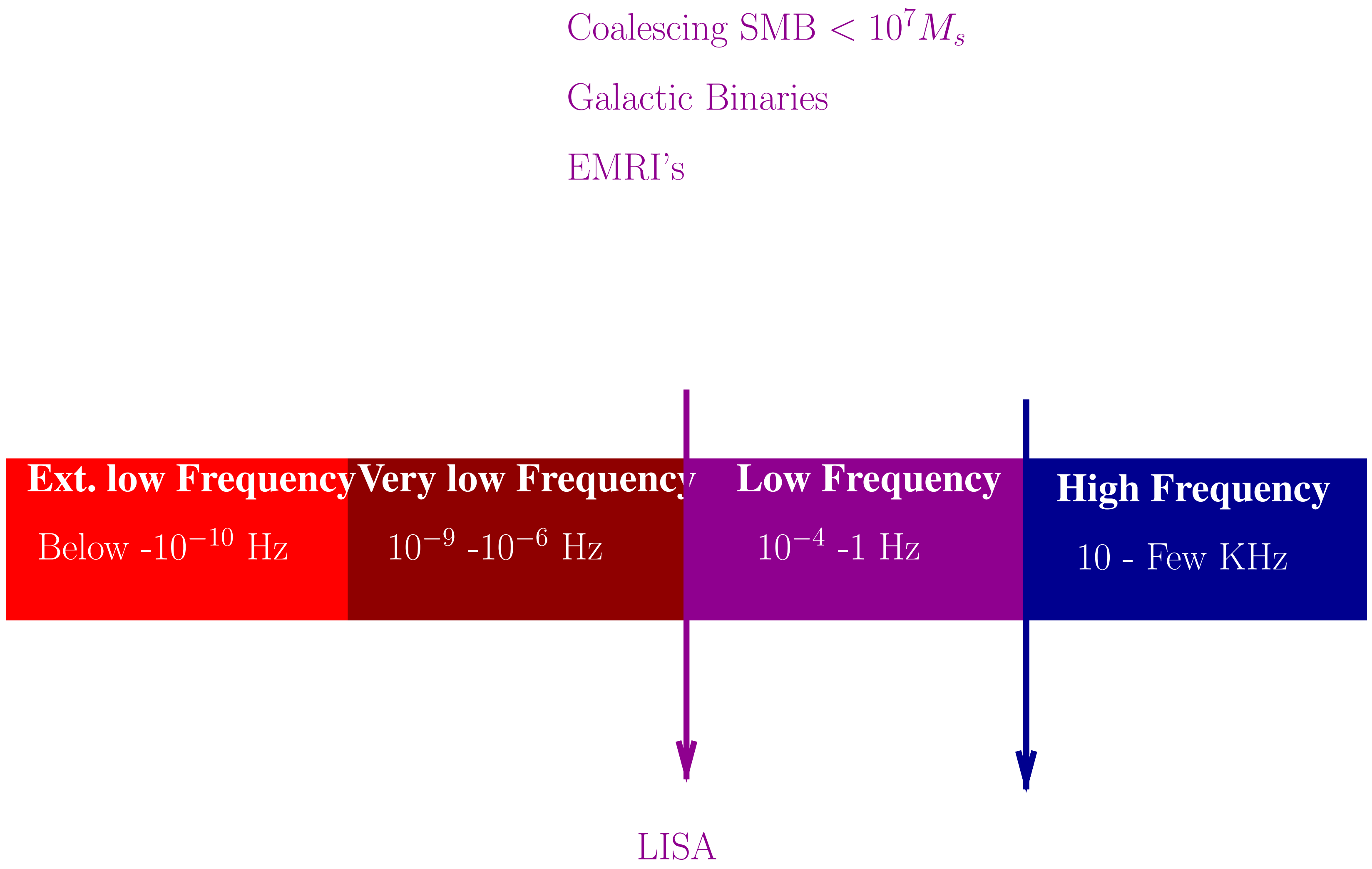
High Frequency
10 - Few KHz
Ground Based Detectors
Such as LIGO, VIRGO etc



Coalescing SMB $< 10^7 M_s$
Galactic Binaries
EMRI's
Ext. low Frequency
Below -10^{-10} Hz
Very low Frequency
10^{-9} -10^{-6} Hz
Low Frequency
10^{-4} -1 Hz
High Frequency
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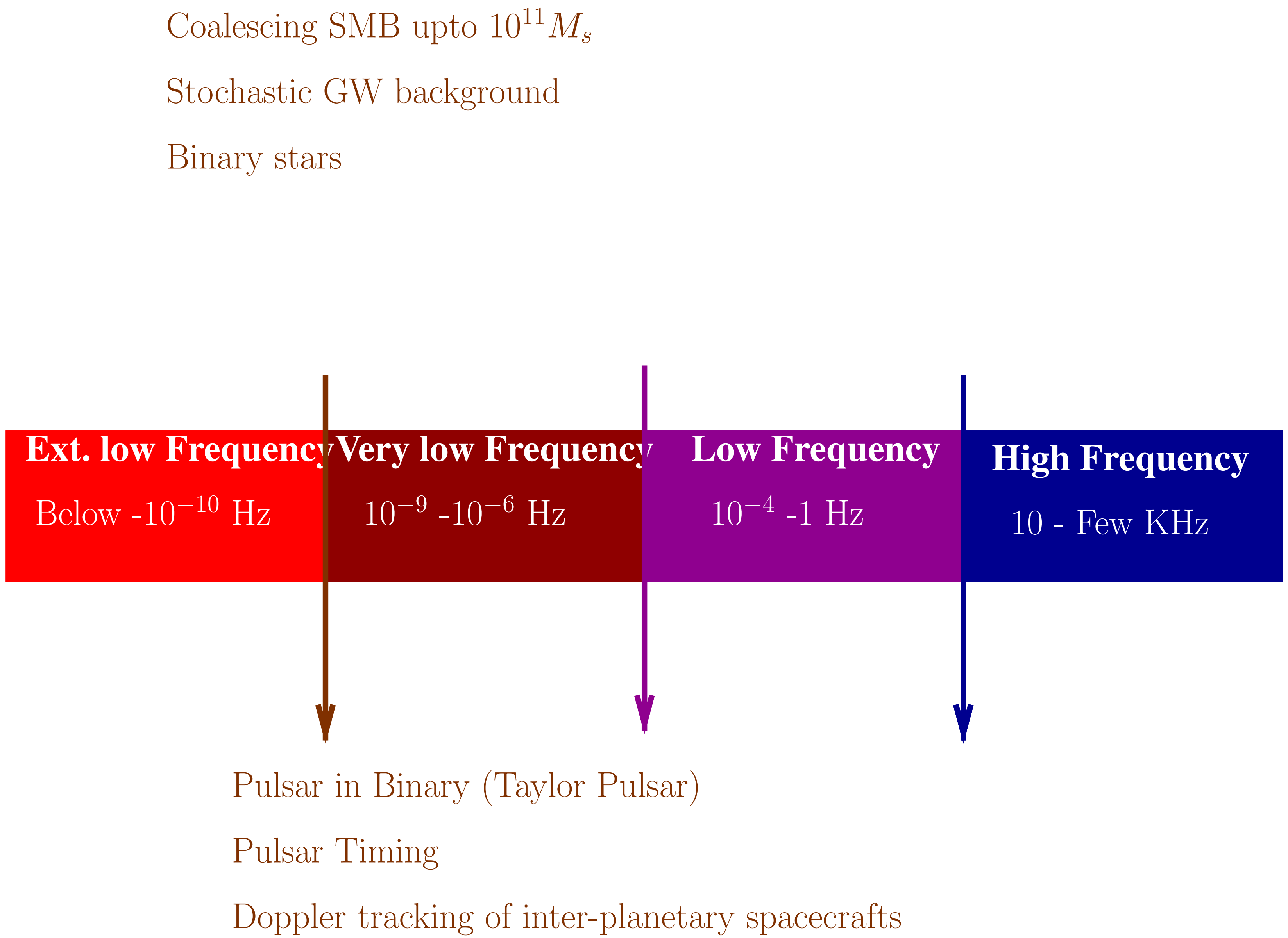
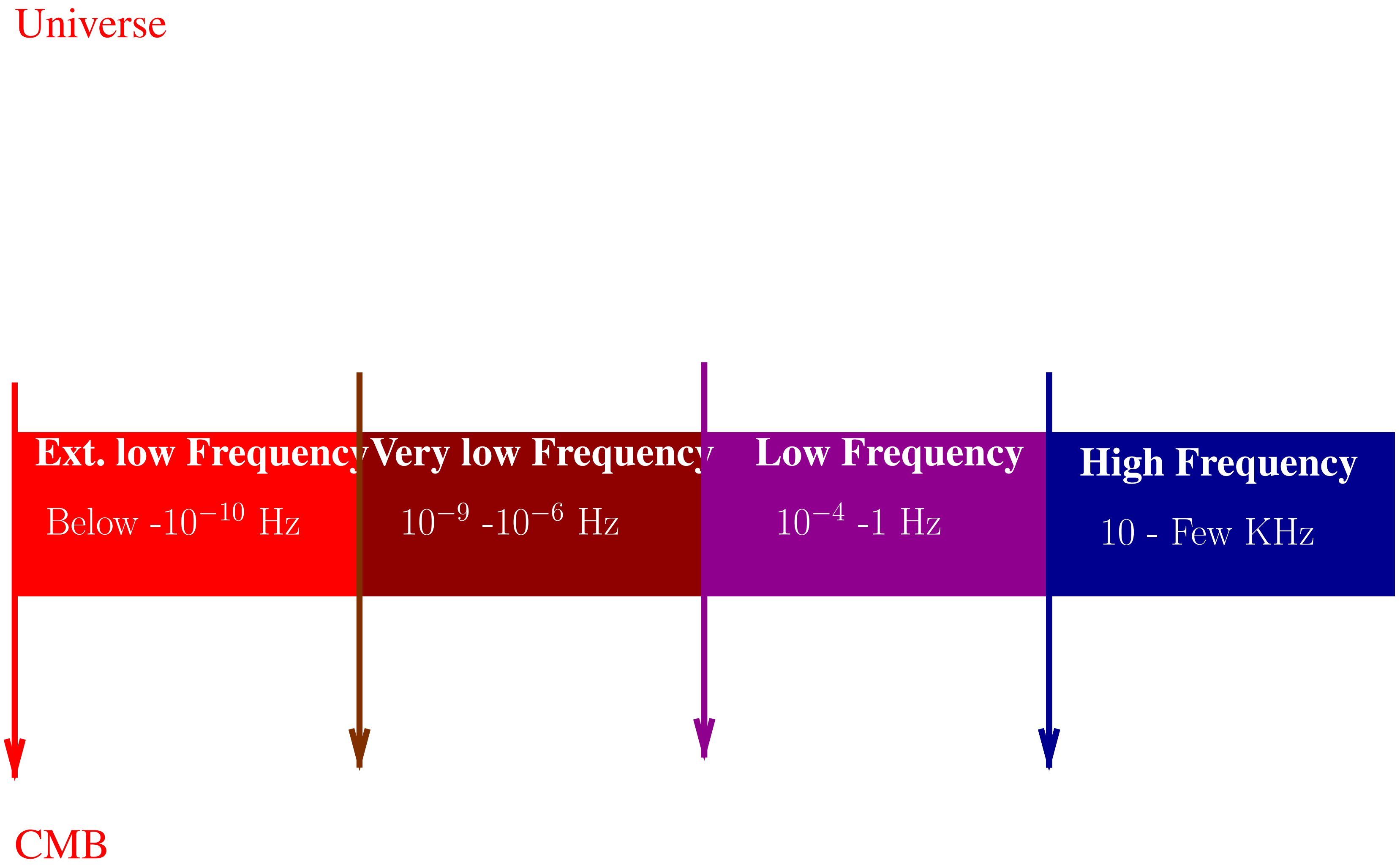
LISA

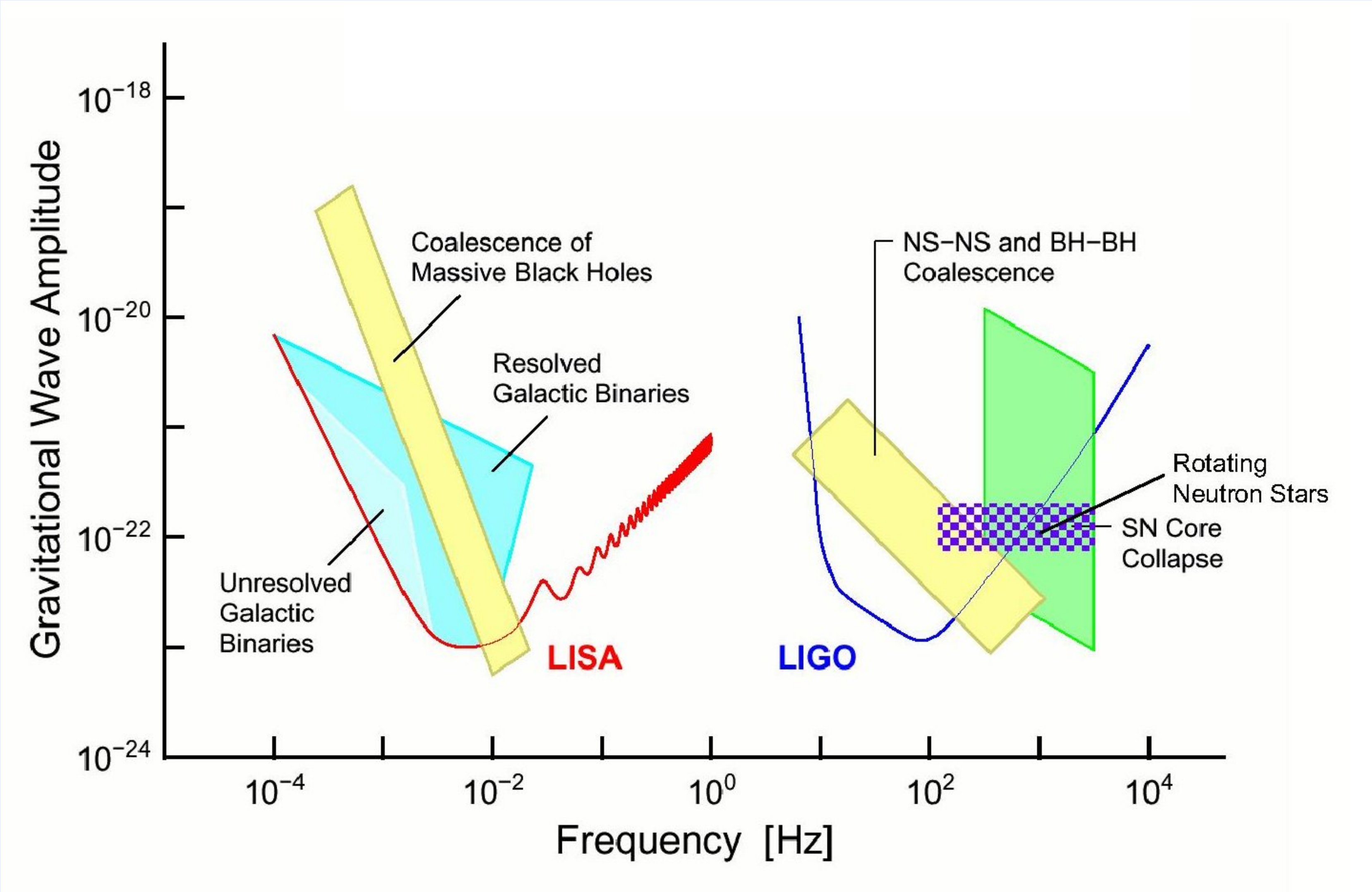

Coalescing SMB upto $10^{11} M_s$
Stochastic GW background
Binary stars
Ext. low Frequency
Below -10^{-10} Hz
Very low Frequency
10^{-9} -10^{-6} Hz
Low Frequency
10^{-4} -1 Hz
High Frequency
10 - Few KHz
Pulsar in Binary (Taylor Pulsar)
Pulsar Timing
Doppler tracking of inter-planetary spacecrafts



Universe
Ext. low Frequency
Below -10^{-10} Hz
Very low Frequency
10^{-9} -10^{-6} Hz
Low Frequency
10^{-4} -1 Hz
High Frequency
10 - Few KHz
CMB

Sensitivity Curve

☞ LISA is sensitive to gravitational waves in the frequency range 10^{-4} Hz to 1 Hz.



Galactic Binaries

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- In our Galaxy there 10^{11} stars and about 50% of them are binary system.

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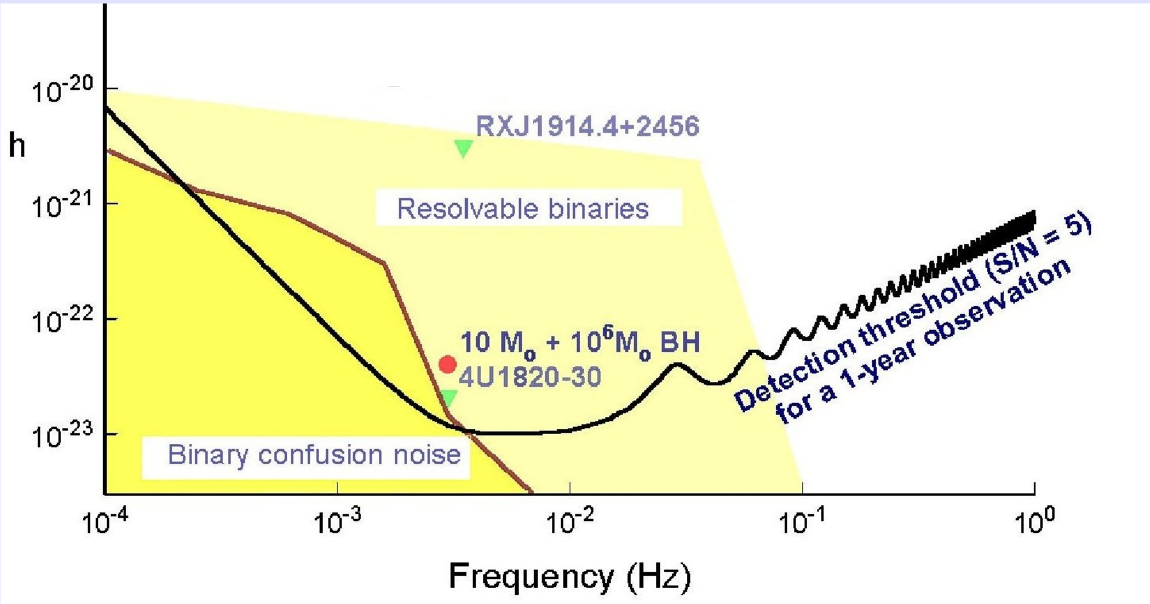

Sensitivity curve for binary confusion noise

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- ☞ LISA observation can give distribution of them in our Galaxy and one can constrain the Galactic models based on LISA observations.

Super massive blackhole binaries

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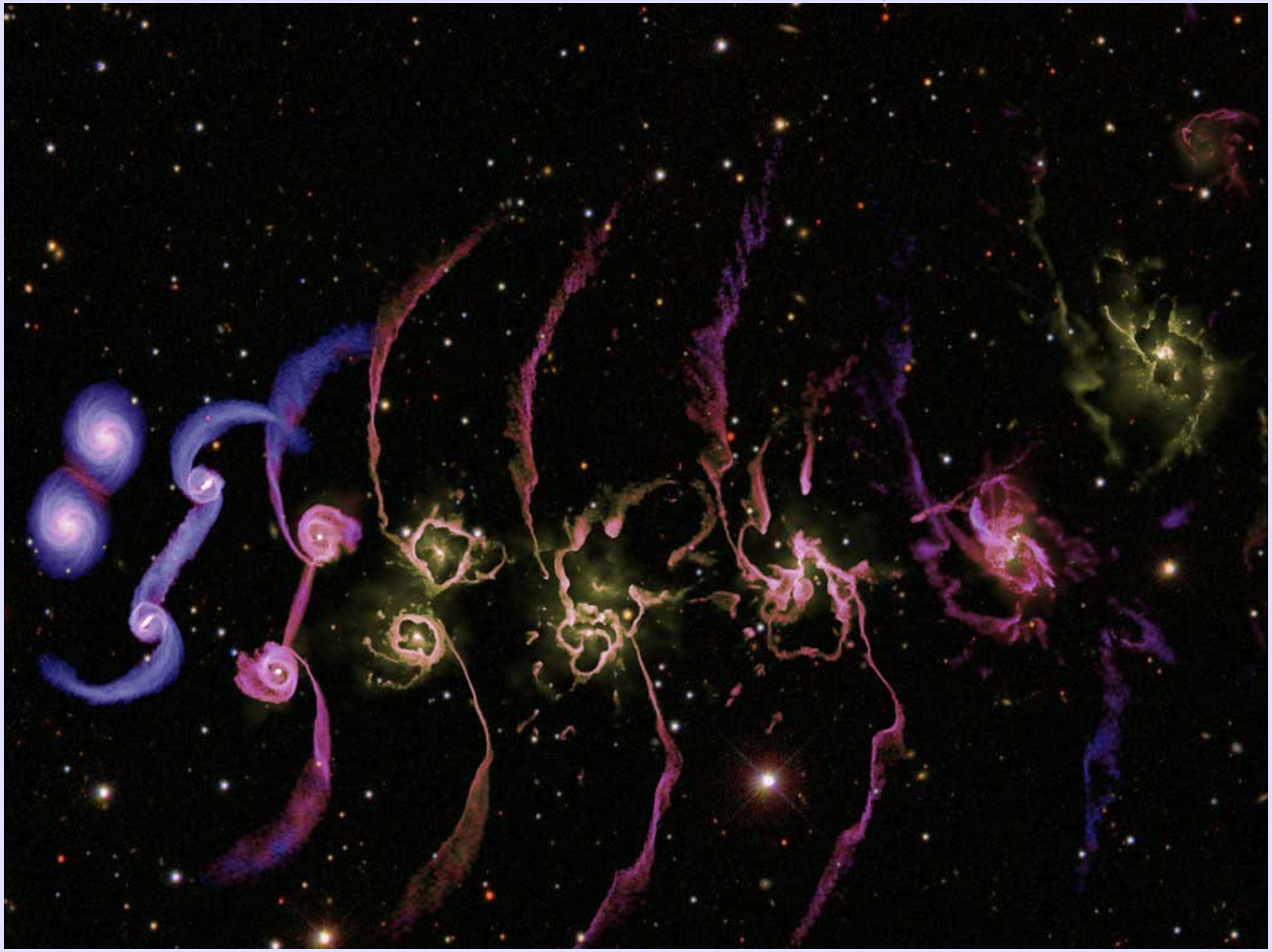
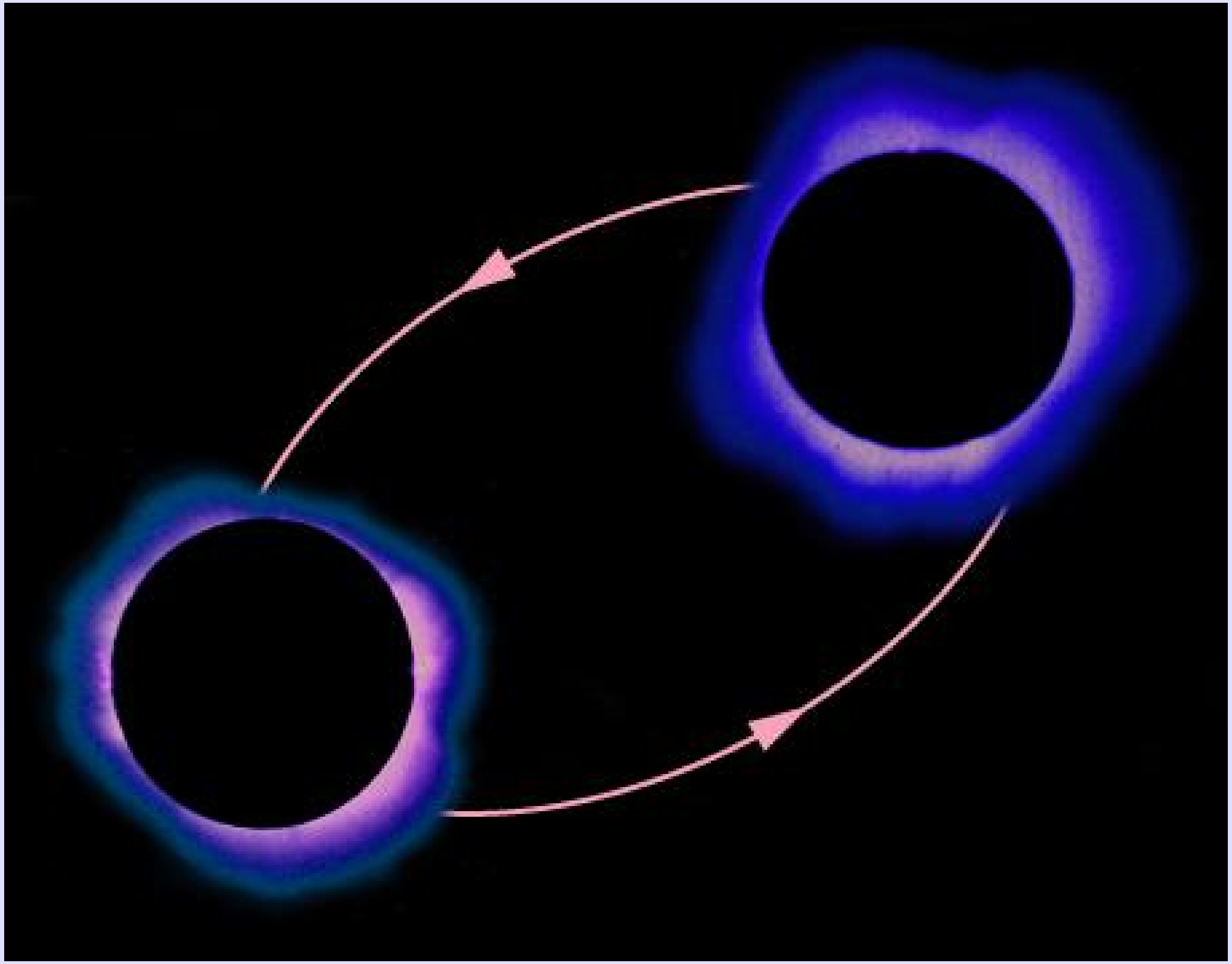
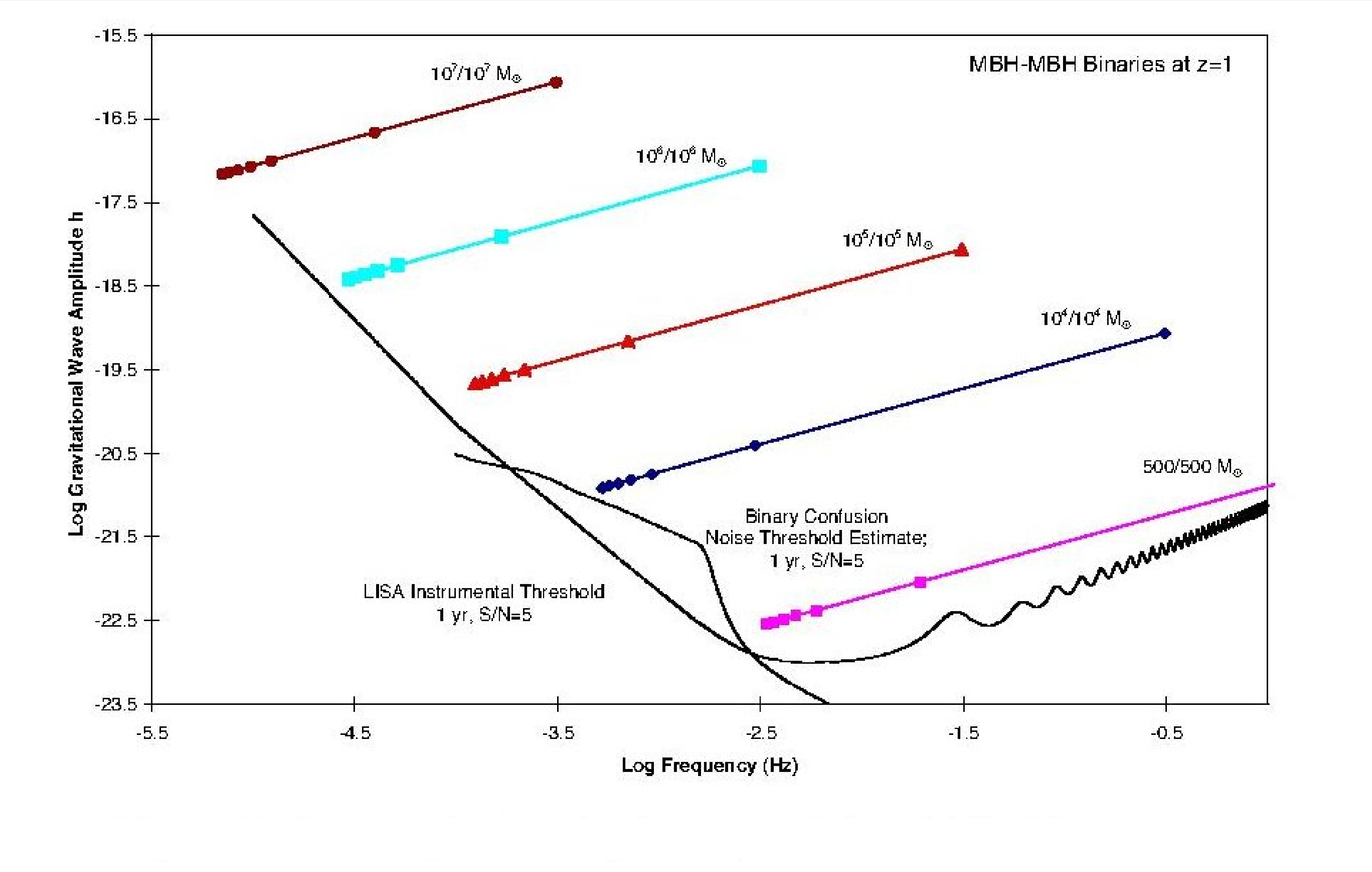
- ☞ It is seen that Galaxies collide,



Binary blackhole's are one of promising sources in the LISA band, can seen almost any where in the Universe.



Sensitivity curve for blackhole binaries:



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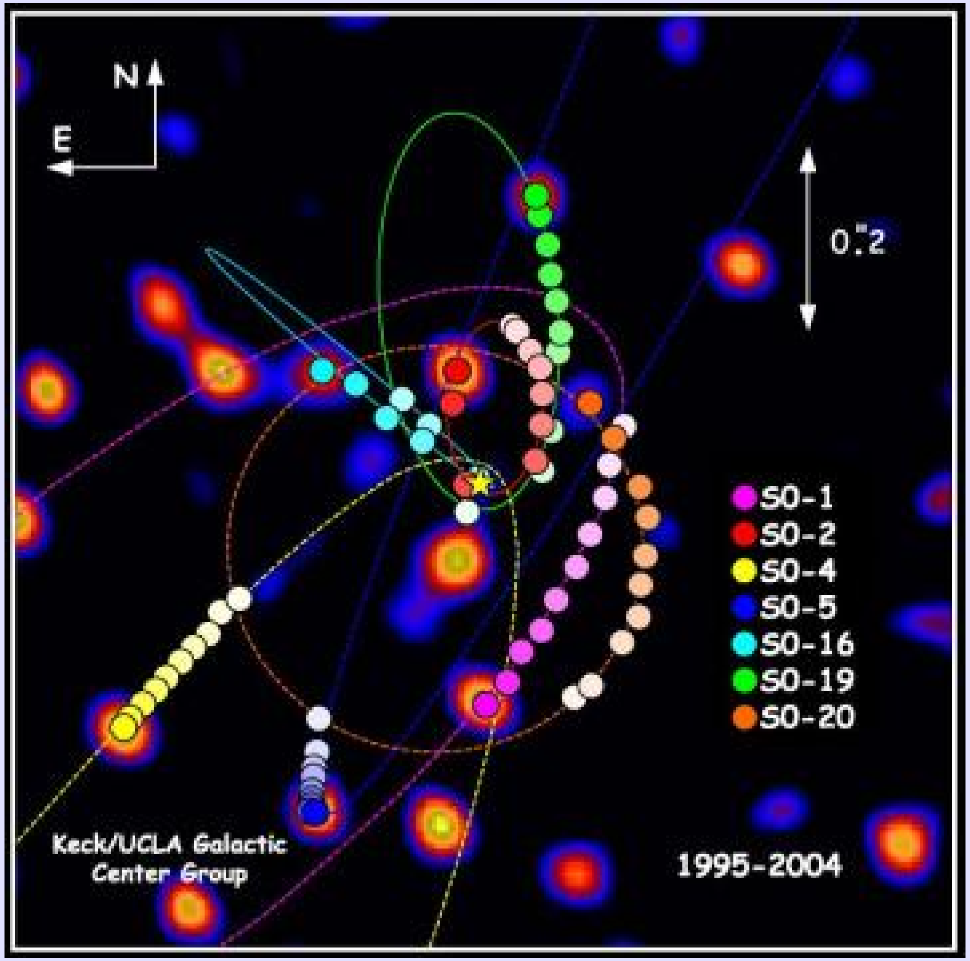
☞ They serve as test for Einstein theory in the strong field limit.

☞ Can be used to test the alternative theory of gravity.

Extreme mass ratio insprial(EMRI's)

☞ Galactic centers, such as our Galaxy hosts blackhole of mass $10^5\ M_{\odot}$. Some of the stars orbiting around them can fall in to blackholes emitting gravitational waves:



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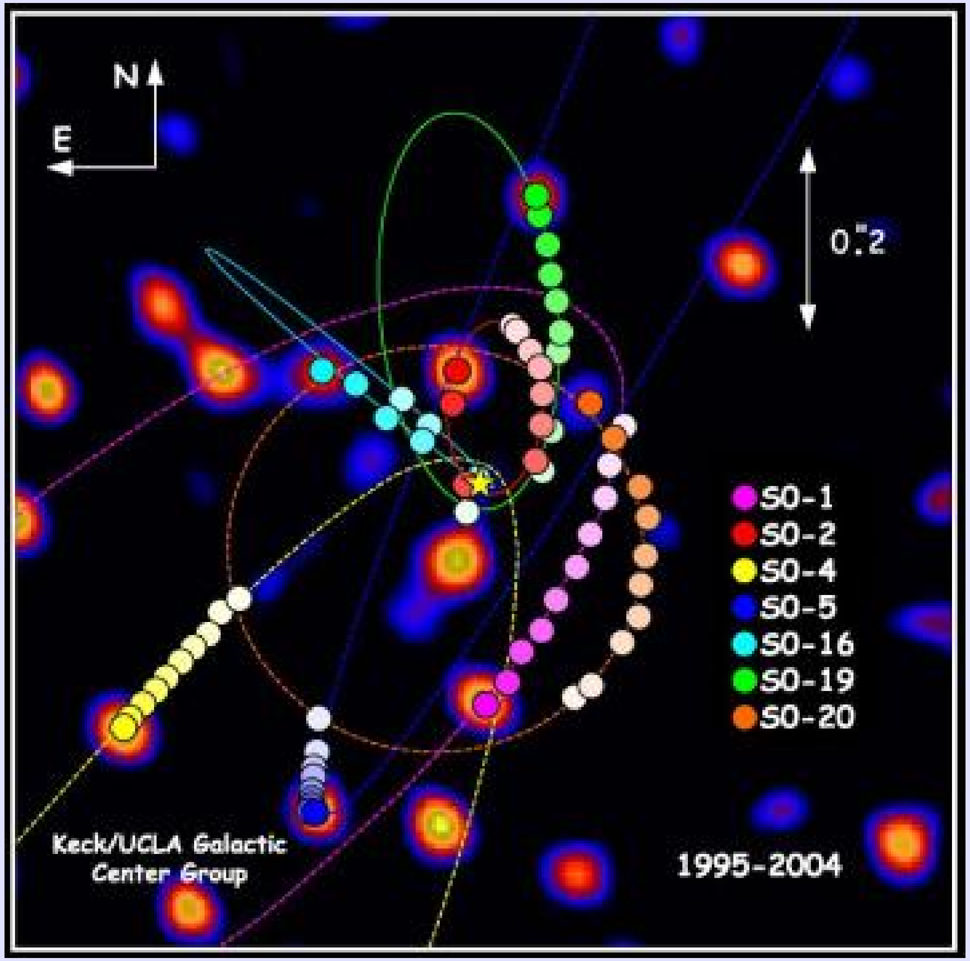
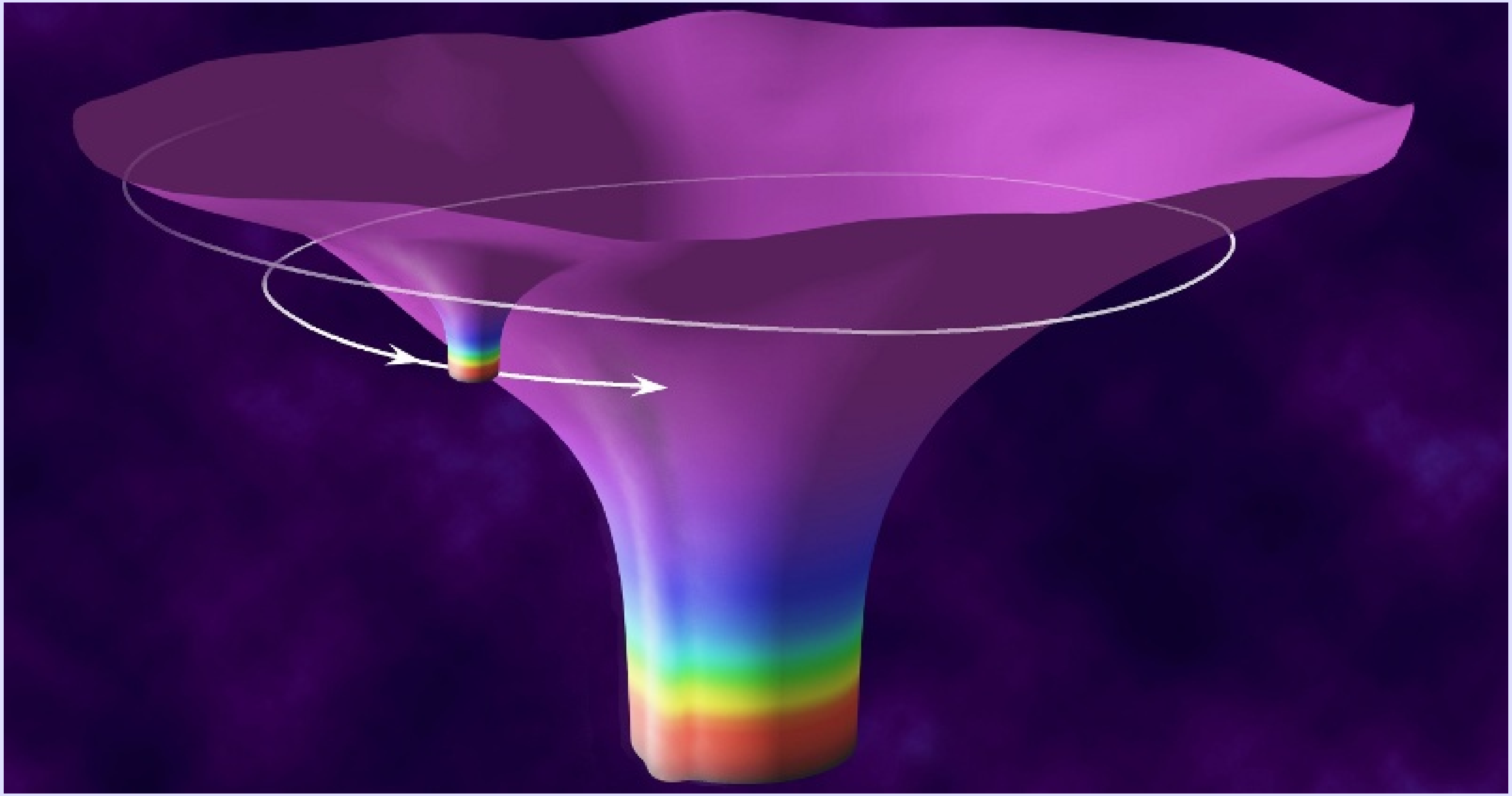
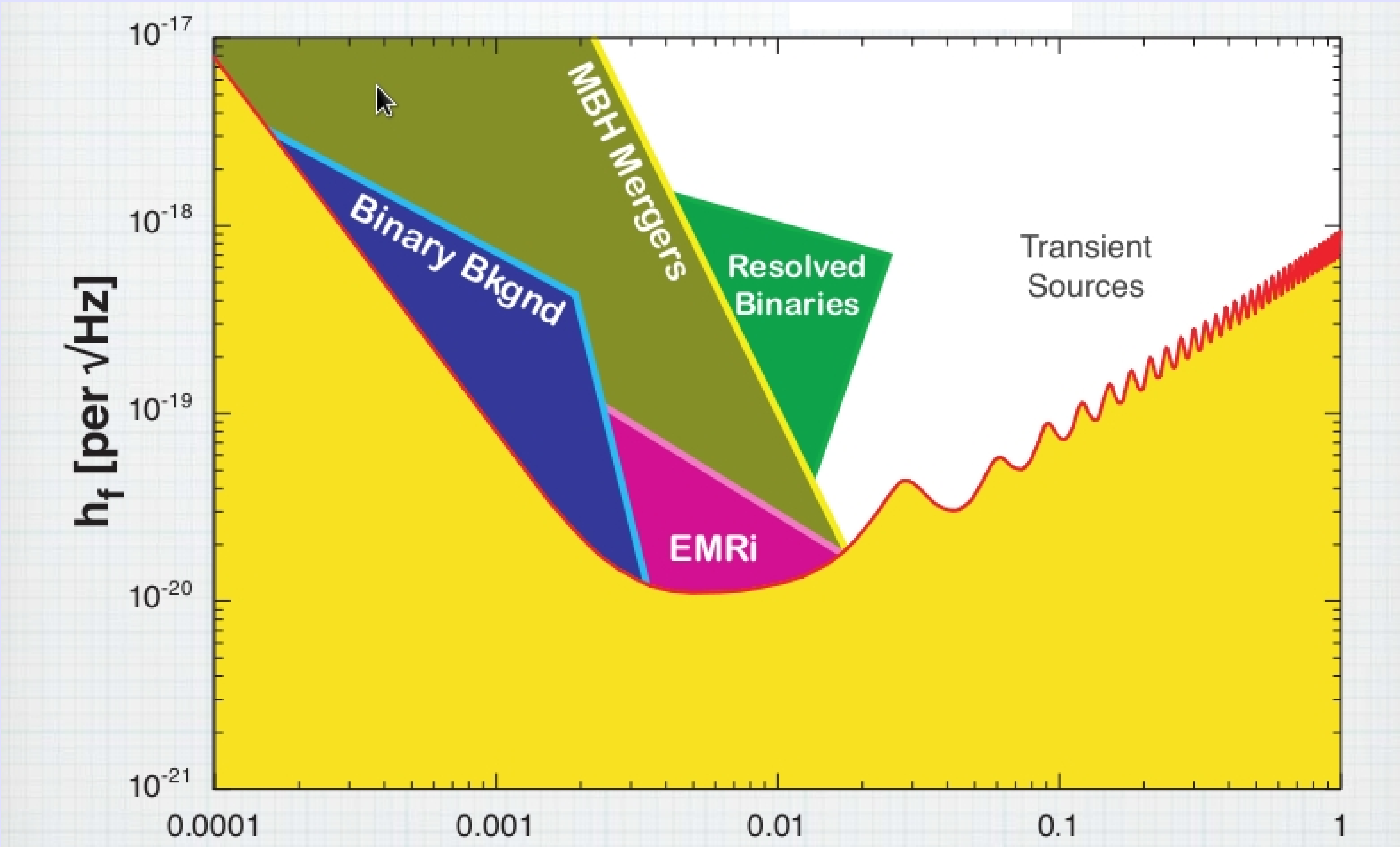


☞ Or one can have blackholes of mass $1 - 10\ M_\odot$ falling into central blackhole

☞ We have small objects of mass 1 to $50\ M_\odot$ falling in to black-holes of mass $10^6\ M_\odot$ is called EMRI



Sensitivity curve for EMRI:



In order to observer astrophysically important sources such as these, one needs a space mission such as LISA.

Mission overview

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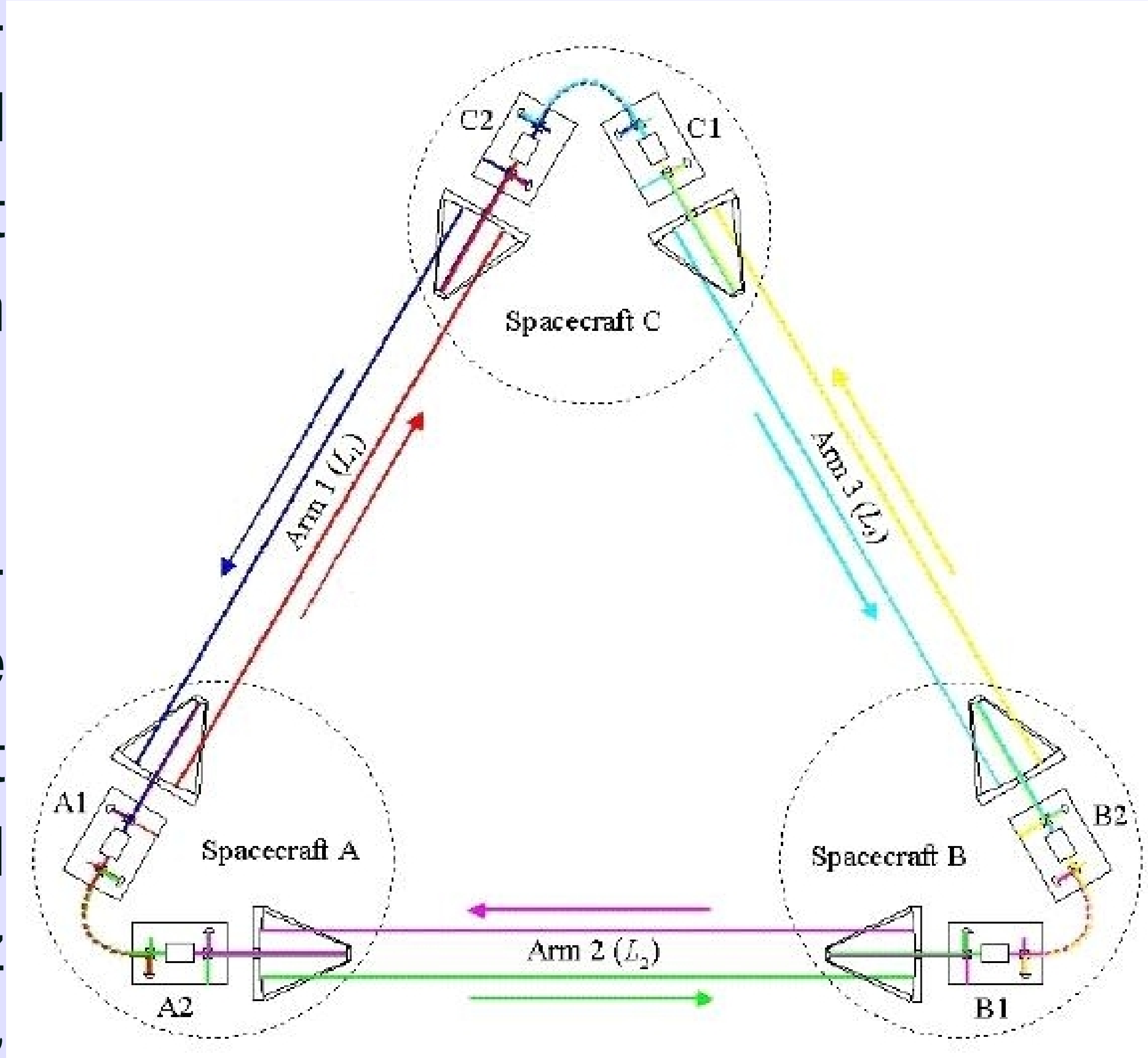
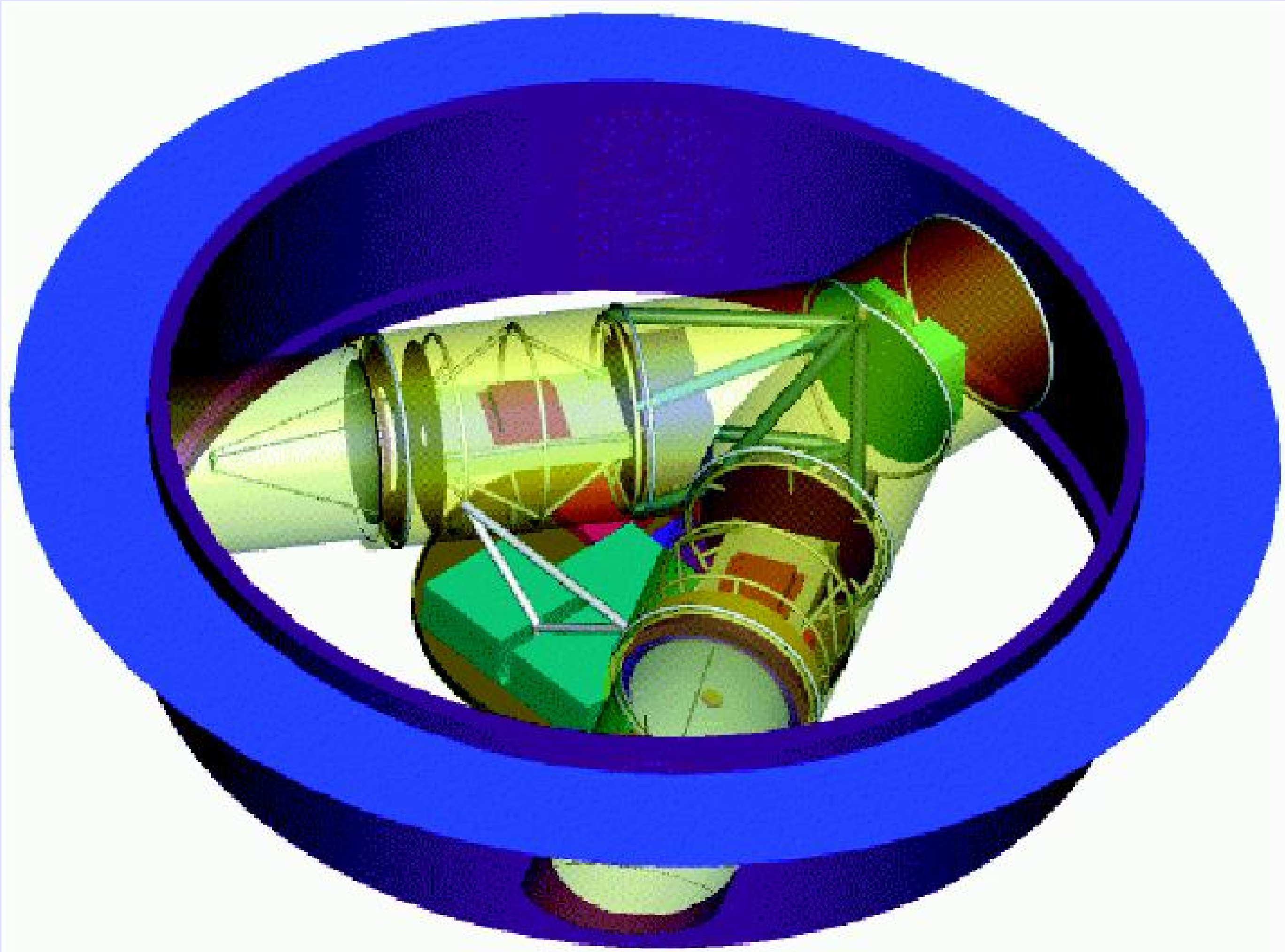
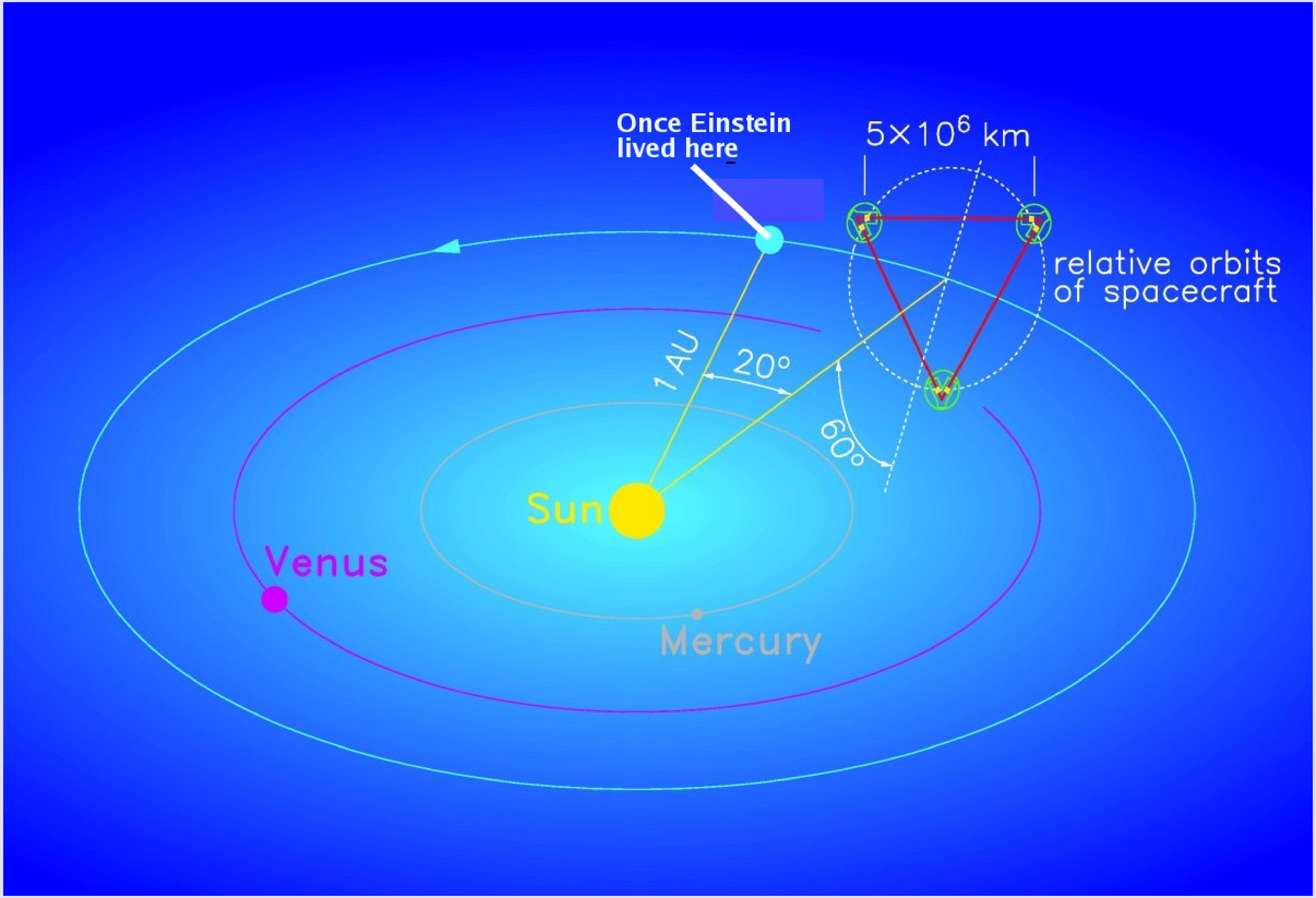
- Six laser beams are exchanged between these three spacecraft to detect gravitational wave signal in the band 10^{-4} - 1 Hz with peak sensitivity of $h \sim 10^{-23}$.



Each spacecraft hosts, two test masses and optical system to communicate with the distance spacecraft



LISA orbits

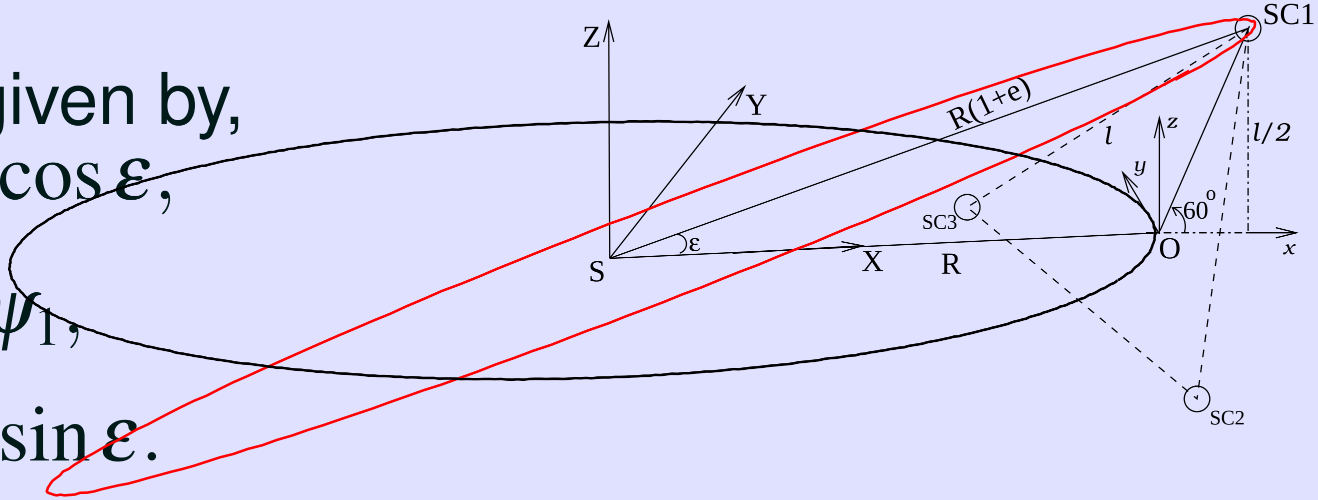


Once Einstein lived here
5×10^6 km
relative orbits of spacecraft
1AU
20°
60°
Sun
Venus
Mercury

LISA orbits

- ☞ Each of the LISA spacecraft go around Sun, in elliptical orbits such that entire formation remain almost equilateral triangle.
- ☞ The plane of the LISA triangle makes an angle of 60^o with the ecliptic plane.
- ☞ The center of this LISA constellation moves around the Sun in an earth-like orbit (R = 1AU), 20^o behind the Earth.
- ☞ The triangular constellation revolves once around its center as it completes one orbit in the course of one year.

A choice of orbits



A choice of orbits are given by,

$$X_1 = R(\cos\psi_1 + e)\cos\varepsilon,$$
$$Y_1 = R\sqrt{1-e^2}\sin\psi_1,$$
$$Z_1 = R(\cos\psi_1 + e)\sin\varepsilon.$$

$$\tan\varepsilon = \frac{\alpha}{1+\alpha/\sqrt{3}}, \qquad \alpha = \frac{l}{2R}$$

$$e = \left(1 + \frac{2}{\sqrt{3}}\alpha + \frac{4}{3}\alpha^2\right)^{1/2} - 1$$

$$\psi_1 + e\sin\psi_1 = \Omega t$$

Orbits for other two spacecraft can obtained by rotating this by 180^o about $z - axis$

CW-Frame and Equations

Clohessy and Wiltshire or Hill's equations are linearised dynamical equations for test-particles in the neighborhood of reference point, in our case the LISA centroid. These equations are written in a frame which has its origin on the reference orbit and also rotates with angular velocity Ω. The equation for a free test particle are given by,

$$\begin{aligned}
\ddot{x} - 2\Omega\dot{y} - 3\Omega^2 x &= 0, \\
\ddot{y} + 2\Omega\dot{x} &= 0, \\
\ddot{z} + \Omega^2 z &= 0.
\end{aligned}$$

The solutions:

$$x(t) = \frac{\dot{x}_0}{\Omega}\sin\Omega t - \left(3x_0 + \frac{2\dot{y}_0}{\Omega}\right)\cos\Omega t + 2\left(2x_0 + \frac{\dot{y}_0}{\Omega}\right)$$

$$y(t) = \left(6x_0 + \frac{4\dot{y}_0}{\Omega}\right)\sin\Omega t + \frac{2\dot{x}_0}{\Omega}\cos\Omega t - 3\left(2\Omega x_0 + \dot{y}_0\right)t + \left(y_0 - \frac{2\dot{x}_0}{\Omega}\right)$$

$$z(t) = z_0\cos\Omega t + \frac{\dot{z}_0}{\Omega}\sin\Omega t$$

Ignoring runaway solutions and offset solution the condition for stable configuration is given by

$$z_0 = \mu\sqrt{3}x_0 \qquad \text{and} \qquad \frac{\dot{z}_0}{\Omega} = \frac{1}{2}\mu\sqrt{3}y_0, \qquad \mu = \pm 1$$

The solutions are given by,

$$
\begin{aligned}
x(t) &= \frac{1}{2}\rho_0 \cos\left(\Omega t - \phi_0\right), \\
y(t) &= -\rho_0 \sin\left(\Omega t - \phi_0\right), \\
z(t) &= \mu \rho_0 \frac{\sqrt{3}}{2} \cos\left(\Omega t - \phi_0\right),
\end{aligned}
$$

where

$$
\rho_0 = \sqrt{4x_0^2 + y_0^2} \qquad \tan\phi_0 = \frac{y_0}{2x_0}
$$

The orbits given earlier when when approximated to first order in $\alpha = l/(2R)$ and transformed to CW frame we get:

$$
\begin{aligned}
x_k &= eR\cos\left[\Omega t - (k-1)\frac{2\pi}{3}\right] \\
y_k &= -2eR\sin\left[\Omega t - (k-1)\frac{2\pi}{3}\right] \\
z_k &= \sqrt{3}eR\cos\left[\Omega t - (k-1)\frac{2\pi}{3}\right]
\end{aligned}
$$

we identify $\rho_0 = 2eR$ and $\phi_0 = 2\pi(k-1)/3$

The general result is

In the CW frame there are just two planes which make angles of $\pm\pi/3$ with the (x-y) plane, in which test particles obeying CW equations perform rigid rotations about the origin with angular velocity $-\Omega$.

S. V. Dhurandhar et. al, “Fundamentals of the LISA Stable Flight Formation”, *Class. Quantum Grav.* **22** 481(2005).

Time-Delay interferometer

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- ☞ For locking Michelson like interferometer, distance between the end points must be integral multiple of $\frac{\lambda}{2}$, which is impossible in space.
- ☞ Because different lasers are used at eand points, laser noise play important role.

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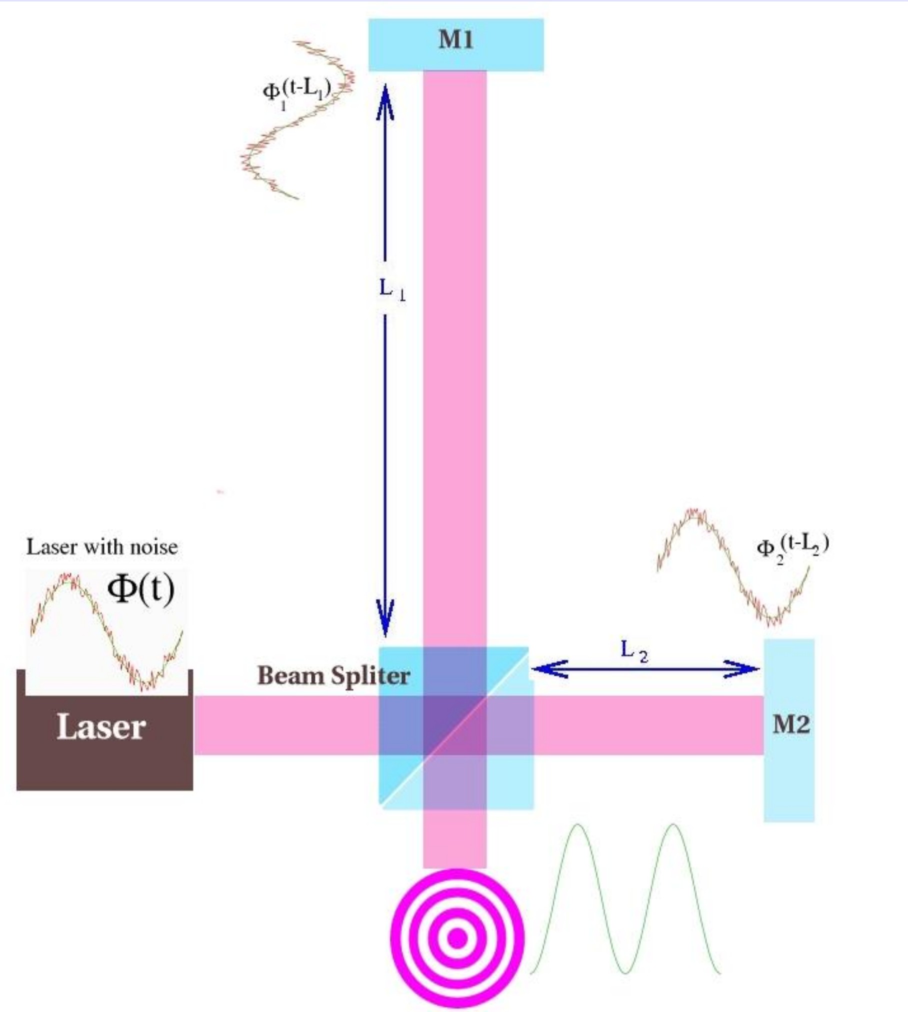
- ☞ In the case of ground based detectors, where both arms are exactly of same length (integral multiple of $\frac{\lambda}{2}$)and the laser noise cancels out.

- ☞ Time-Delay interferometry is one of novel technique suggested by Armstrong et. al (J.W. Armstrong, F.B Estabrook and M. Tinto, *Astrophys. J.* **527**, 814(1999).) for constructing unequal arm interferometer in space

- ☞ In this method the individual beams are combined off-line after introducing suitable time delay corresponding to the light travel time across the arms to simulate the interferometer.

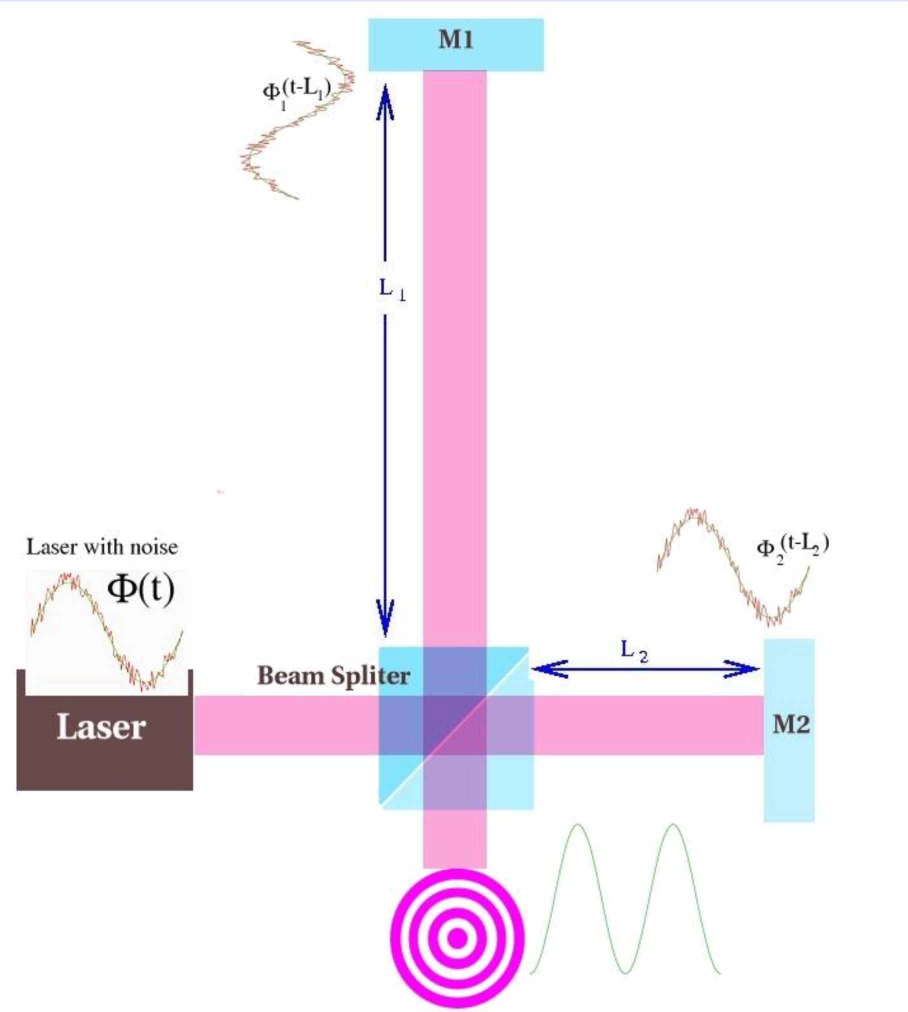
Unequal Arm Interferometer

Unequal Arm Interferometer



Let $\Phi(t)$ be the Phase fluctuation of laser

Unequal Arm Interferometer



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If $L_1 \neq L_2$

$$\begin{aligned} \Phi_1(t) &= \Phi(t - 2L_1) - \Phi(t) \\ \Phi_2(t) &= \Phi(t - 2L_2) - \Phi(t) \end{aligned}$$

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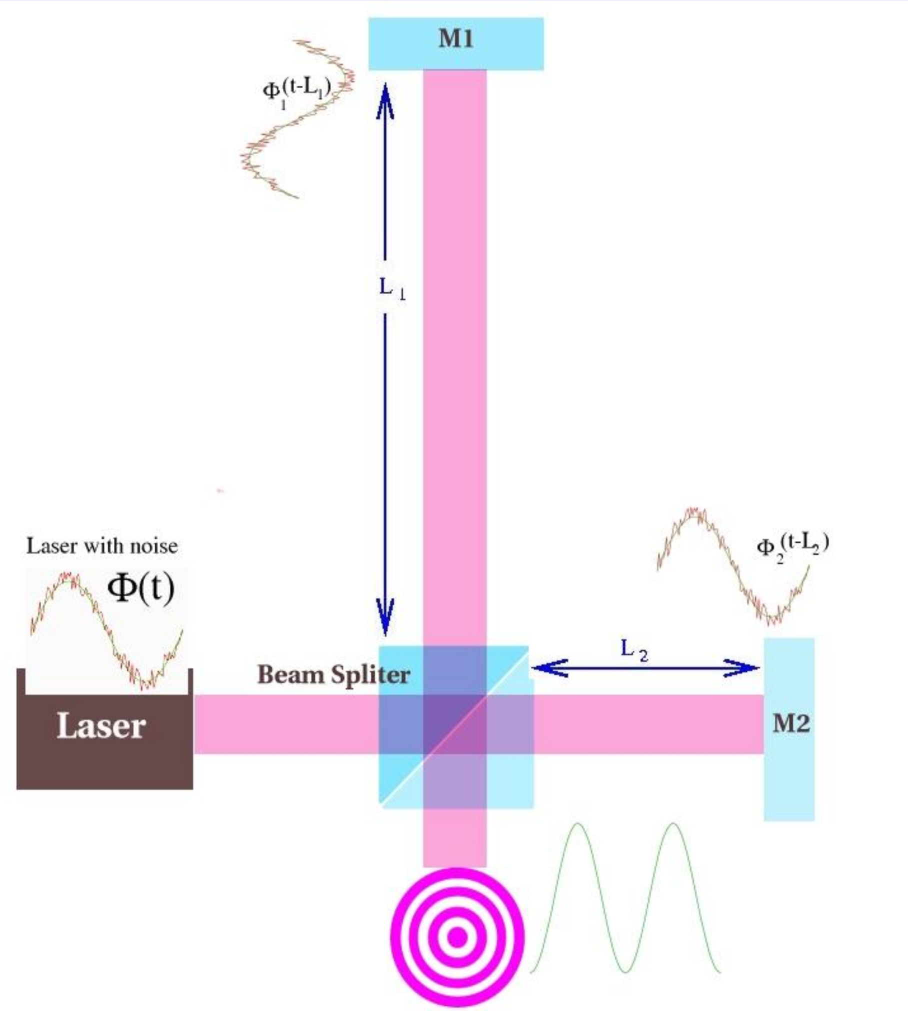

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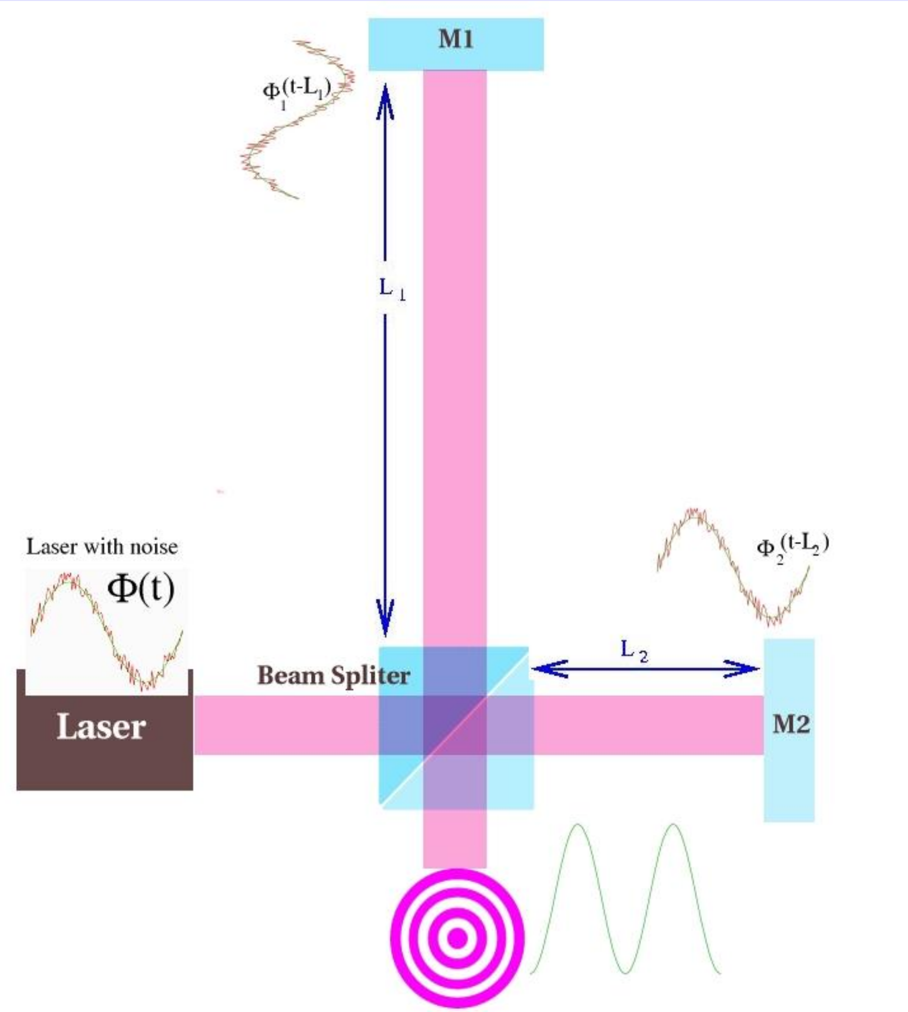
Clearly,

$$\Phi_1(t)-\Phi_2(t)\neq 0$$

We can still cancel-out the laser phase noise by taking the combination,



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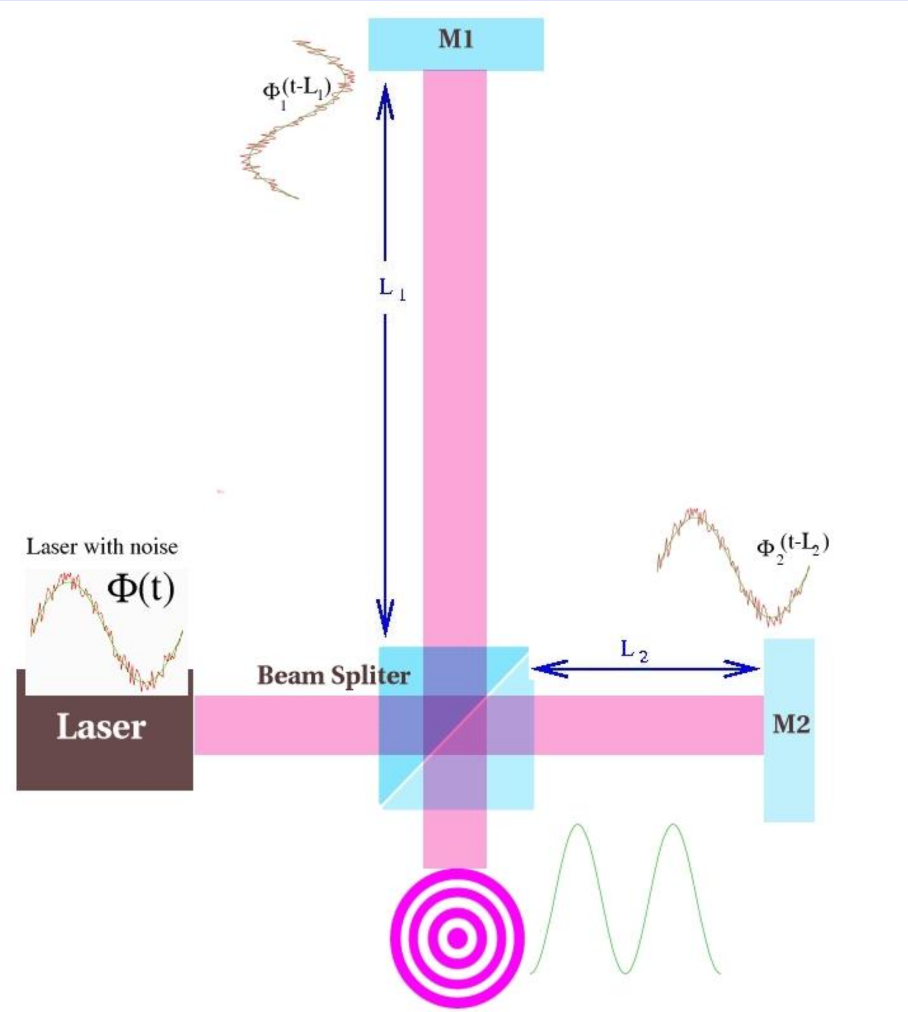

$$X_1 = [\Phi_1(t - 2L_2) - \Phi_1(t)]$$

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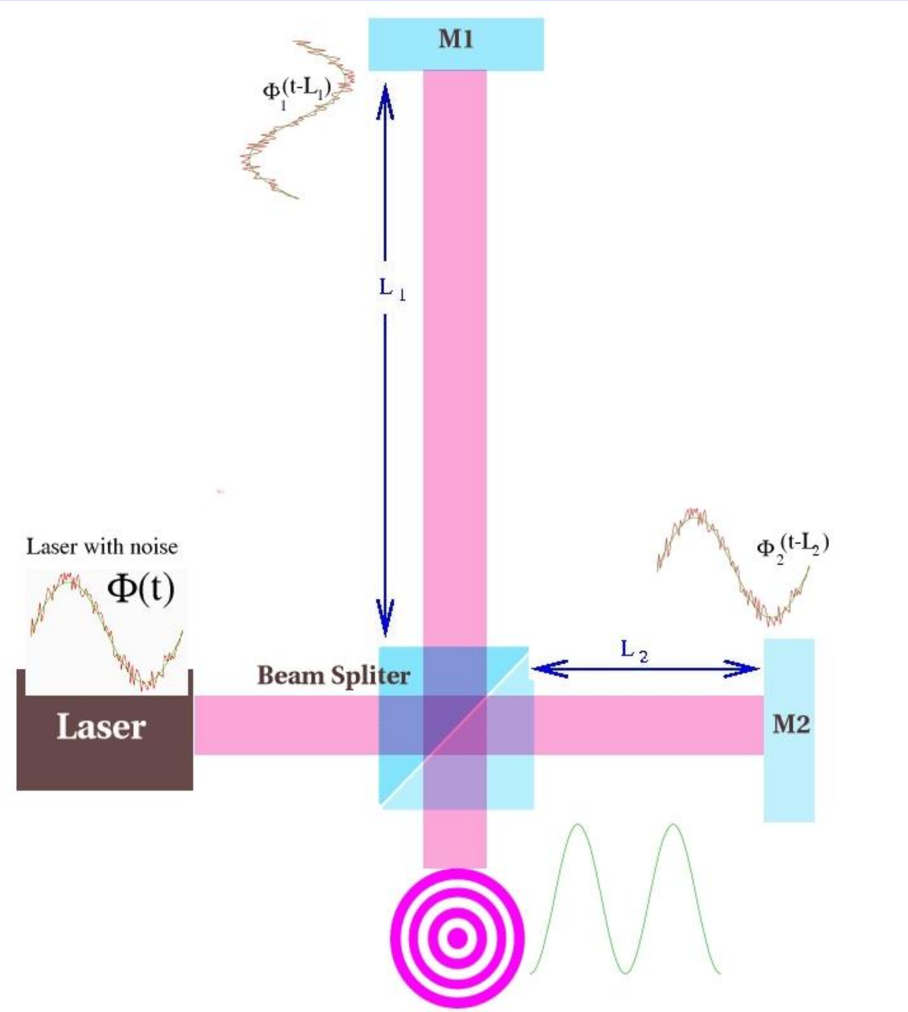

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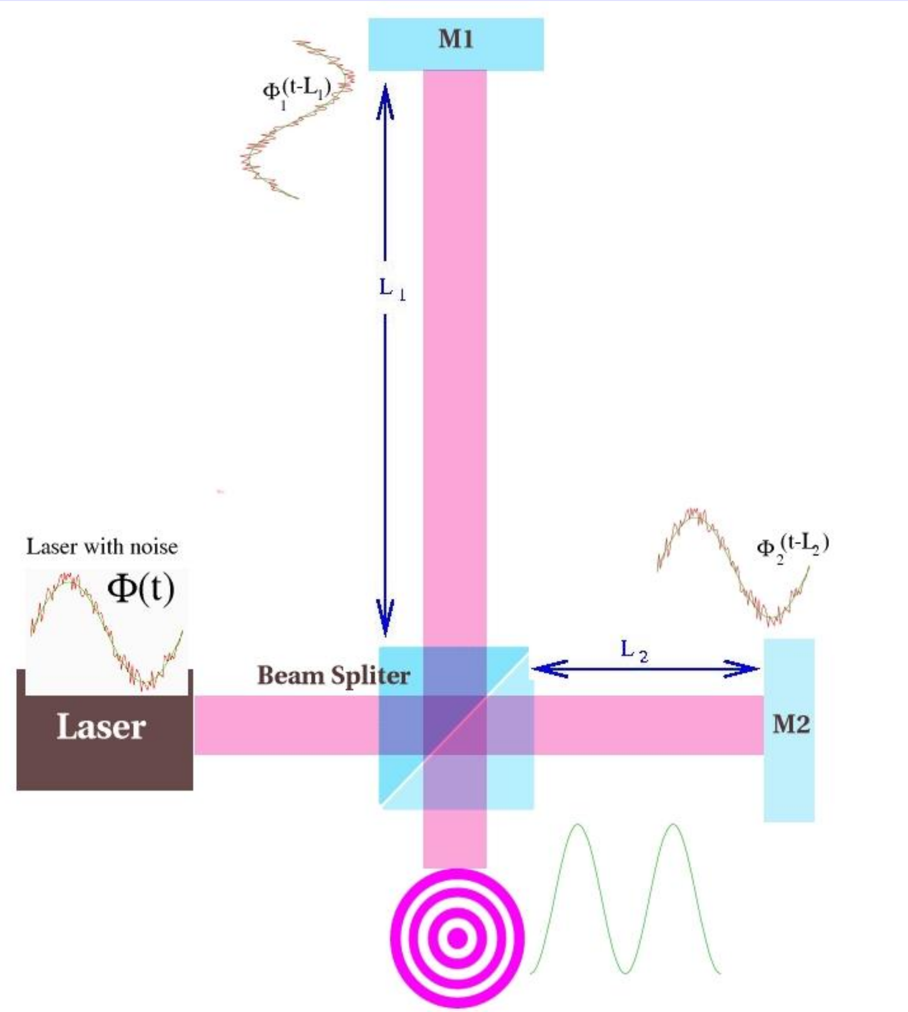

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LISA is a more complex system. There are three arms and six beams.

Delay Operator

S. V. Dhurandhar et. al, “Algebraic approach to time-delay data analysis for LISA”, *Phs Rev. D65*, 102002(2002), gr-qc/0112059.

Let $a(t)$ be any arbitrary function of time and L_k be the length of the kth arm, then we define time delay operator for the arm k as,

$$D_k a(t) = a(t - L_k)$$

$c = 1$ and all the distances are measured in the unit of time.

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$D_1^l D_2^m D_3^n$
This is equivalent to successively applying the delay operator

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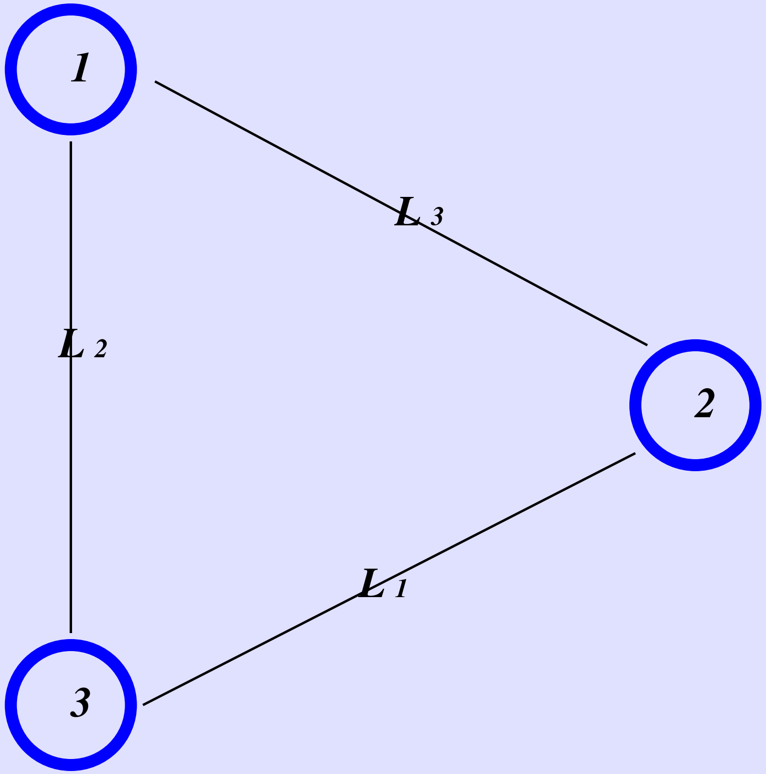
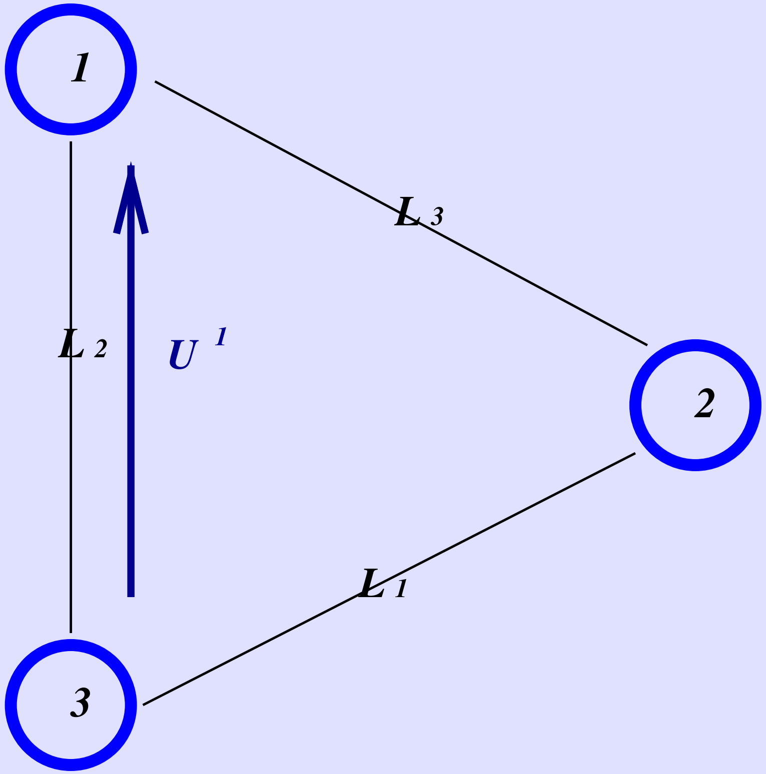
They commute: $D_1^k D_2^l = D_2^l D_1^k$

Back to LISA geometry and beams



Back to LISA geometry and beams

$U^1 = D_2\, C_3 - C_1$



Back to LISA geometry and beams

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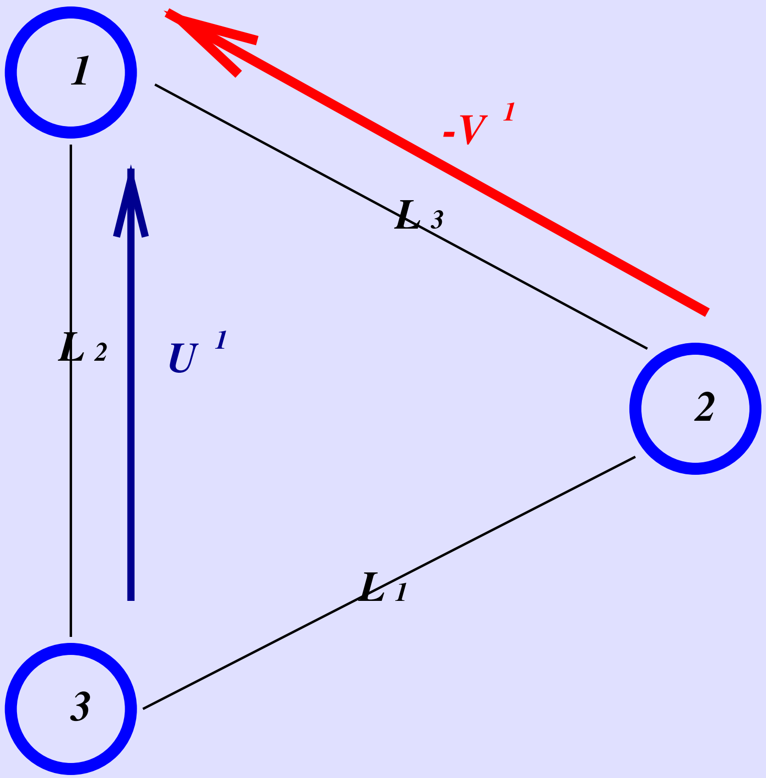

$V^1 = C_1 - D_3\ C_2$

Back to LISA geometry and beams

Ui's in +ve direction

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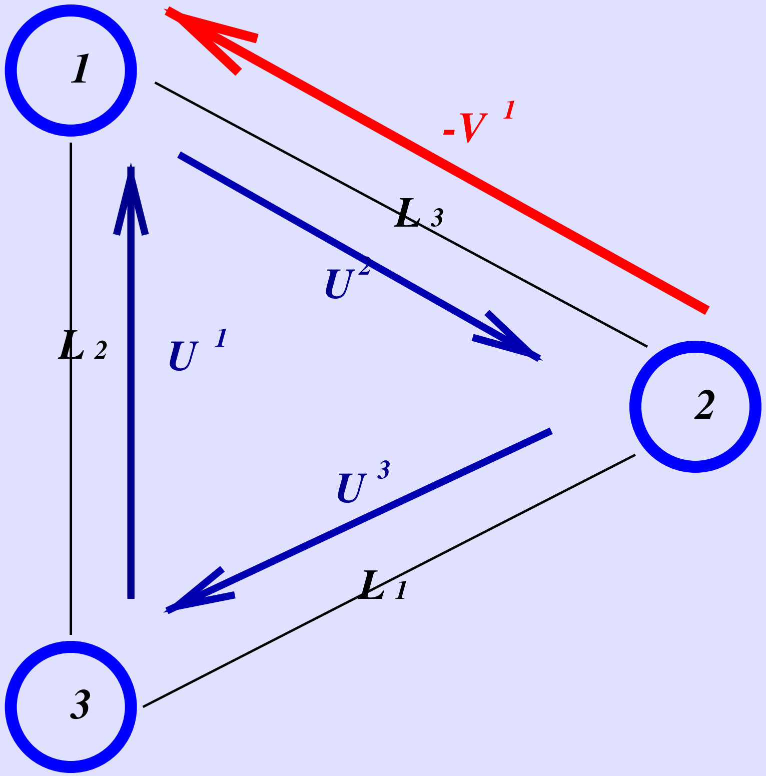

Back to LISA geometry and beams

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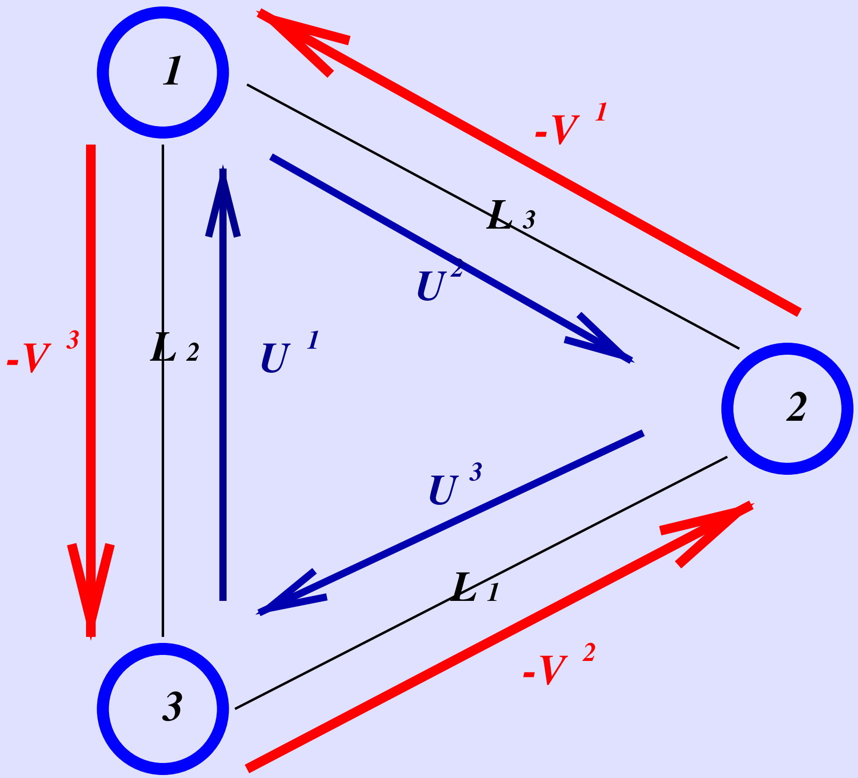

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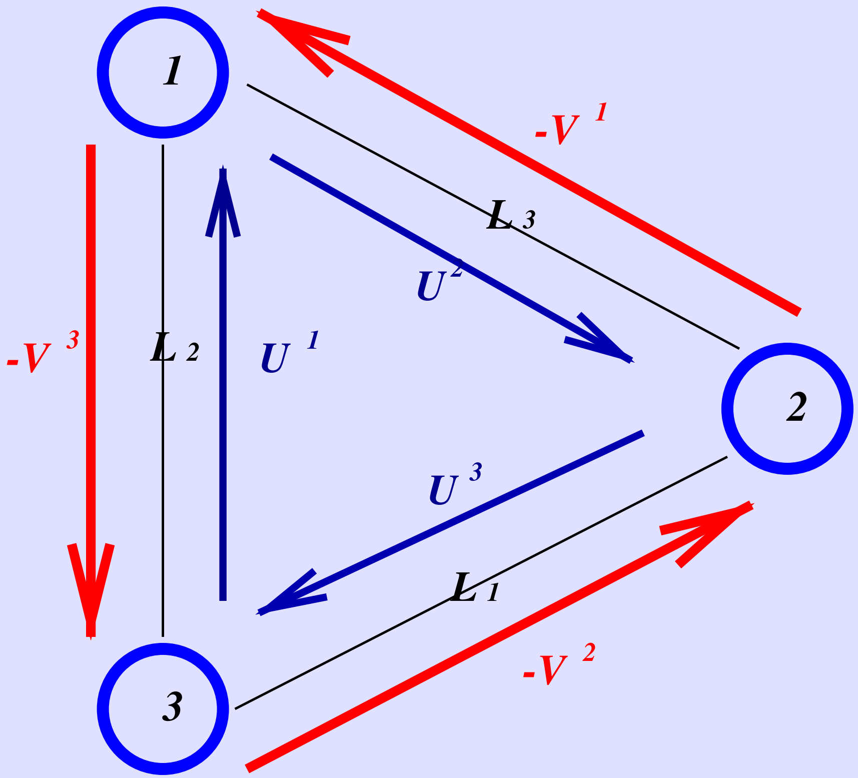

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There are six beams

⇓

Redundancy in Data

Back to LISA geometry and beams

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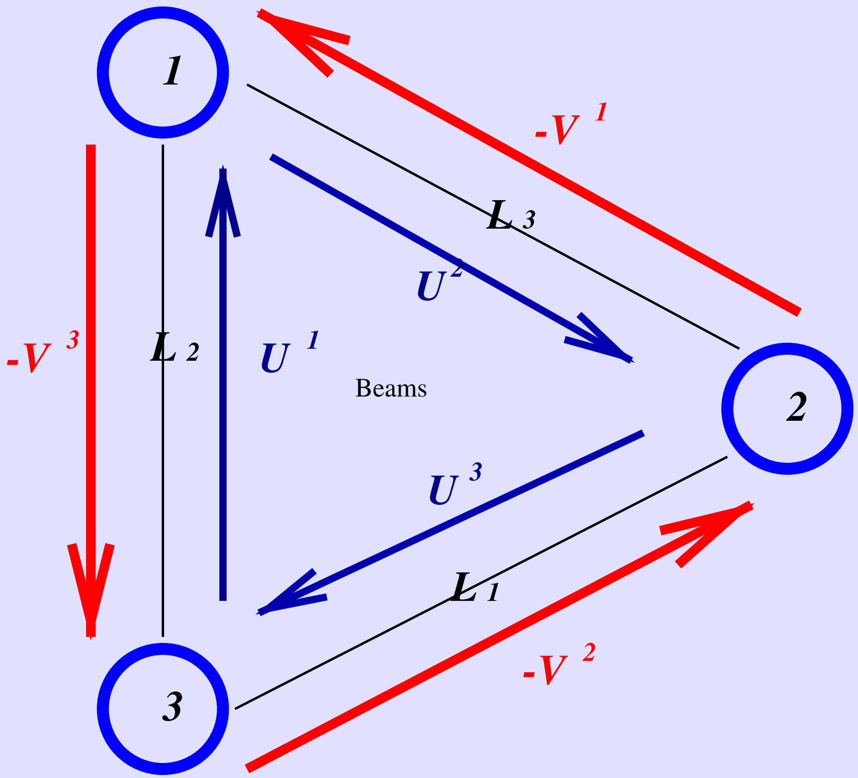

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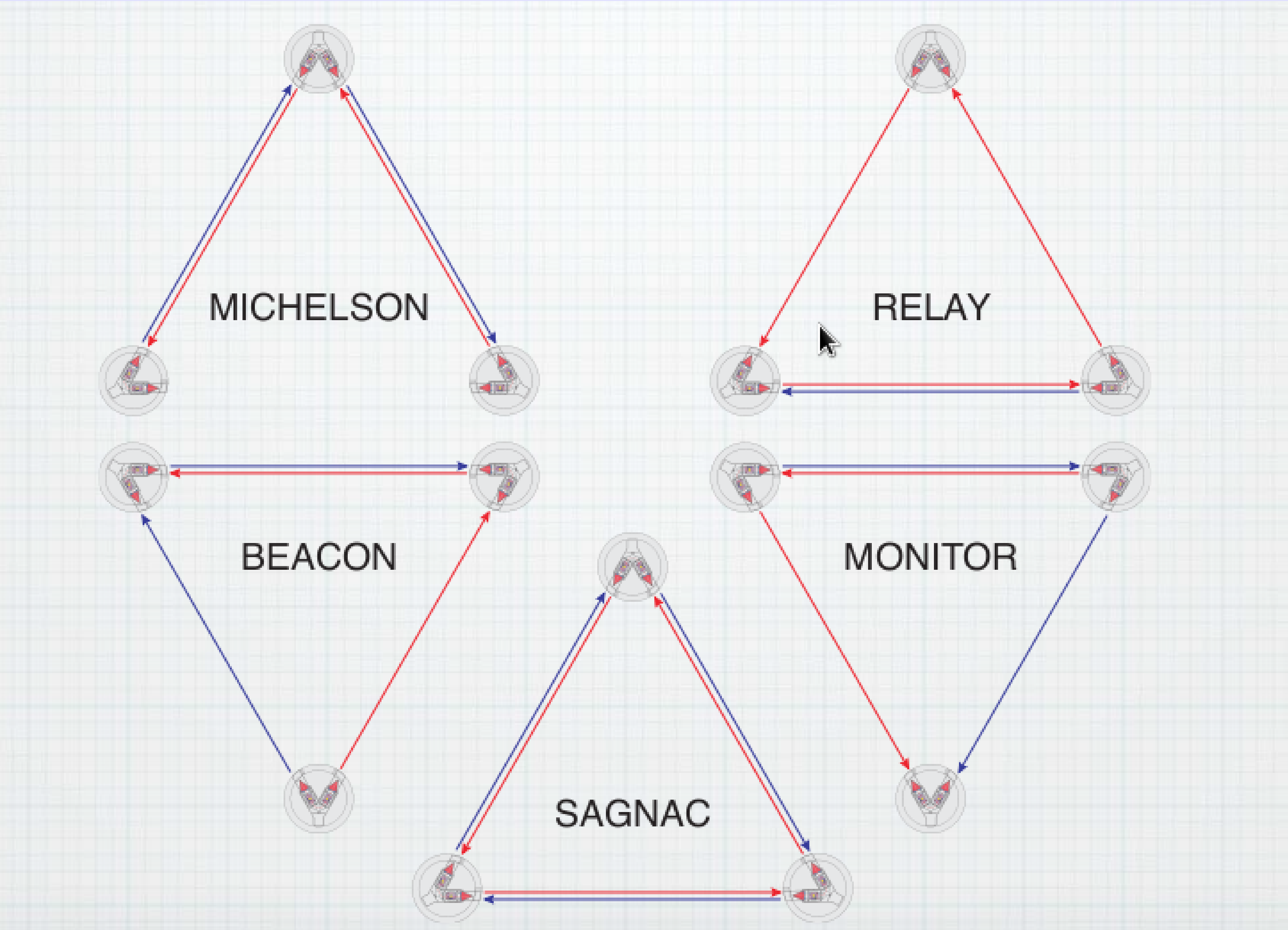
Redundancy in Data

Known Noise cancellation solution



Laser noise Cancellation Data combination

A general data combination is given by the combination of U^i and V^i of the form

$$X = \sum_{i=1}^{3} p_i V^i + q_i U^i$$

p_i and q_i are polynomial in D_i, This is clear from the properties of D_i

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A noise cancellation Data combination, we need to determine p_i and q_i such that,

$$\sum_{i=0}^{3} p_i V^i + q_i U^i = 0$$

We need to solve for (p_i, q_i) as functions of D_i.

The solution to this equation is well known in the algebra and forms a module called "***First Module of Syzygies***" , over the polynomial ring D_i.

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☞ Vector spaces are defined over field $\mathscr{F}$. Modules are defined over ring $\mathscr{R}$.

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  Set of polynomials are important example for rings. No inverse for Polynomial !

- ☞ Basis can generate the complete vector space by taking the combination $\sum a_i v^i$ where $a_i \in \mathscr{F}$ and v^i's are basis elements.

  They are called generators! and any element of module can be written of the form $\sum p_i g^i$ where $p_i \in \mathscr{R}$ and g^i's are the generators.

The Generators

The solutions are represented by generator with (p_i, q_i),

$$
\begin{aligned}
X^{(1)} = \alpha &= \left(1, \ D_3, \ D_1D_3, \ 1, \ D_1D_2, \ D_2 \right), \\
X^{(2)} = \beta &= \left(D_1D_2, \ 1, \ D_1, \ D_3, \ 1, \ D_2D_3 \right), \\
X^{(3)} = \gamma &= \left(D_2, \ D_2D_3, \ 1, \ D_1D_3, \ D_1, \ 1 \right), \\
X^{(4)} = \zeta &= \left(D_1, \ D_2, \ D_3, \ D_1, \ D_2, \ D_3 \right).
\end{aligned}
$$

With these generator any solution can be expressed as:

$$
X(p_i, q_i) = \sum_{I=1}^{4} \alpha_{(I)} X^{(I)}
$$

In general, $\alpha_{(I)}$ can be polynomials in D_i

Advantages of the Formalism

☞ All the Noise cancellation combination can be obtained using a set of generators. Any noise cancellation data combination can be written as,

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