Gravitational Wave Data Analysis

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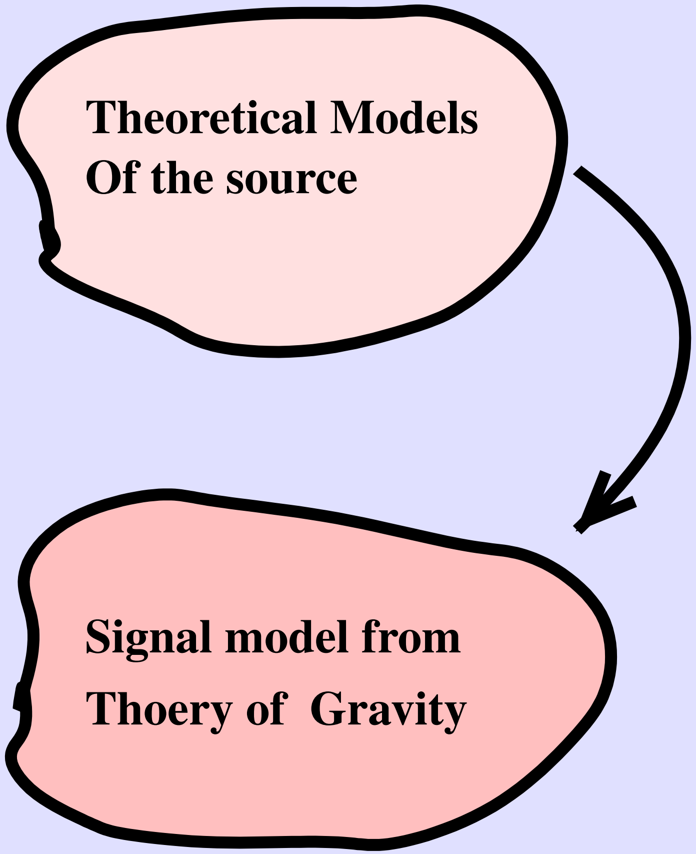
- ☞ Source parameter estimation from the signal is one of the very important problem in the data analysis.

- ☞ Data from the detector consists of a time series from which all the information about sources have to be extracted or filtered!

- ☞ This is done through an efficient method called match-filtering.

Theoretical Models
Of the source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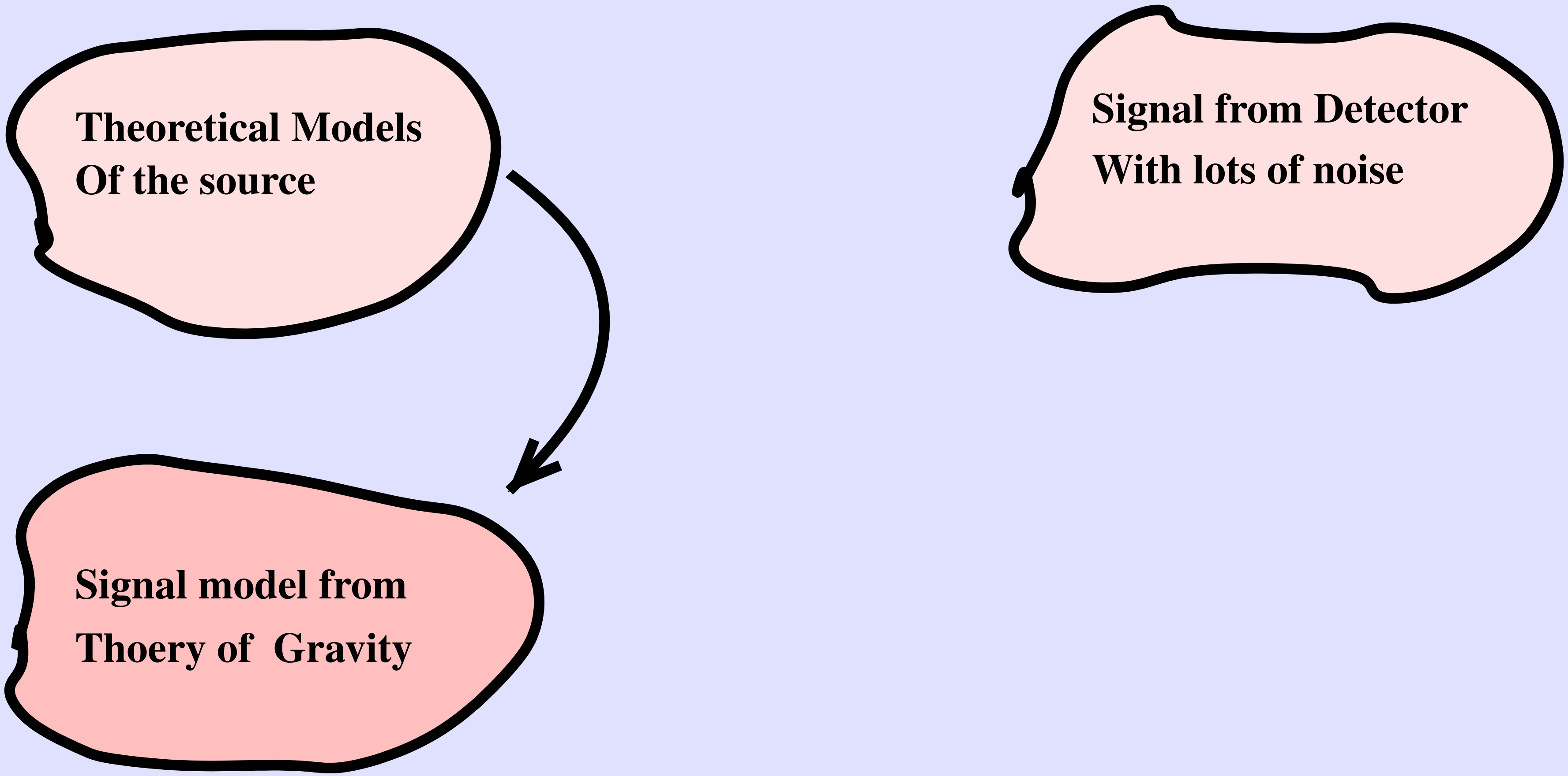
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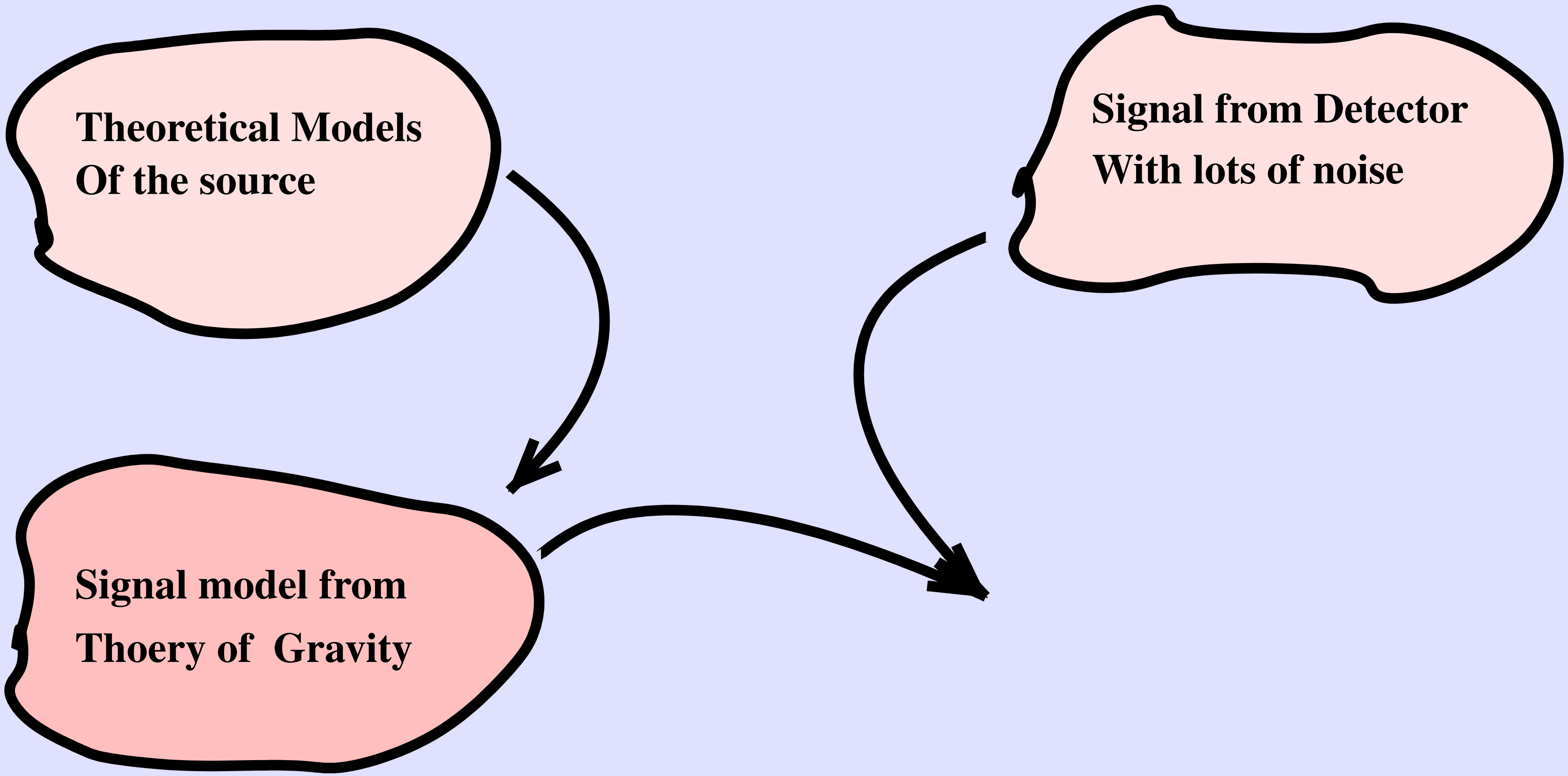
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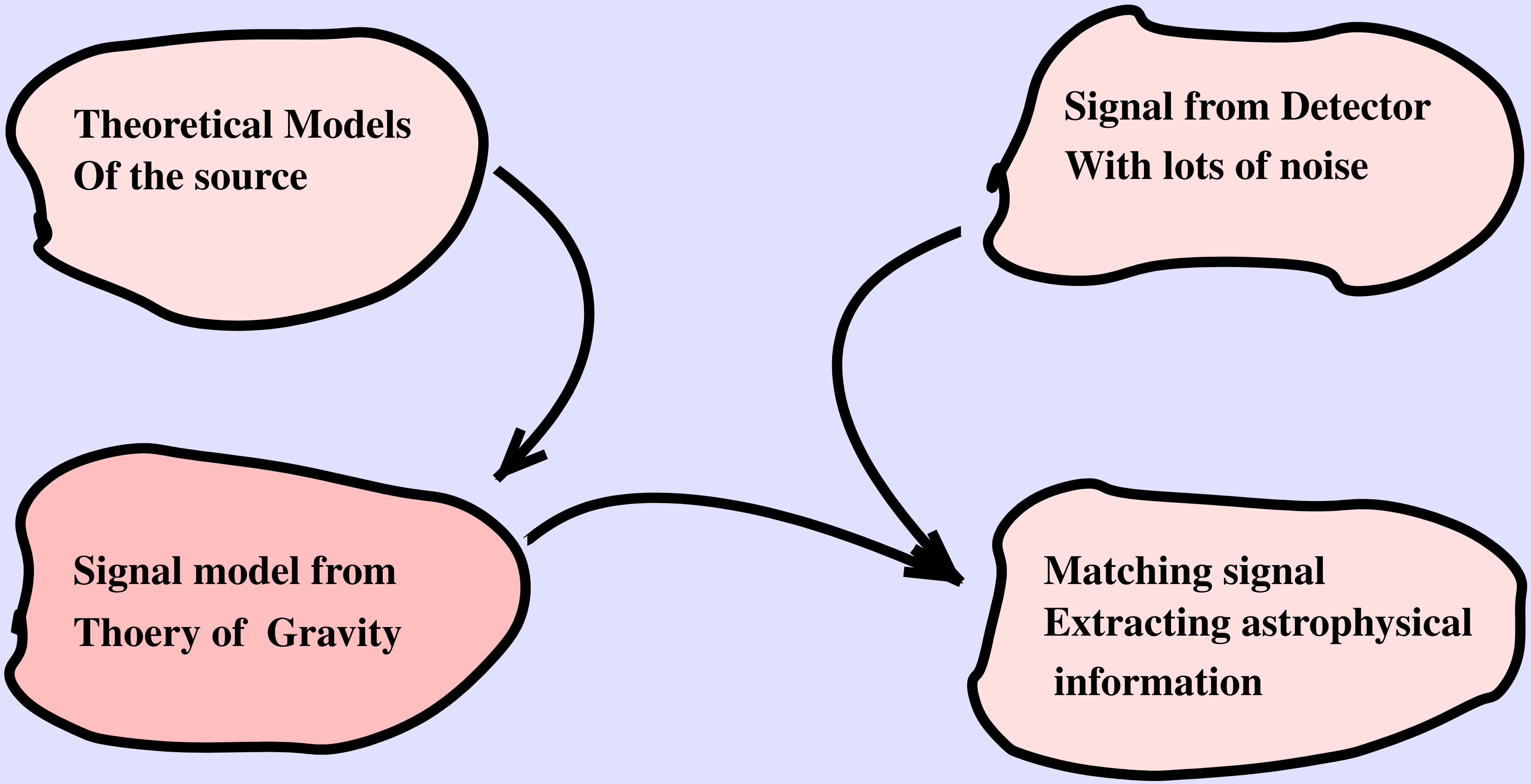
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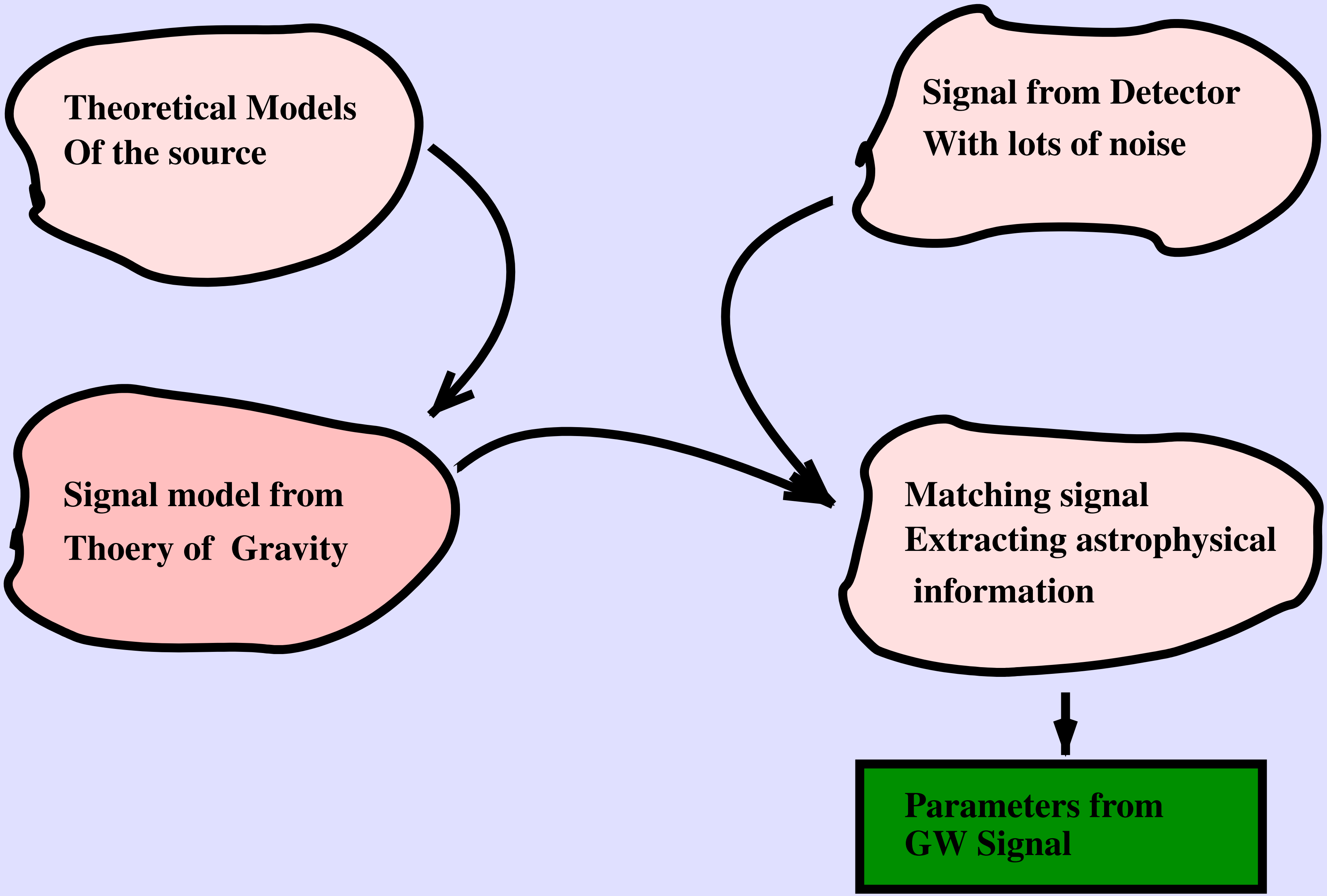
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Parameters from
GW Signal

Fourier Series

Any seasonally smooth function $f(x)$ in an interval $[-L, L]$, can be expressed in terms of Fourier series as:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) .$$

where the coefficients a_n and b_n are given by,

$$\begin{aligned} a_n &= \frac{1}{L}\int_{-L}^{+L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx \, , \\ b_n &= \frac{1}{L}\int_{-L}^{+L} f(x) \sin\left(\frac{n\pi x}{L}\right) \, , \end{aligned}$$

EXAMPLE : Express the function

$$f(x) = x \qquad -\pi < x < \pi$$

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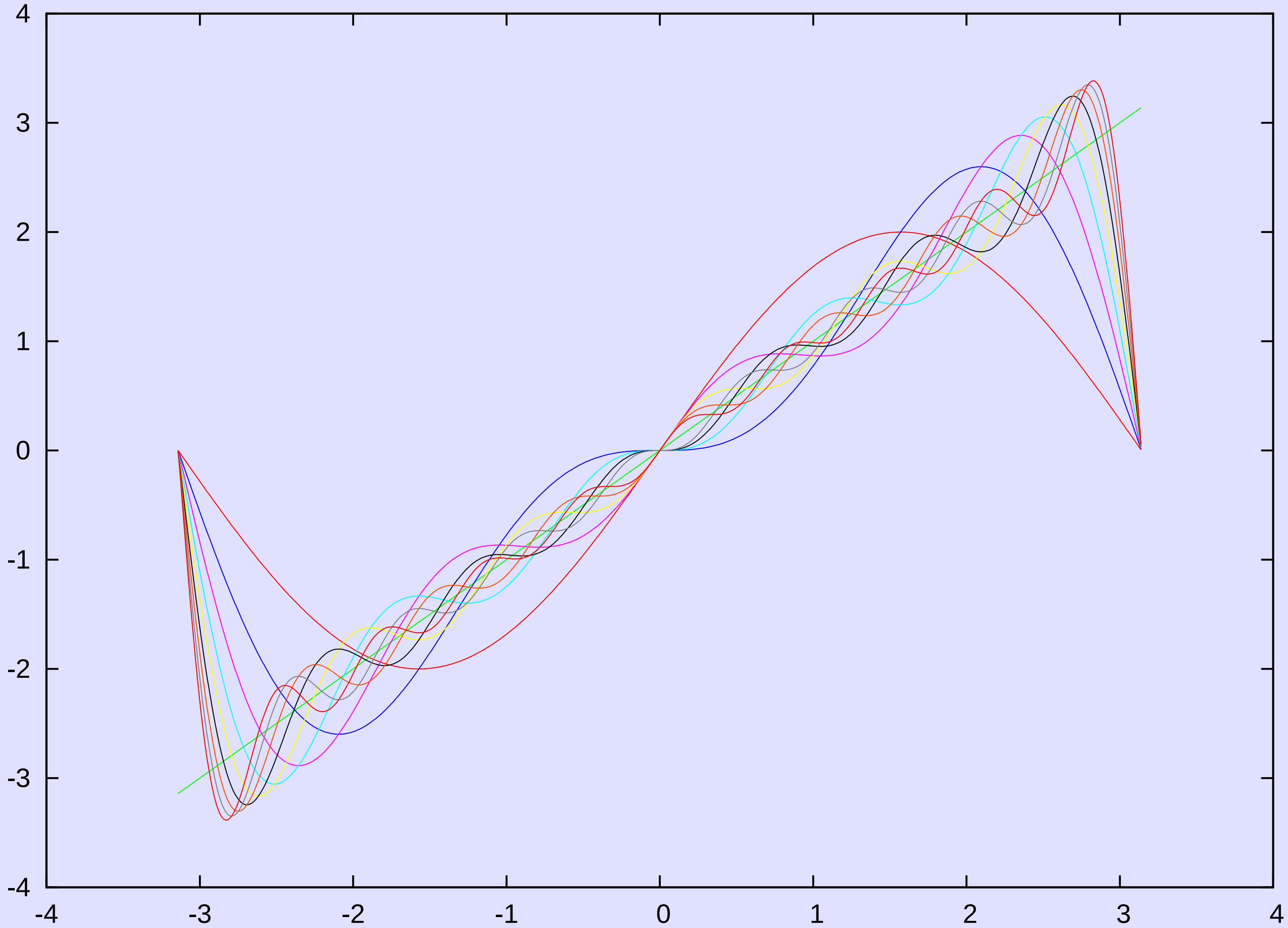
$$\begin{aligned}
a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} x\,dx = 0 \\
a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos nx\,dx = 0 \\
b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin nx\,dx = -\frac{2\pi}{n}\cos n\pi \\
&= \frac{2}{n}(-1)^{n+1}
\end{aligned}$$

and the function can be written as:

$$f(x) = 2\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}.$$

The plots of various terms of this series is shown in figure



Let us look at series

$$f(x, x_0) = \sum_{n=1}^{\infty} \sin(nx_0) \sin(nx)$$

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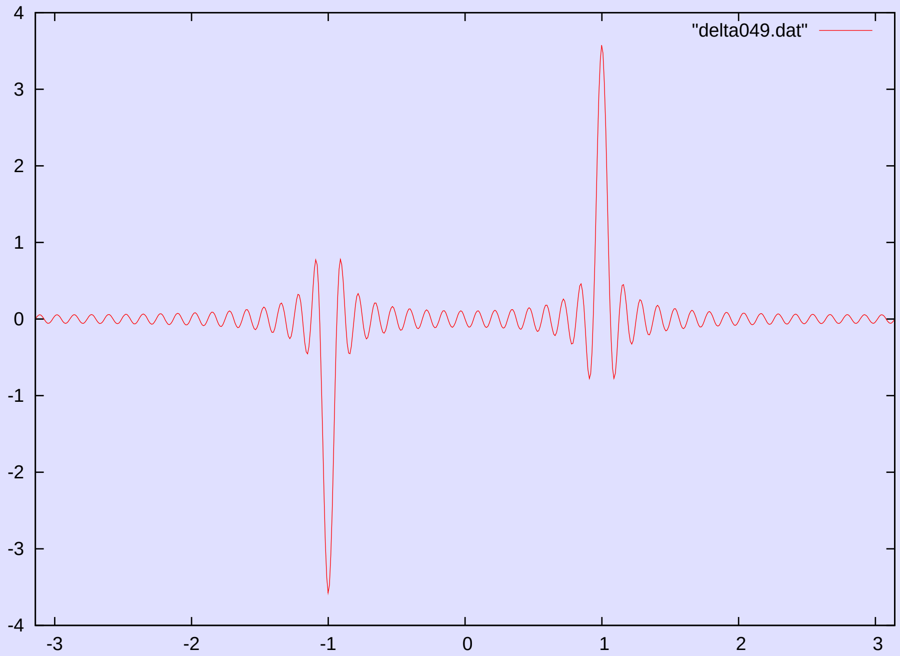

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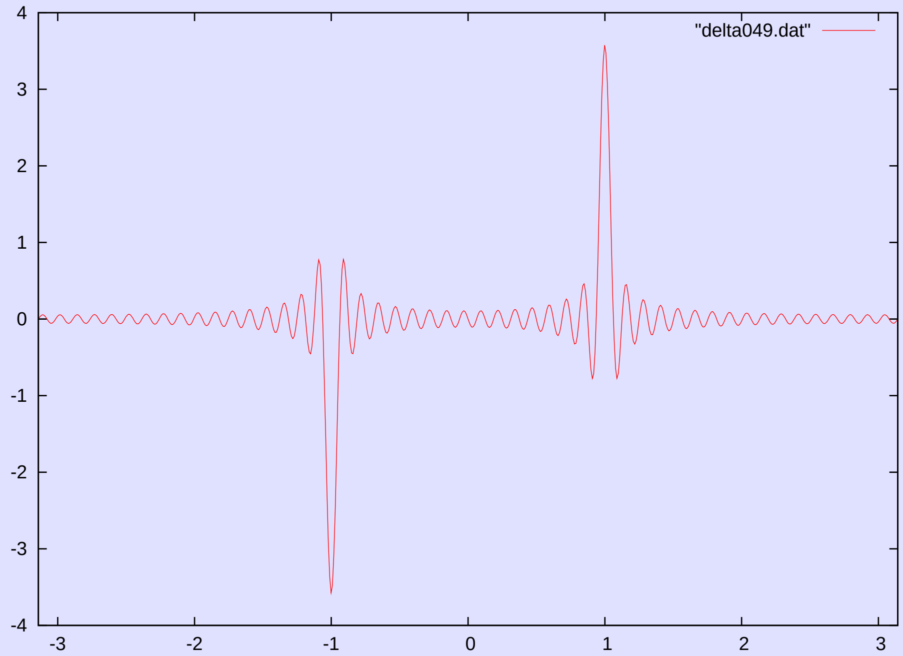


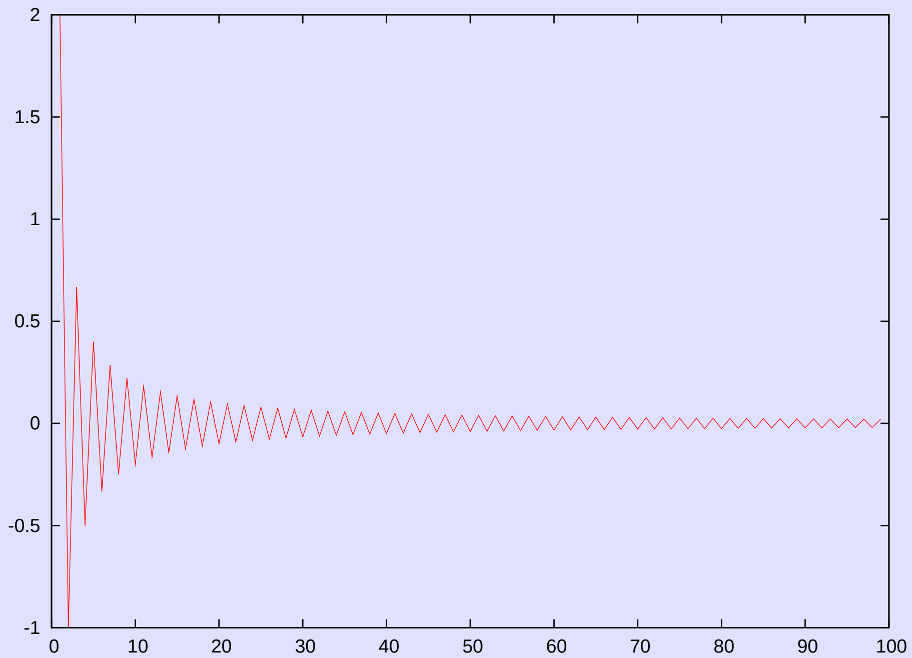
We have the condition

$$\delta(x - x_0) = \sum_{n=1}^{\infty} \sin(nx_0) \sin(nx)$$

is known as **completeness or Parseval's Theorem**

Fourier representation

The coefficients b_n can fully describe the function, because basis function $\sin(nx)$ are known. This is called Fourier representation of function



Fourier domain representation

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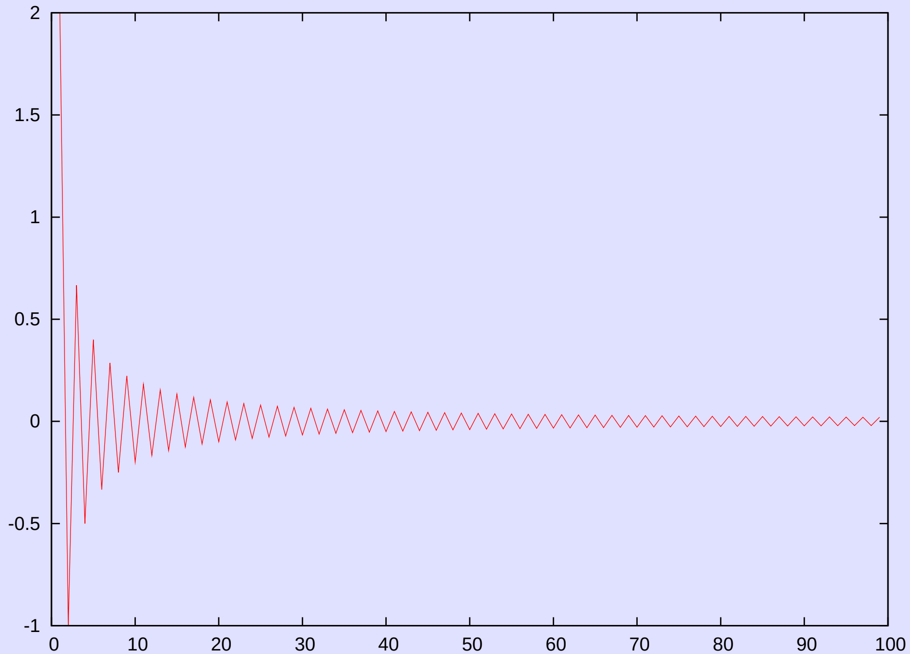

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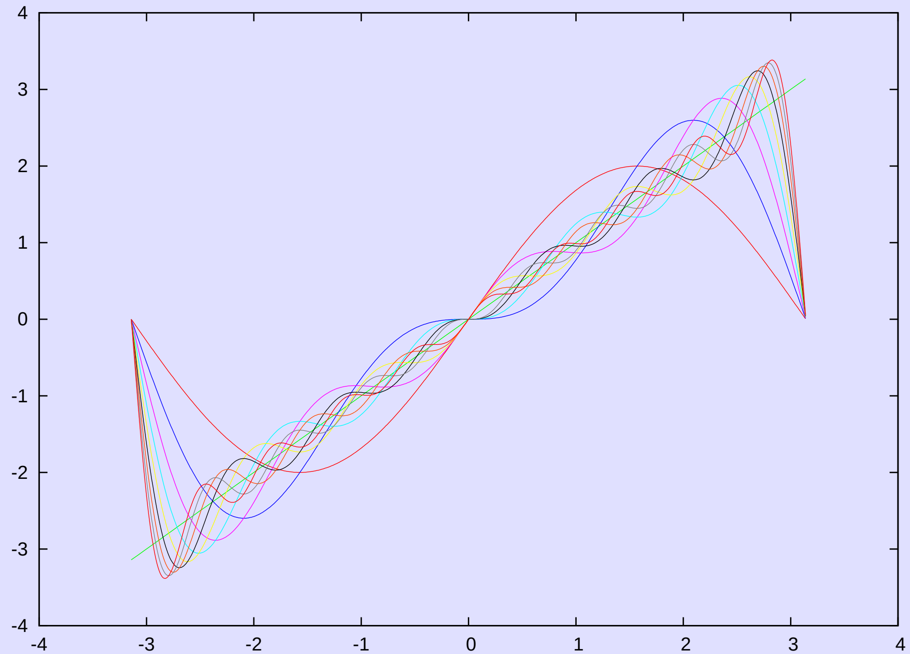

Time domain representation

Sampling interval

☞ Output from the detector have to be sampled at the regular interval. $\{x_0, x_1, x_2, \cdots x_n\}$ $\delta x = x_i - x_{i-1}$

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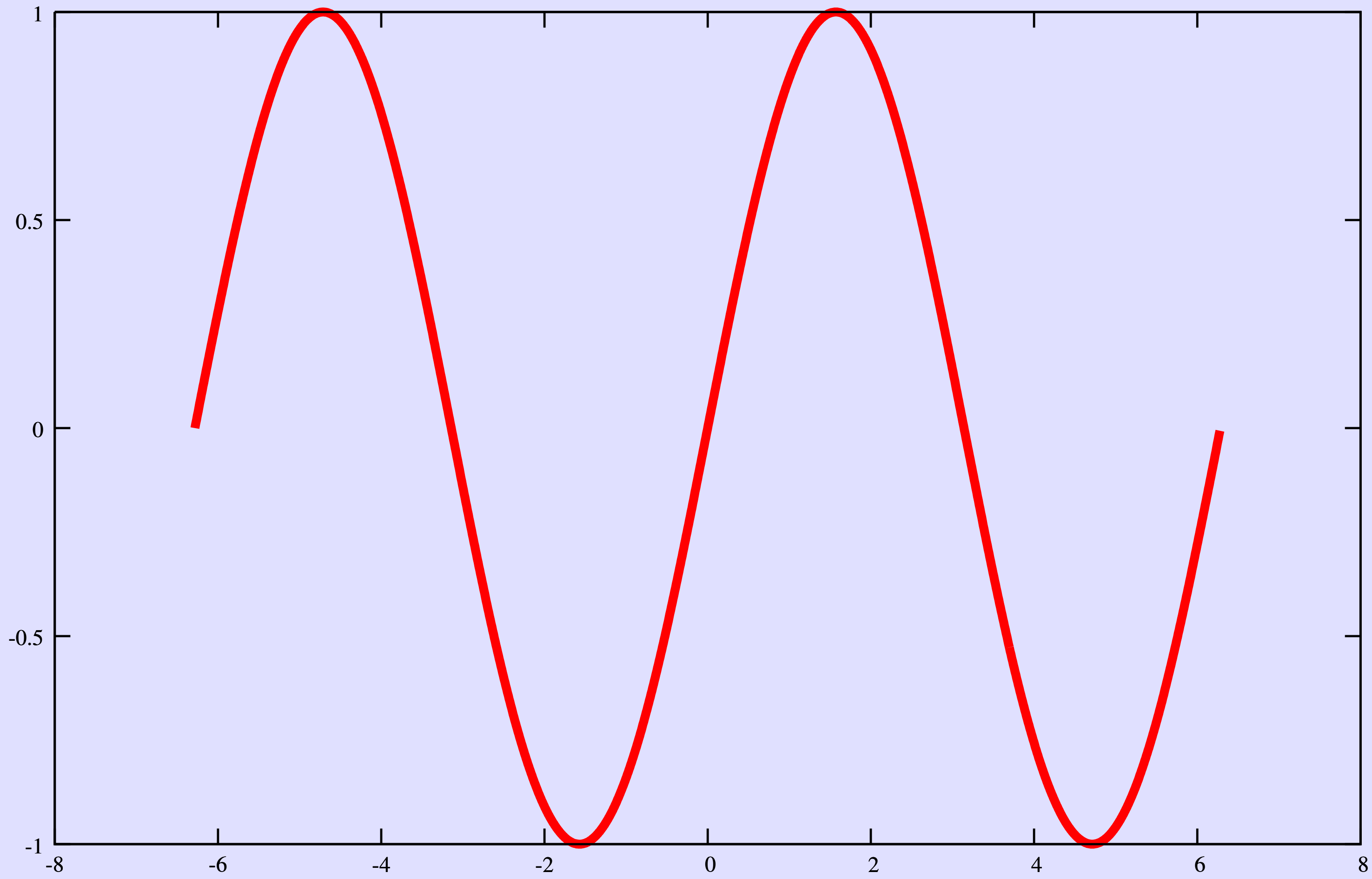

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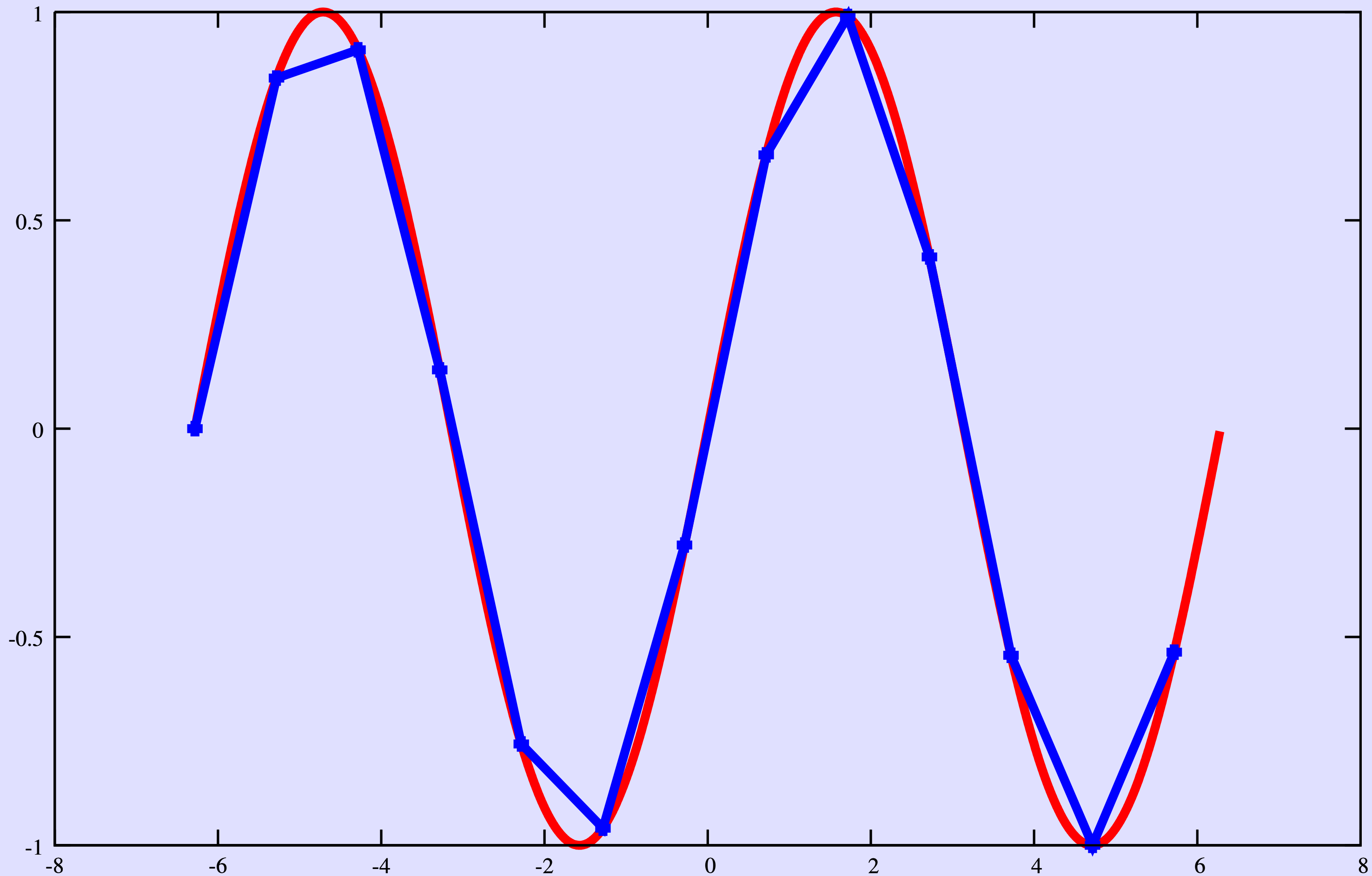

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$$f(x) \approx f(x_0) + f'(x_0)\,\delta x + \frac{1}{2} f''(x_0)\,\delta x^2 + \cdots$$

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we get

$$f(x) \approx sin(wx_0) + w\cos(wx_0)\,\delta x - \frac{1}{2} w^2 \sin(wx_0)\,\delta x^2$$

Since $|\sin(wx_0)| \leq 1$ and $|\cos(wx_0)| \leq 1$ we need

$$\delta x \ll \frac{1}{w}$$

Nyquist Theorem

Nyquist Theorem-2cm

If a function has maximum Fourier frequency f_m , the sampling interval δx such that

$$\delta x < \frac{1}{2 f_m}$$

Fourier transform

In the Fourier series we can replace $\sin(nx)$and $\cos(nx)$ by e^{inx}
Then we have:

$$g(x) = \sum_{n=0}^{\infty} \tilde{g}_n e^{i\left(\frac{n\pi x}{L}\right)}$$

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If we take limit $L \longrightarrow \infty$

Fourier Transform

Fourier transform a function $f(x)$ is defined as,

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The inverse Fourier transform is given by:

$$g(x) = \int_{-\infty}^{+\infty} \tilde{g}\left(f\right) e^{2\pi i f x} dx$$

Inverse Fourier Transform maps the series of frequencies (their amplitudes and phases) back into the corresponding time series.

☞ Show that for real function $r(x)$ $\tilde{r}(-f) = \tilde{r}(f)*$

Discrete Fourier Transform(DFT)

For a time series data of N samples , $\{x_0, x_1, x_2, \cdots x_{N-1}\}$ discrete Fourier transform is defined as:

$$\tilde{x}_k = \sum_{j=0}^{N-1} x_j e^{-2\pi i jk/N}$$

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$$x_j = \frac{1}{N} \sum_{j=0}^{N-1} \tilde{x}_k e^{2\pi i\, jk/N}$$

Here complex numbers $\tilde{x}_k$ (Fourier series) are transformed back to (time series) x_j

Comparing with continuous version, we get:

$$f_k = \frac{k}{(N\delta x)} = \frac{k}{X}$$

where $X = x_{n-1} - x_0$,

The frequency resolution of the data is

$$\delta f = \frac{1}{X}$$

DFT is very useful because they reveal periodicity in input data as well as the relative strengths of any periodic components.

Parseval Theorem

The relation between the Fourier transform and the original time-series data is given by,

$$\sum_{k=0}^{N-1} |\tilde{x}_k|^2 = N \sum_{j=0}^{N-1} |x_j|^2$$

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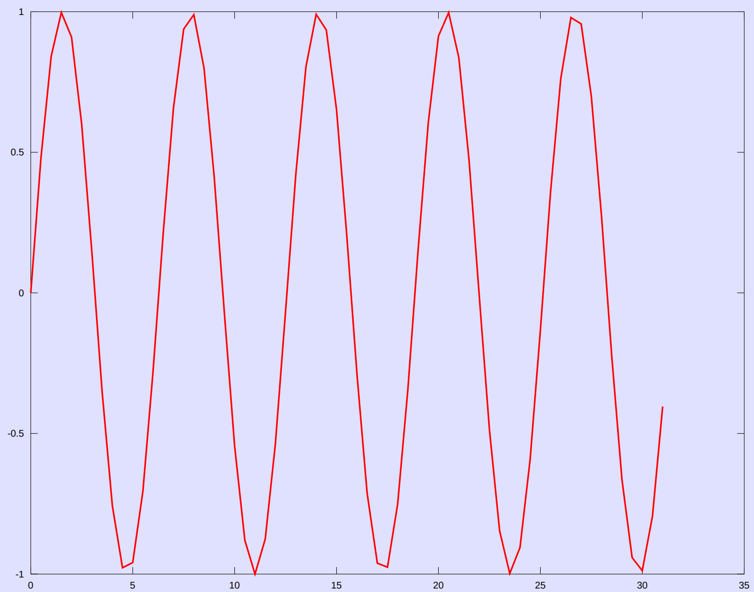

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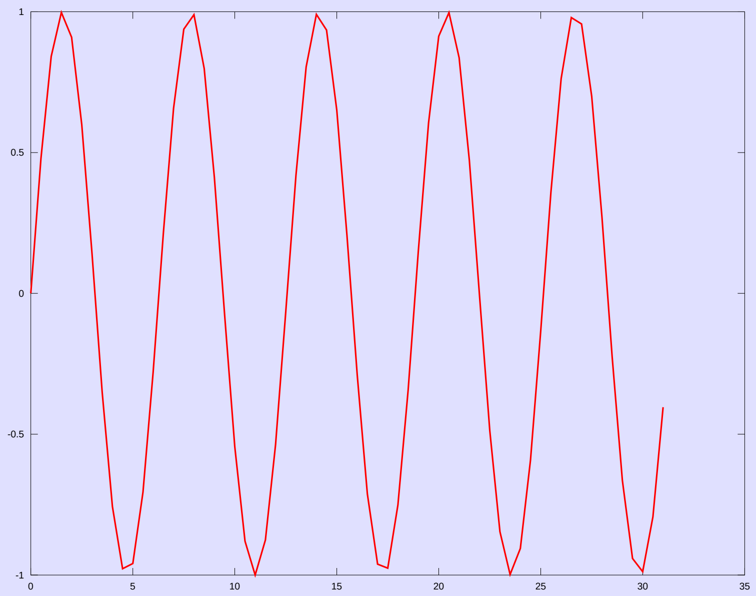
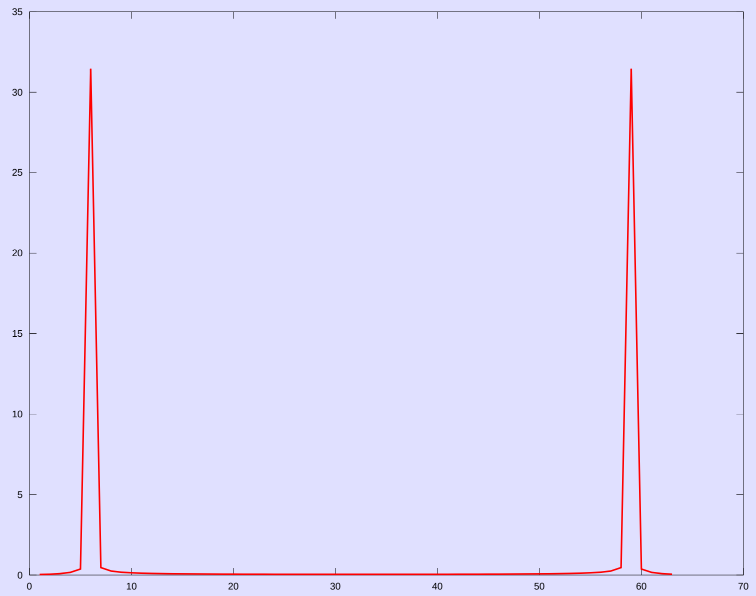
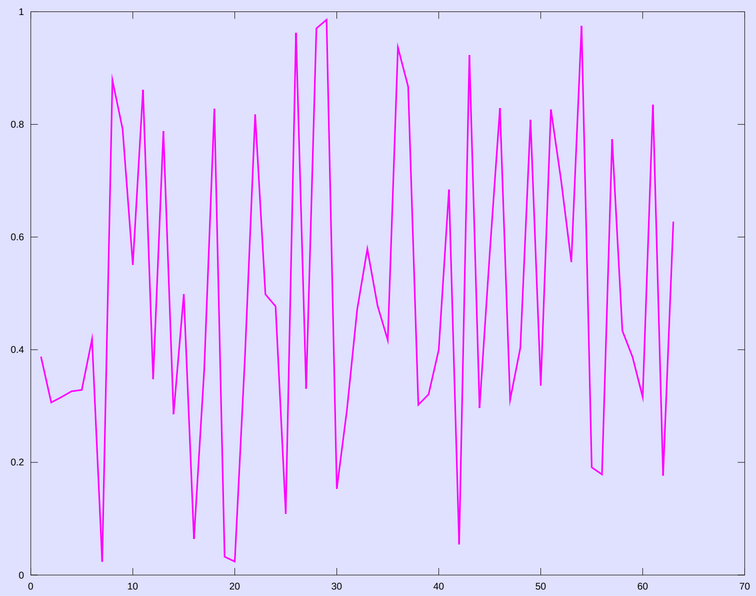




Fourier Transform as filter

Fourier transform of noise



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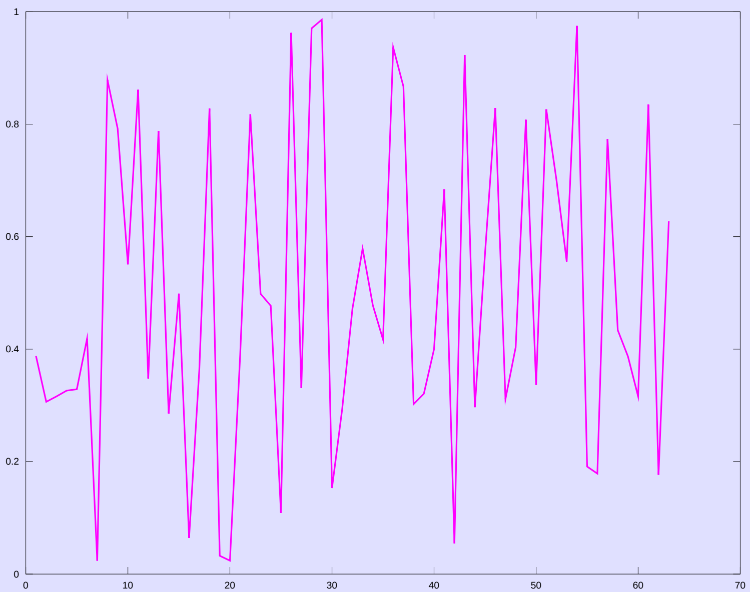

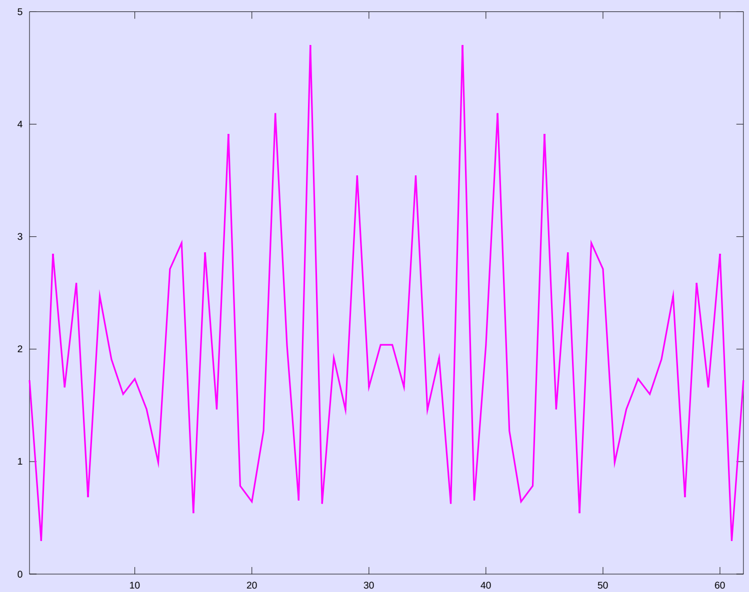

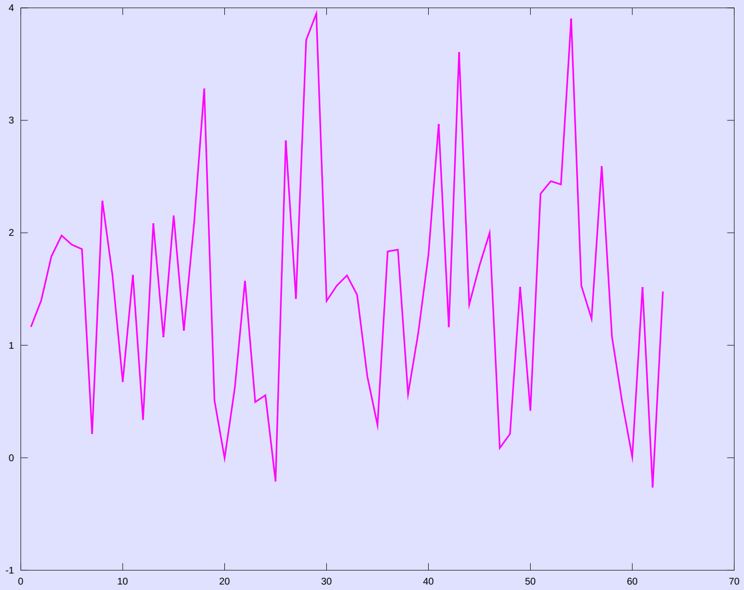

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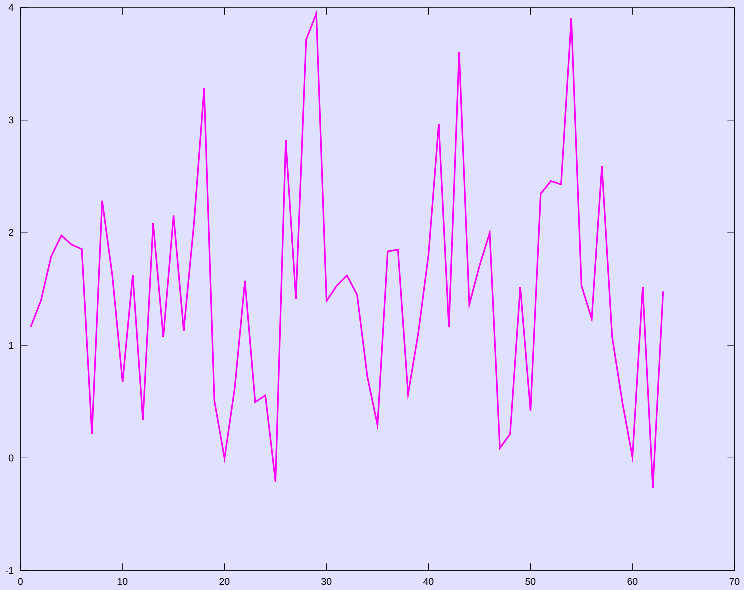
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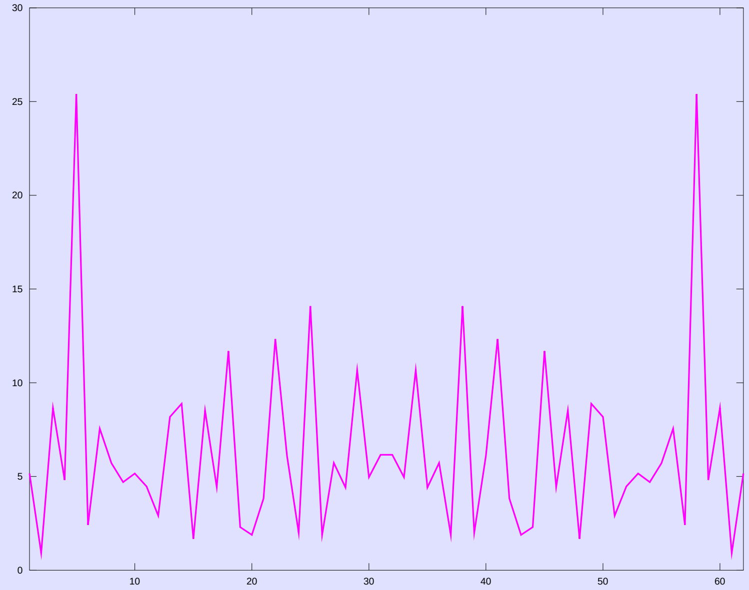
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