EINSTEIN AND THE ENIGMA OF TIME

“Everything you wanted to know about TIME, but simply did not have TIME to ask !”

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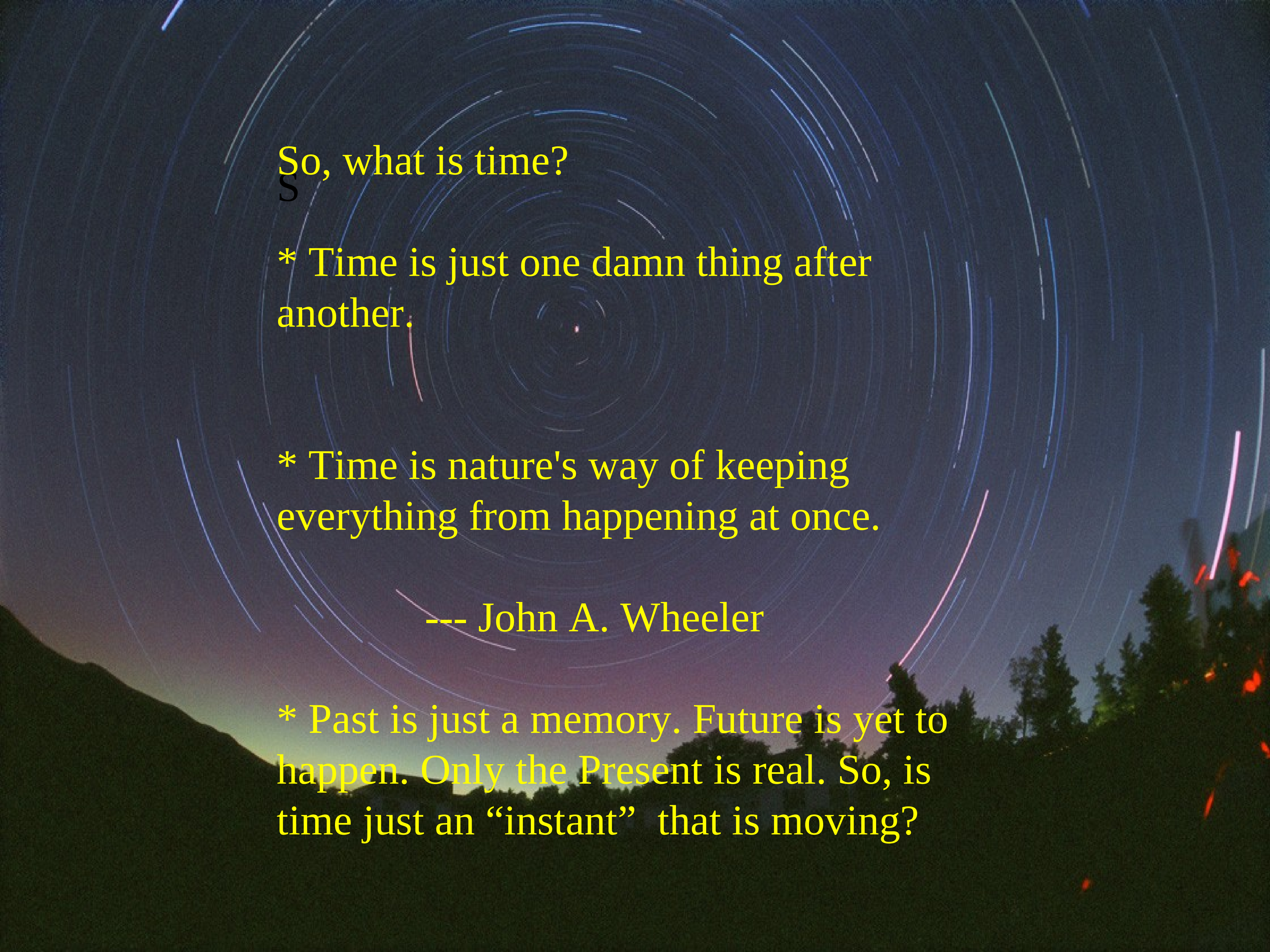
University of Delhi, Delhi – 110 007

September 21, 2005


TOURNEAU



So, what is time?
S

* Time is just one damn thing after another.

* Time is nature's way of keeping everything from happening at once.

--- John A. Wheeler

* Past is just a memory. Future is yet to happen. Only the Present is real. So, is time just an “instant” that is moving?

Pre- 1905 days ...... life was simple:

Why?

PHYSICISTS BELIEVED IN GALILIEAN RELATIVITY

(O, space is SPACE and time is TIME,

Never the twain shall mix.

--------- Newtyard Galiling)

From one inertial frame to another:

Spatial coordinates transformed, but time did not

Simultaniety of events was absolute

Lengths, areas, volumes, shapes etc. of physical objects did not transform

Life-times of radio-active nuclei, amoeba, plants, human-beings etc. stayed invariant

Span of THIS LECTURE remains 60 min. no matter from which frame you observe

In short, physicists had it easy!

Came 1905:

A 26 year old patent clerk entered the scene.

Hearing the old poem:

(O, space is SPACE and time is TIME,

Never the twain shall mix.

-------- Newtyard Galiling)

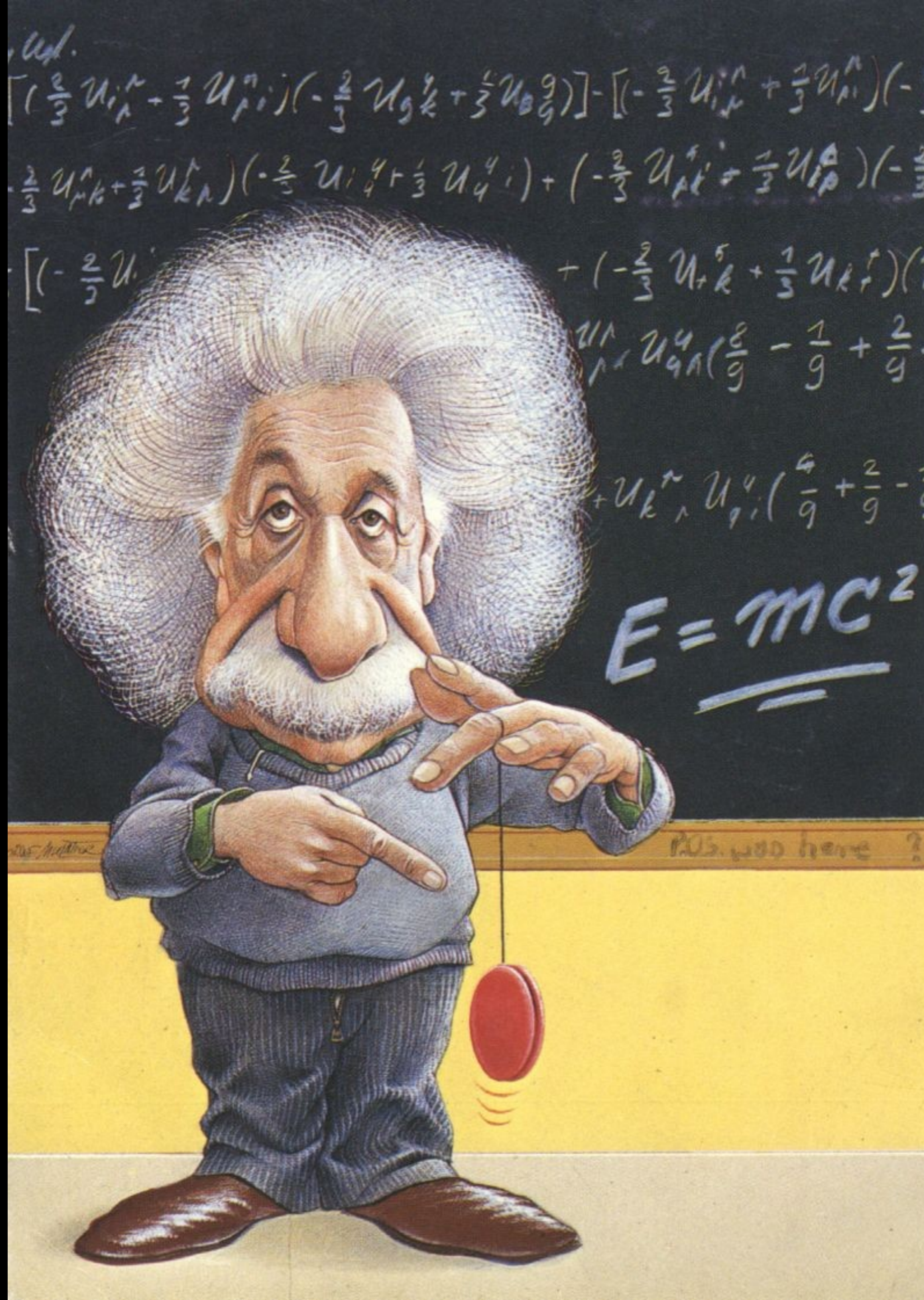
he quipped:

Oh yeah! Their mix is no longer a jinx,

For, I have a Minkowskiian quick-fix!

EINSTEIN'S THEORY OF RELATIVITY

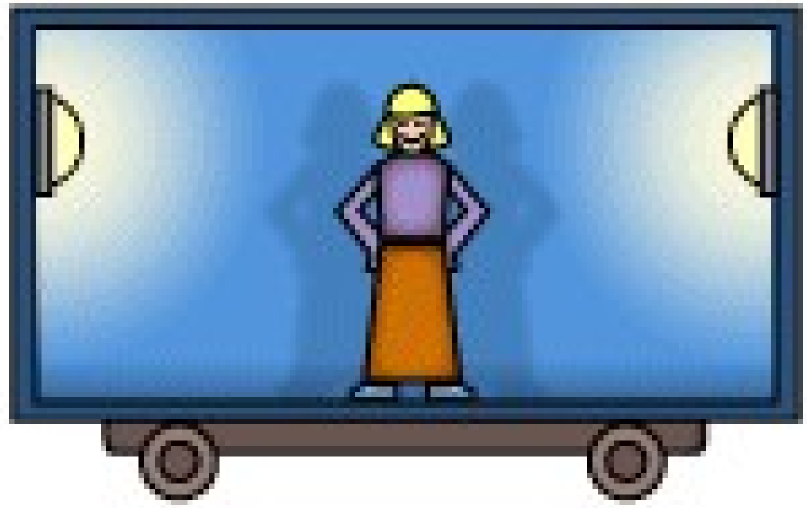
- Every event corresponds to a point in space-time.
- Space and time are just different coordinates of the “event”
- Space and time merge together into a four-dimensional "world".

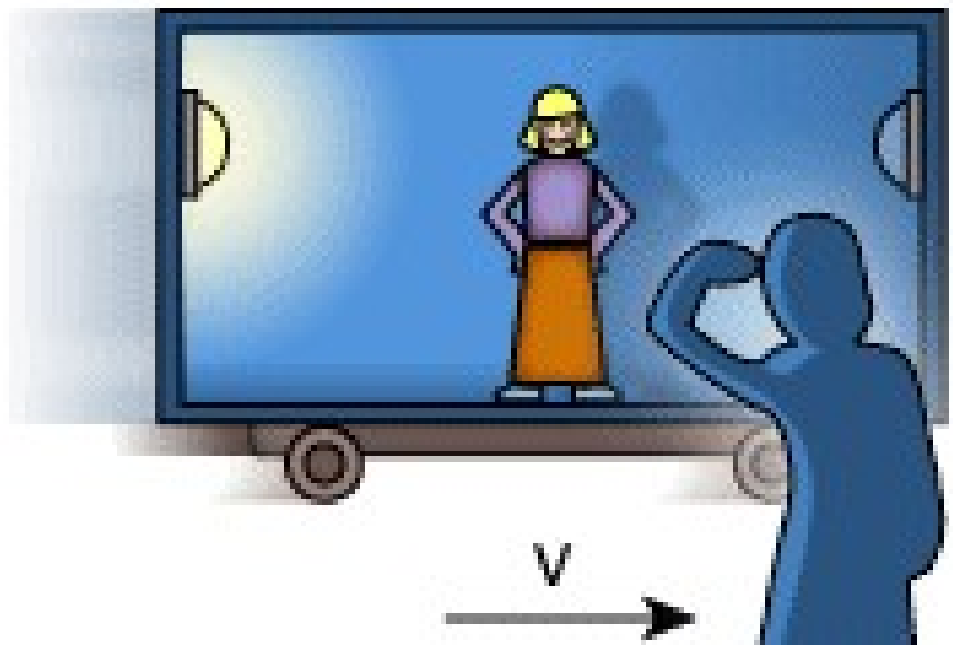

$E=mc^2$

Speed of light in vacuum is an invariant. Therefore, c is a fundamental constant.

Two events simultaneous in one frame may occur at different times in another frame.

The space coordinates and the time coordinate get mixed up by the Lorentz transformations, when one goes from one frame to another that is moving with a relative velocity.

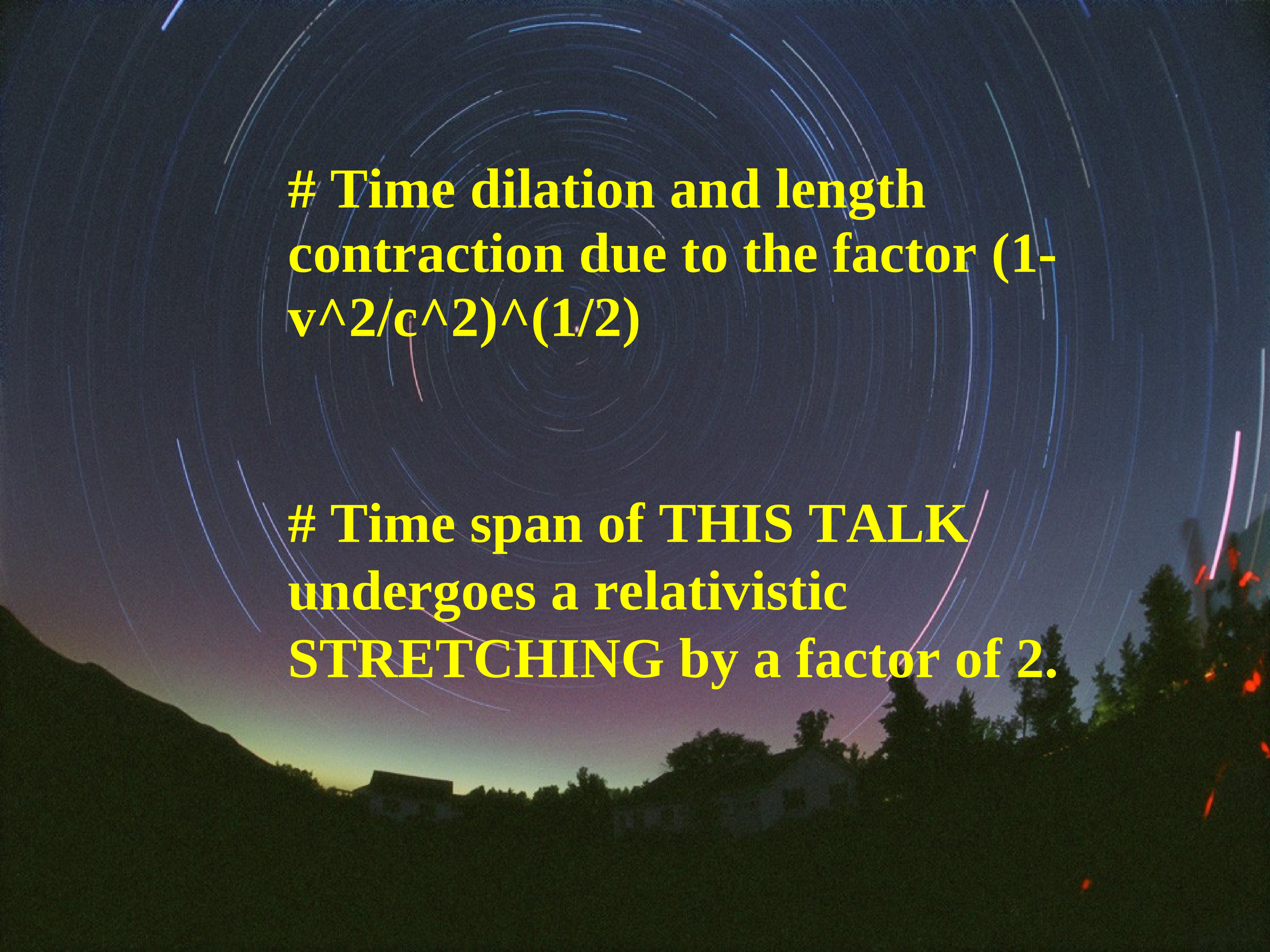



v

No object with a real and positive rest mass can move faster than the speed c= 3 times 10^10 cm/s.

There cannot be a perfect rigid body.

Energy is equivalent to inertial mass (E=mc^2)


Time dilation and length contraction due to the factor (1-v^2/c^2)^(1/2)
Time span of THIS TALK undergoes a relativistic STRETCHING by a factor of 2.

Special Relativity

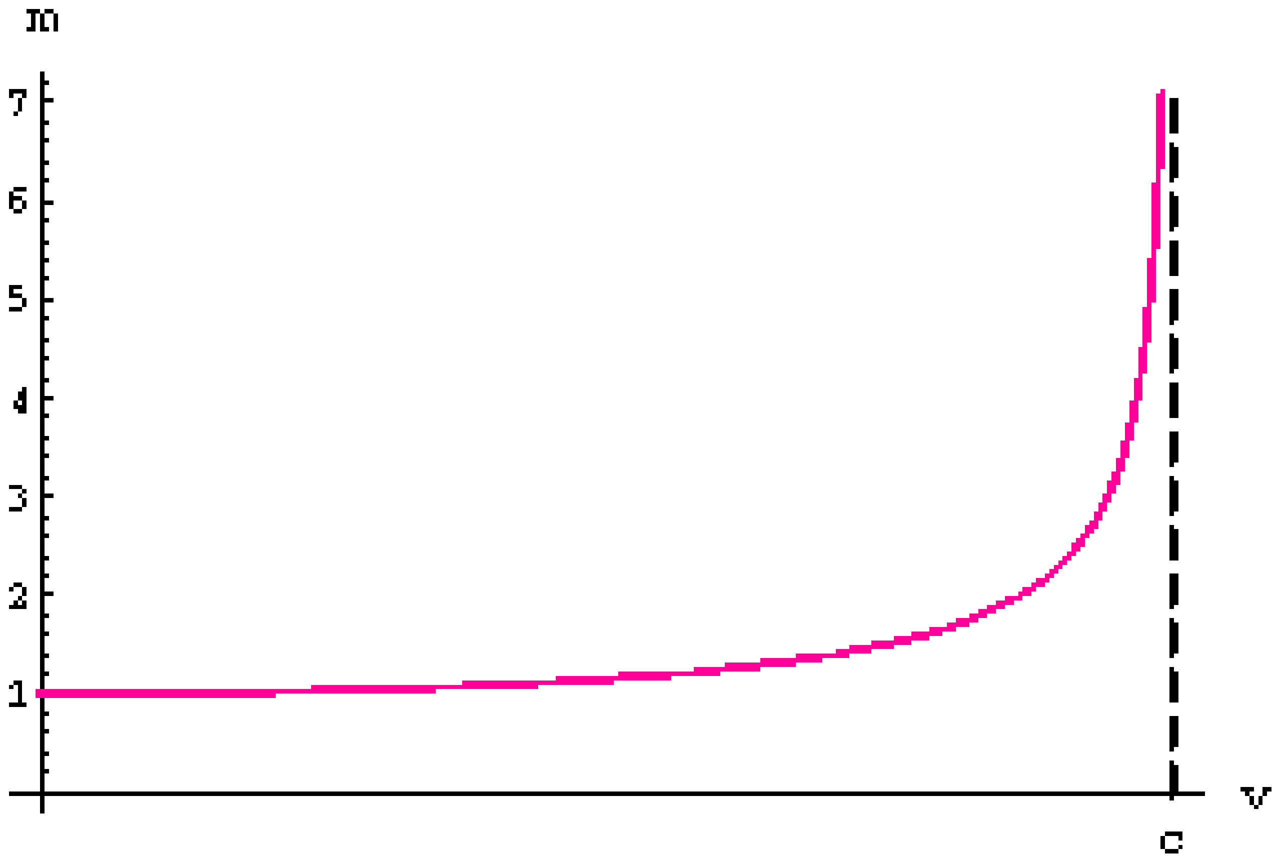
$$t' = \gamma (t - v\, x/c^2)$$
$$x' = \gamma (x - v\, t)$$
$$y' = y$$
$$z' = z$$

Galileo's relativity

$$t' = t$$
$$x' = x - v\, t$$
$$y' = y$$
$$z' = z$$

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz gamma factor = (1 – v^2/c^2)^(-1/2)



m
7
6
5
4
3
2
1
v
c

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \frac{v}{c^2}x)$$

Where γ is defined as

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Special Relativity

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

At $v = 0.42c$, $\gamma = 1.10$, which means the effects of Relativity become noticeable.

Length contraction

$$L' = \frac{L_0}{\gamma}$$

Time dilation

$$t' = \gamma\, t$$

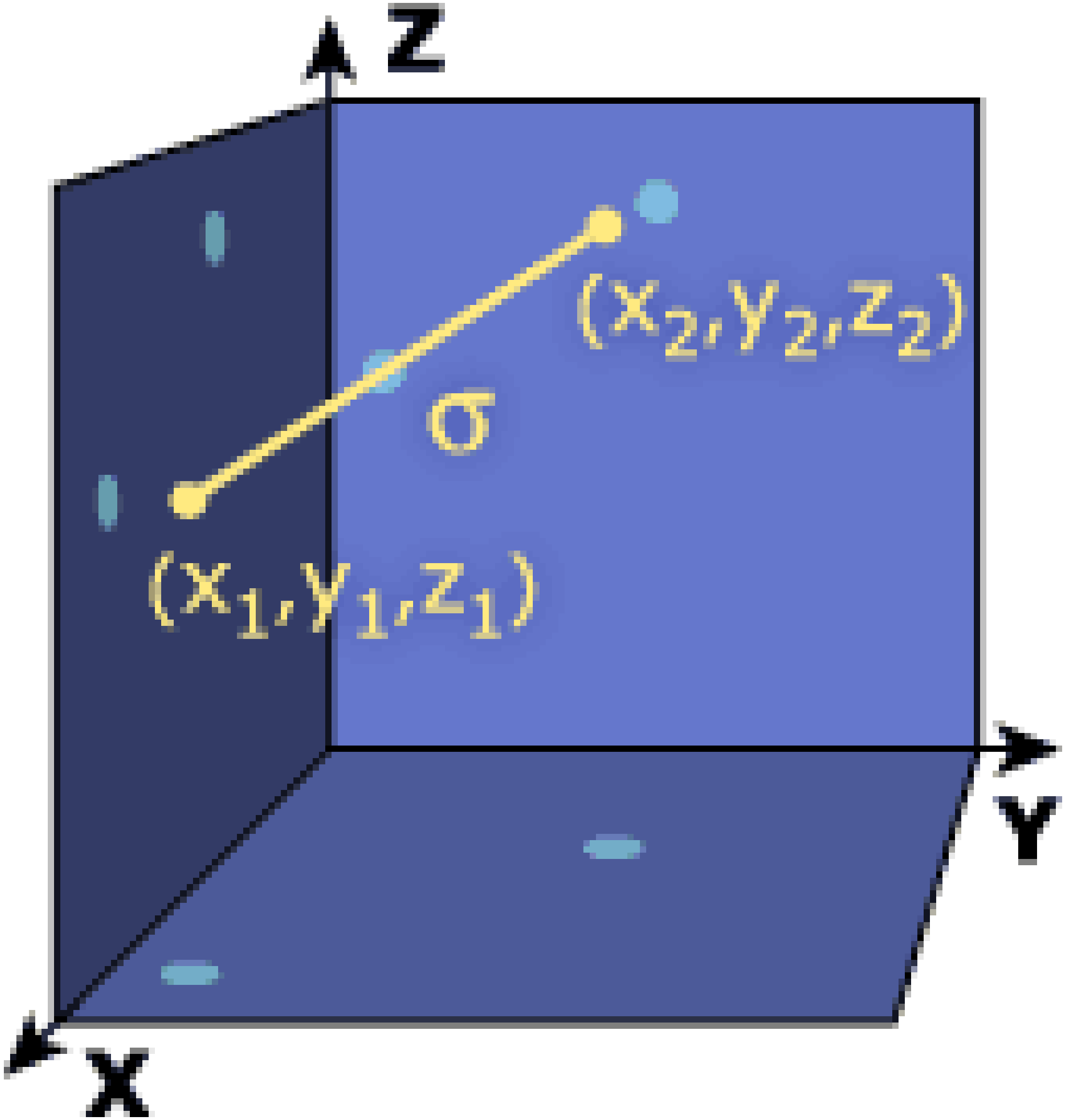
Euclidean Geometry:

 # (length)^2= (sigma)^2 =

(x1 - x2)^2 + (y1 - y2)^2 + (z1 - z2)^2,

where (x1, y1, z1) and (x2, y2, z2) are the coordinates of the two points in 3 D space.

 # Length is invariant under translations and rotations of 3D coordinate system.



Z
(x2,y2,z2)
σ
(x1,y1,z1)
Y
X

Special relativity:

 # The proper interval between two events is defined by

$$(s_1 - s_2)^2 = c^2 (t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2,$$

or, $$-(s_1 - s_2)^2 = - c^2 (t_1 - t_2)^2 + (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2,$$

where (t_1, x_1, y_1, z_1) and (t_2, x_2, y_2, z_2) are the coordinates of the two events.

The proper interval is invariant under Lorentz transformations, space-time translations as well as rotations in 3D space.

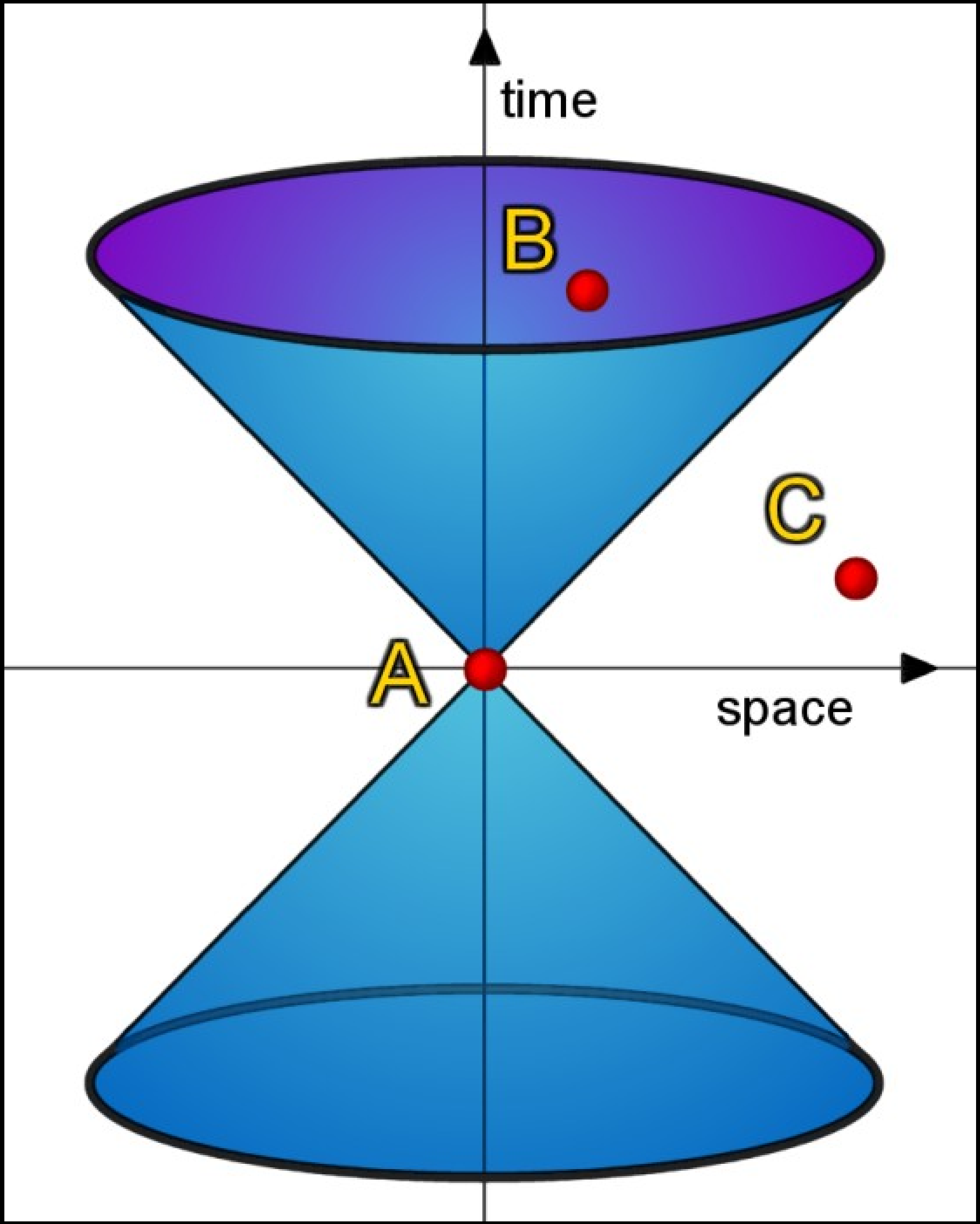
Note that $(s1 - s2)^2$ can be negative for a pair of events! Such events are necessarily causally disconnected.

If we consider two events separated infinitesimally, (t, x, y, z) and $(t + dt, x + dx, y + dy, z + dz)$, then the interval becomes

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2.$$

or, $$-ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$$

This "distance" formula is called the Minkowski line-element and the corresponding four-dimensional "world" is called the Minkowski space-time or just the Minkowski space.


time
B
C
A
space

Minkowski: "Henceforth space by itself, and time by itself are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

$$ds^2 = \sum_{\alpha,\beta=0}^{3} \eta_{\alpha\beta}\, dx^{\alpha}\, dx^{\beta}$$

$$ds^2 = \{\boldsymbol{c\, dt}, \boldsymbol{dx}, \boldsymbol{dy}, \boldsymbol{dz}\} \bullet \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \bullet \begin{Bmatrix} c\,dt \\ dx \\ dy \\ dz \end{Bmatrix}$$

Imaginary time?

Use "i t" instead of "t" !

-ds^2=dx^2 + dy^2 + dz^2 - c^2 dt^2

=dx^2 + dy^2 + dz^2 + d(i c t)^2

Graffiti in I.I.Sc., Bangalore (~ 1981):

Person A: Einstein had an i on time.

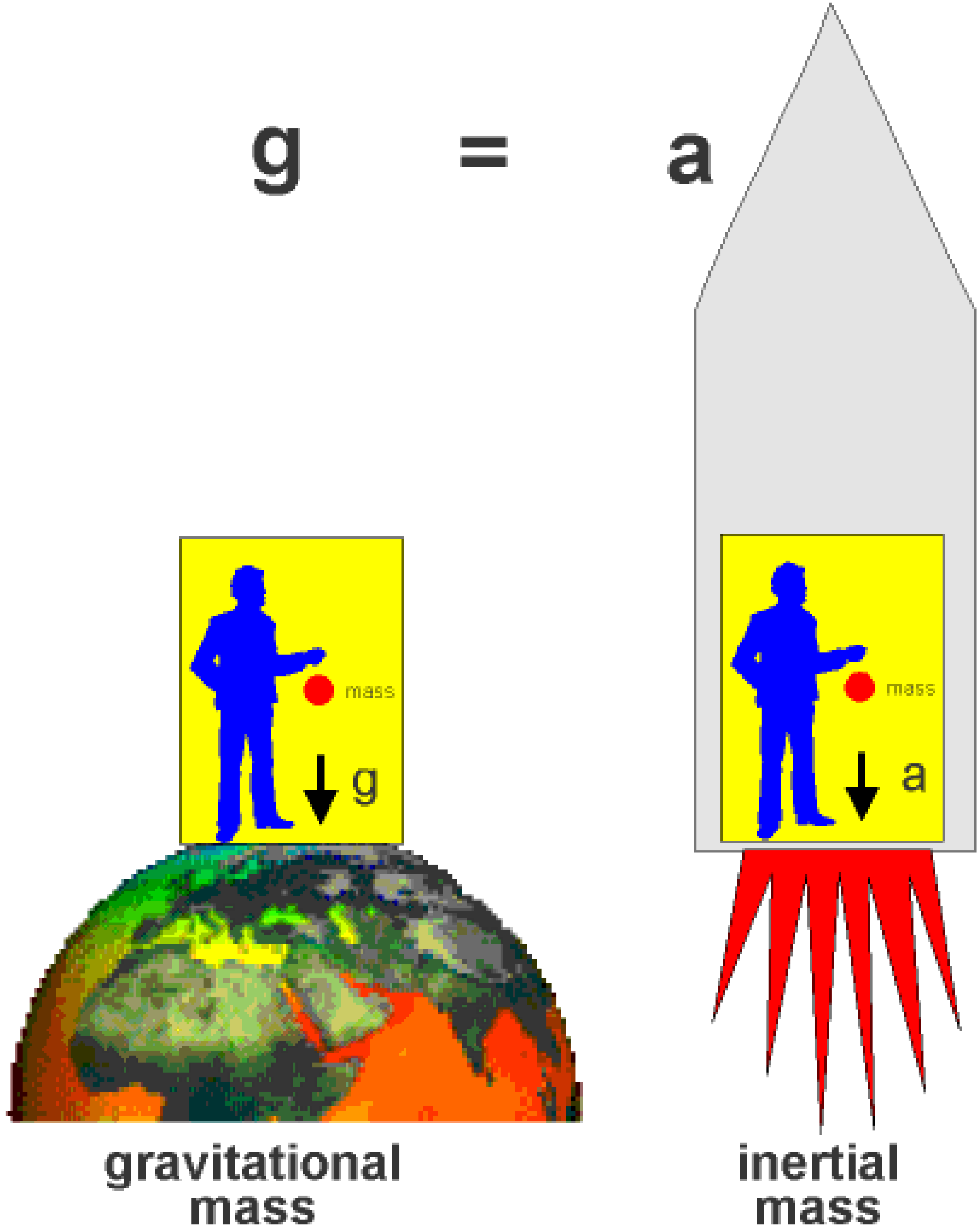
Person B: i c .

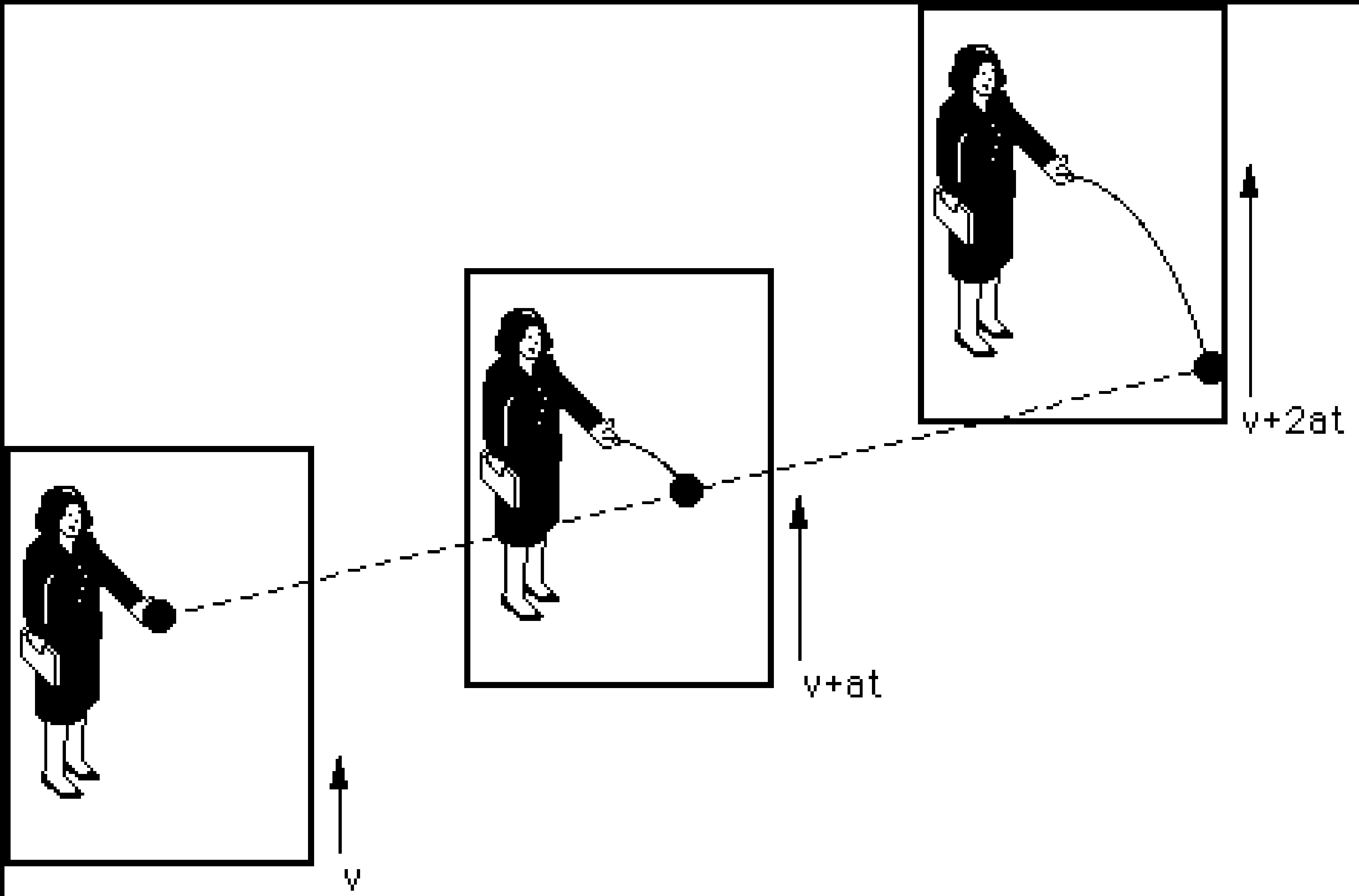
Einstein's relativistic theory of GRAVITATION is based on the EQUIVALENCE PRINCIPLE:

Inertial Mass = Gravitational Mass (Galileo, Eotvos,TIFR-Gauribidnur,....)

Gravitational force disappears in a frame of reference that freely falls in a gravitational field (Einstein: “..... happiest day of my life ….”)

Or, equivalently, in free space (i.e. zero gravity condition), one can simulate a gravitational field by going to an accelerating frame


g = a
mass
g
mass
a
gravitational mass
inertial mass


v
v+at
v+2at

GENERAL THEORY OF RELATIVITY

Salient features:

Gravity is not a force

The apparent gravitational effects are due to Non-Euclidean nature of space-time geometry

Presence of matter makes the geometry curved

GRAVITY is equivalent to space-time GEOMETRY

GENERAL THEORY OF RELATIVITY

 # Geometry is characterized by 10 functions of space and time called the metric tensor g_{ij}

 # Proper interval, $ds^2 = g_{ij}\ dx_i\ dx_j$, (i,j summed over from 0 to 3)

(Special Relativity: $ds^2 = n_{ij}\ dx_i\ dx_j$)

 # x_0 is time coordinate, and x_1, x_2 and x_3 are the spatial coordinates

 # The metric is determined using the Einstein equation

Consequences of GTR:

Redshifting of light → (Test of Equivalence Principle)
(Time runs slower in stronger gravity)

Bending of light → Gravitational Lensing

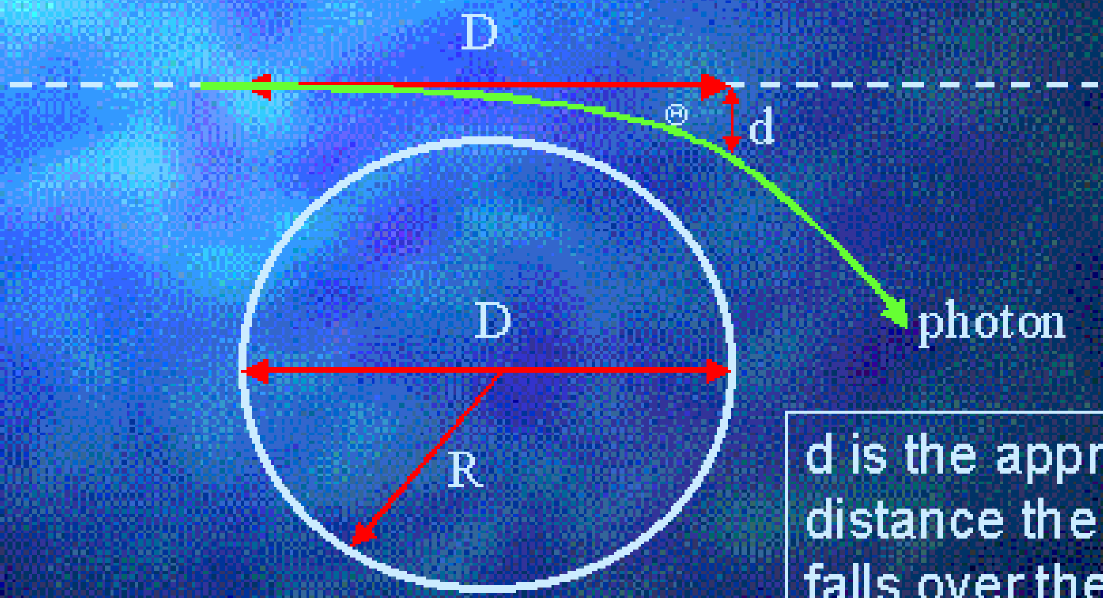
Lense - Thirring Precession → Precession of Gyroscopes

Gravitational Waves → Hulse-Taylor binaries, LIGO, LISA

Blackholes → x-ray sources, supermassive BHs

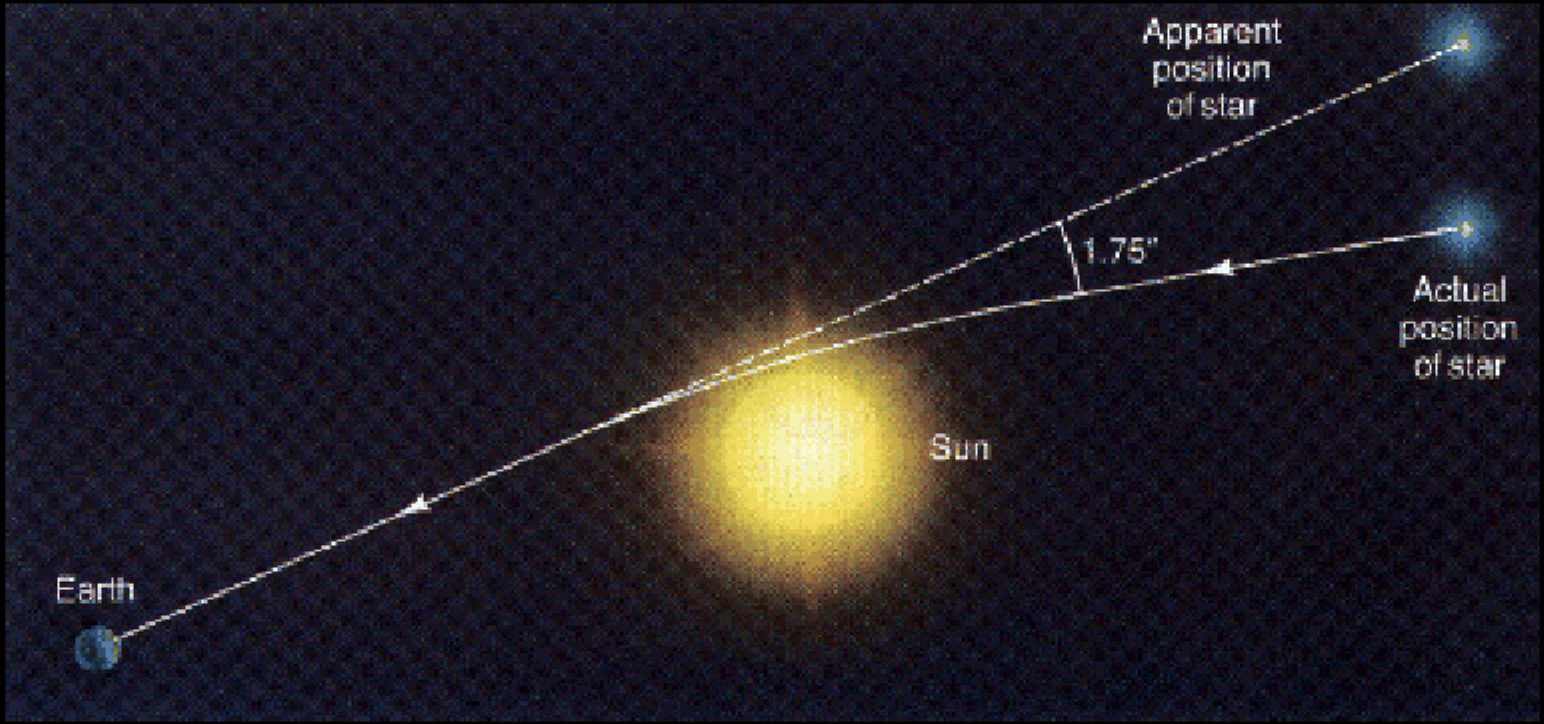
Big-bang model

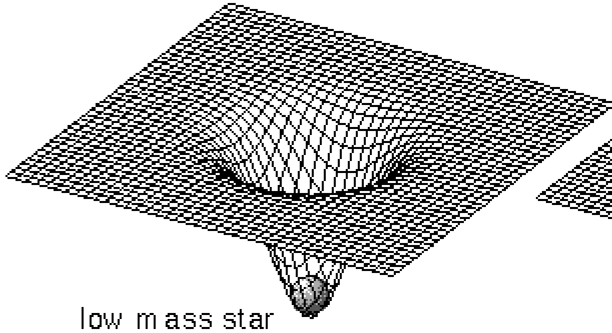
Bending of Light

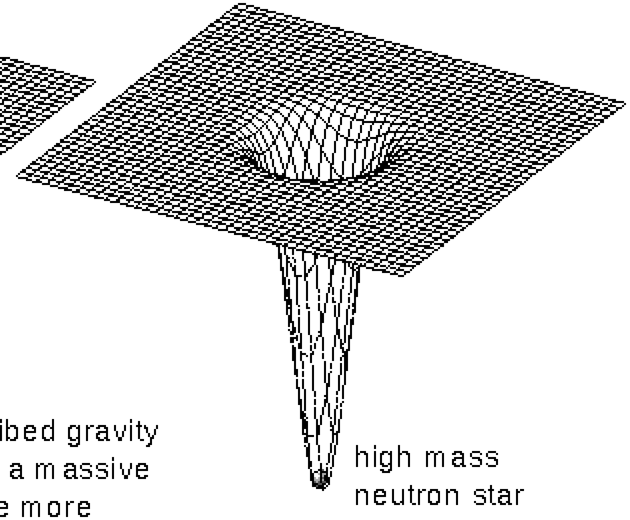


d is the approximate distance the photon falls over the diagram D of an object with mass M

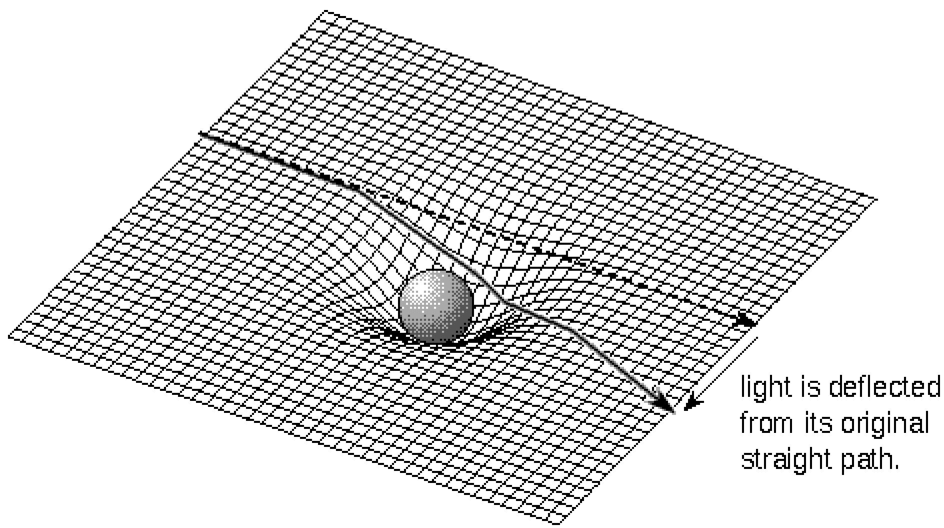
The Bending of Light



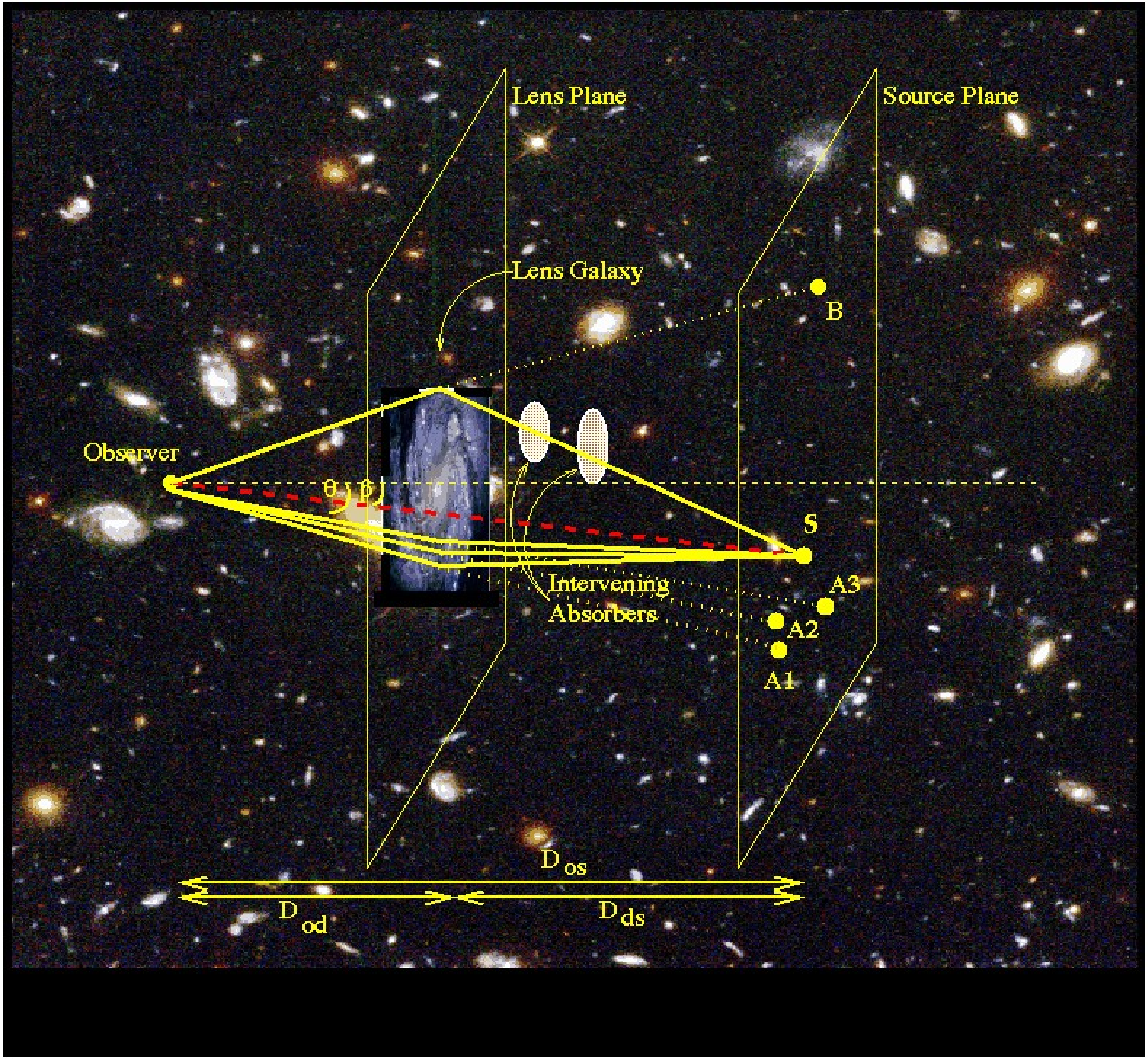




General Relativity: Einstein described gravity as a warping of space-time around a massive object. The stronger the gravity, the more space-time is warped.



General Relativity: Light travels along the curved space taking the shortest path between two points. Therefore, light is deflected toward a massive object! The stronger the local gravity is, the greater the light path is bent.

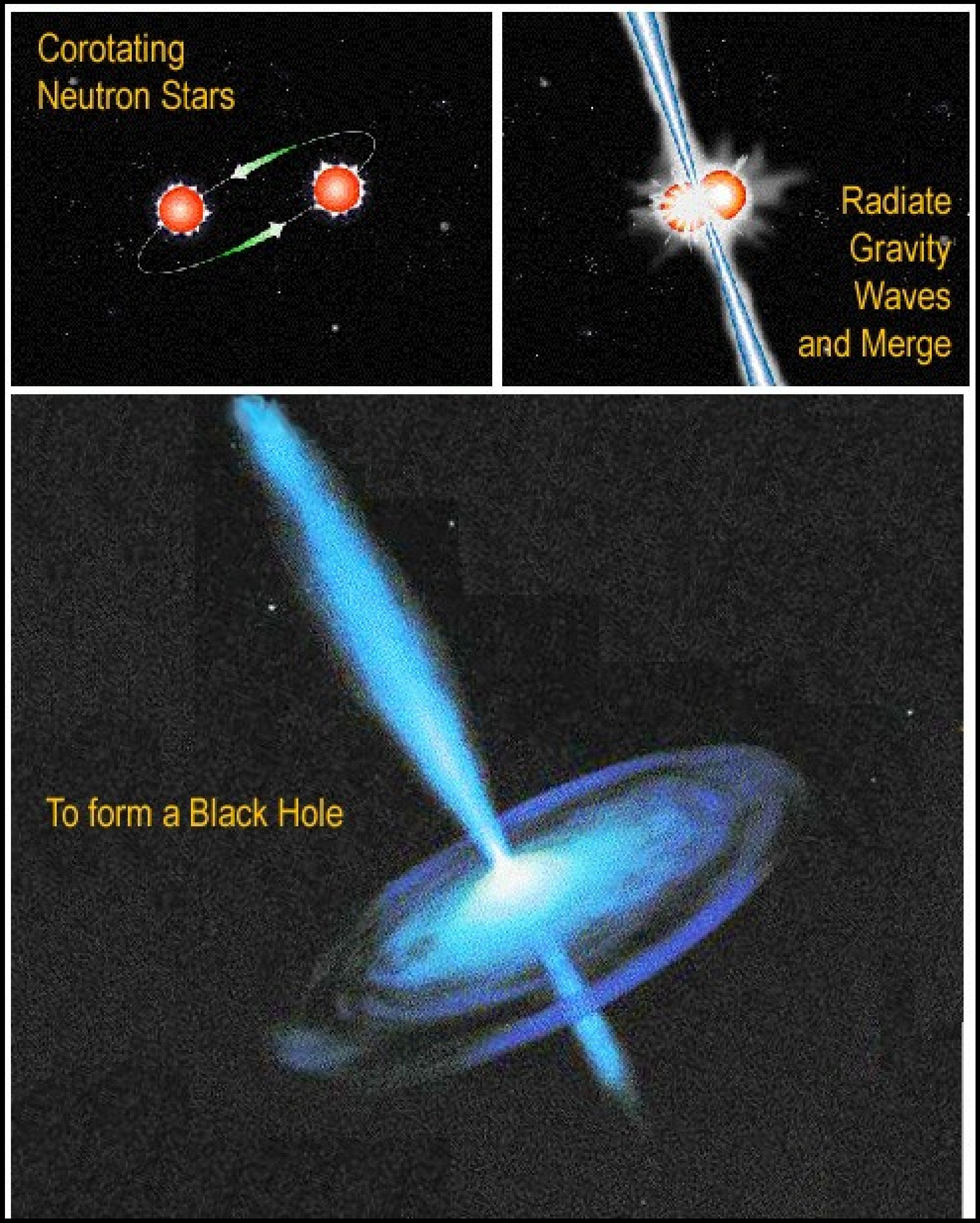

Lens Plane
Source Plane
Lens Galaxy
B
Observer
θ
β
S
Intervening
Absorbers
A3
A2
A1
D os
D od
D ds

Einstein Equation

$$R_{ij} - \frac{1}{2} R g_{ij} - \lambda g_{ij} = \frac{8\pi G}{c^4} T_{ij}$$

T_ij is the energy-momentum tensor of matter.

Lambda is the Cosmological Constant.



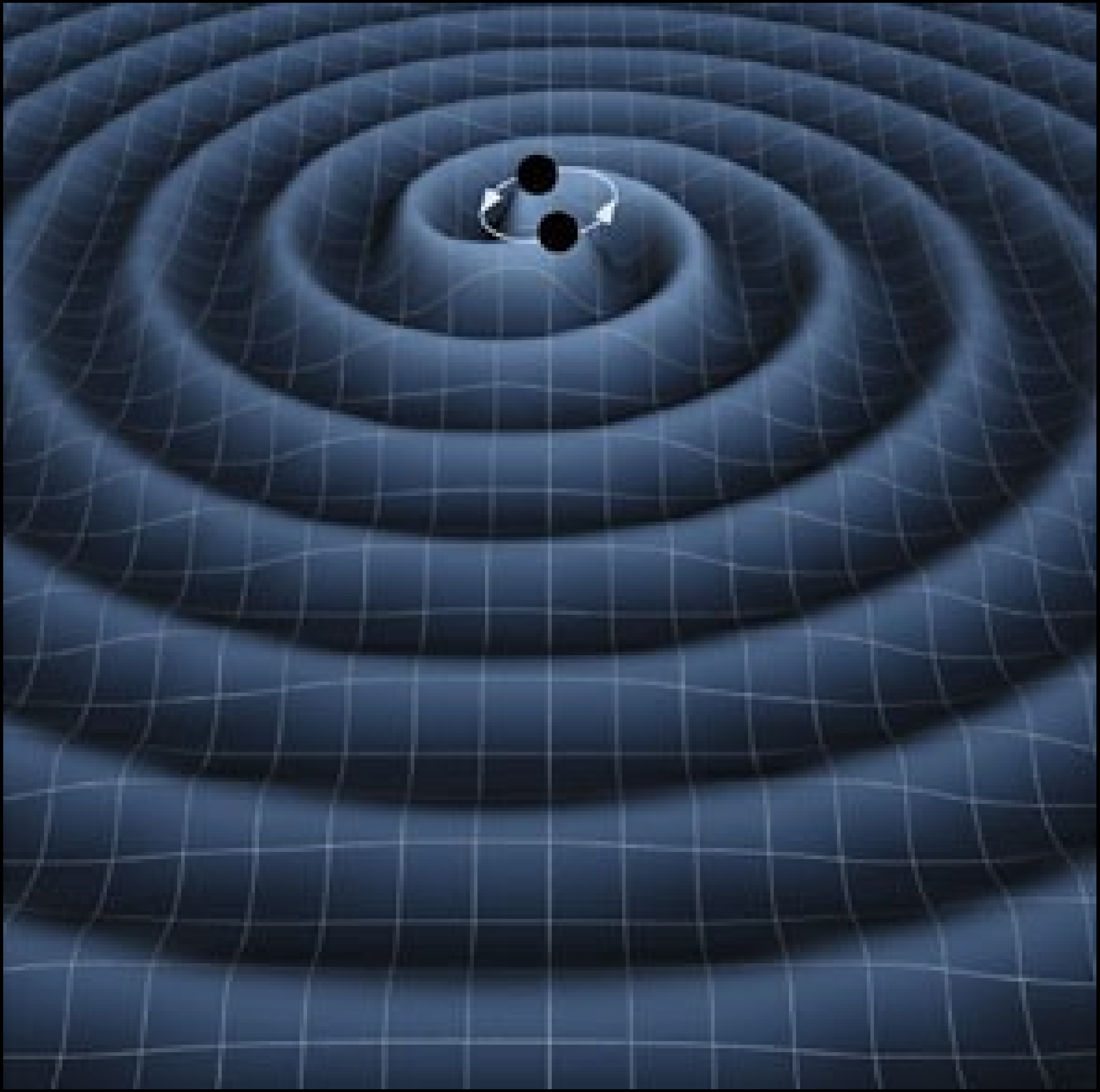
Coalescing

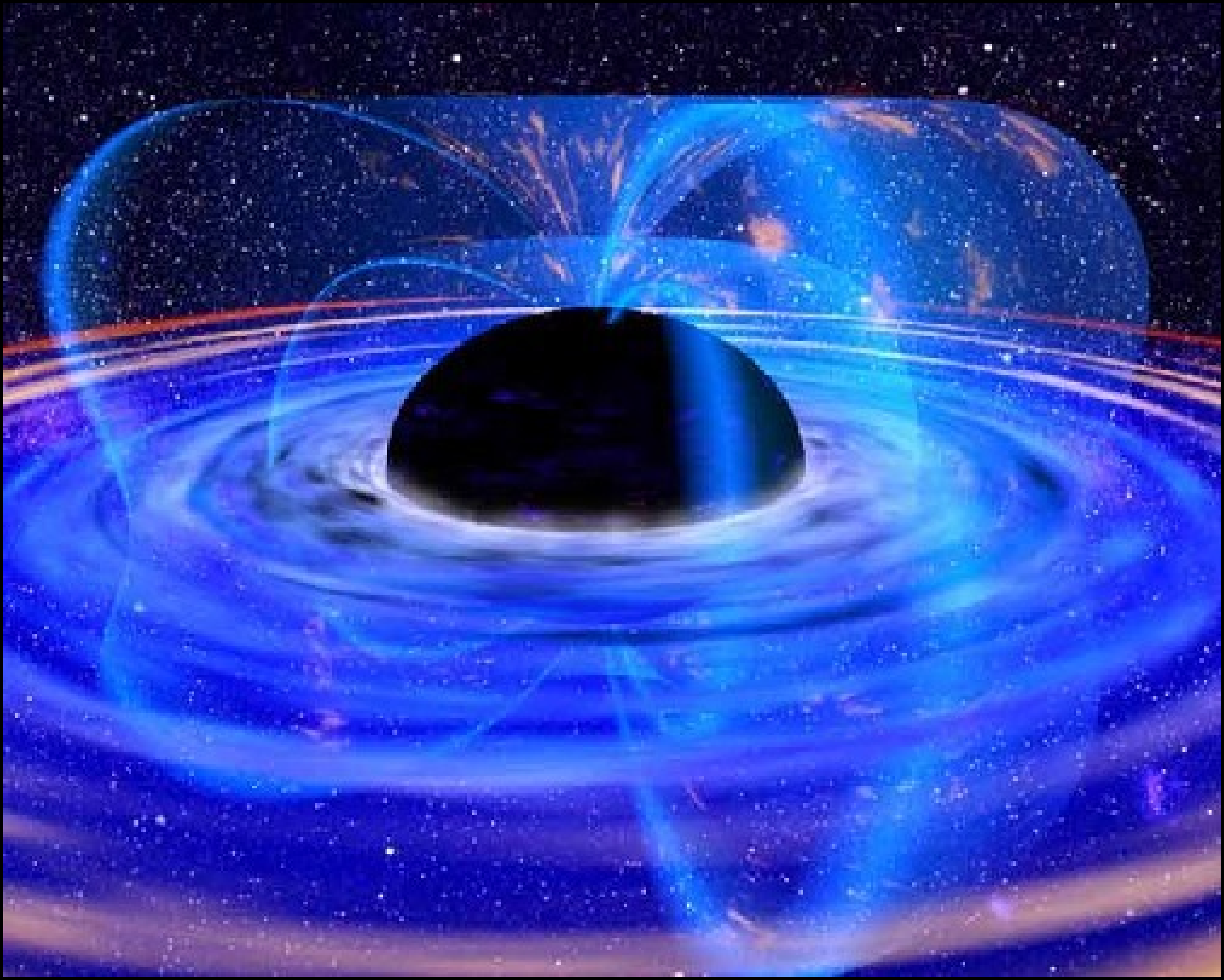
Binary neutron

Stars:

Short duration

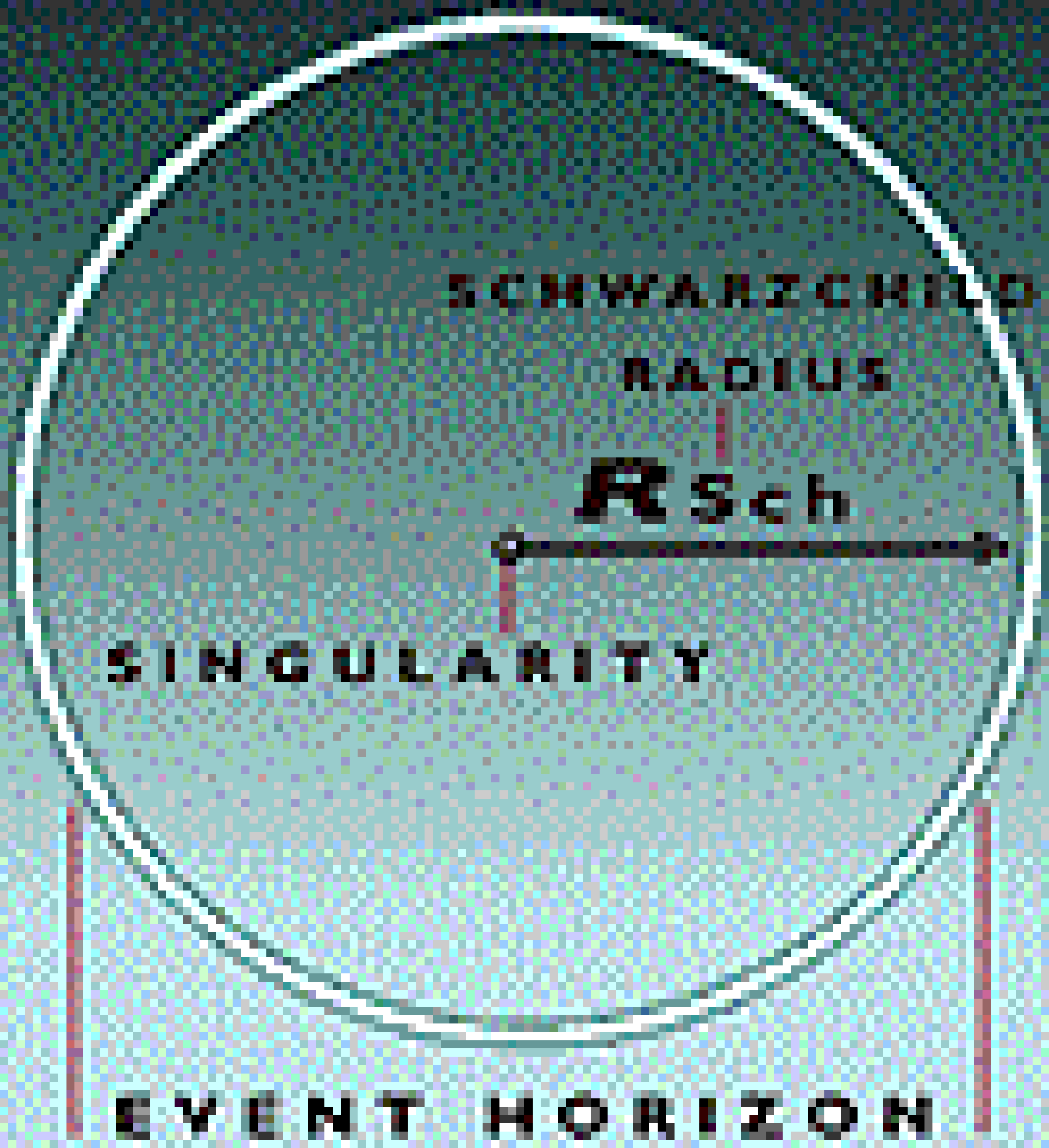
GRBs ?





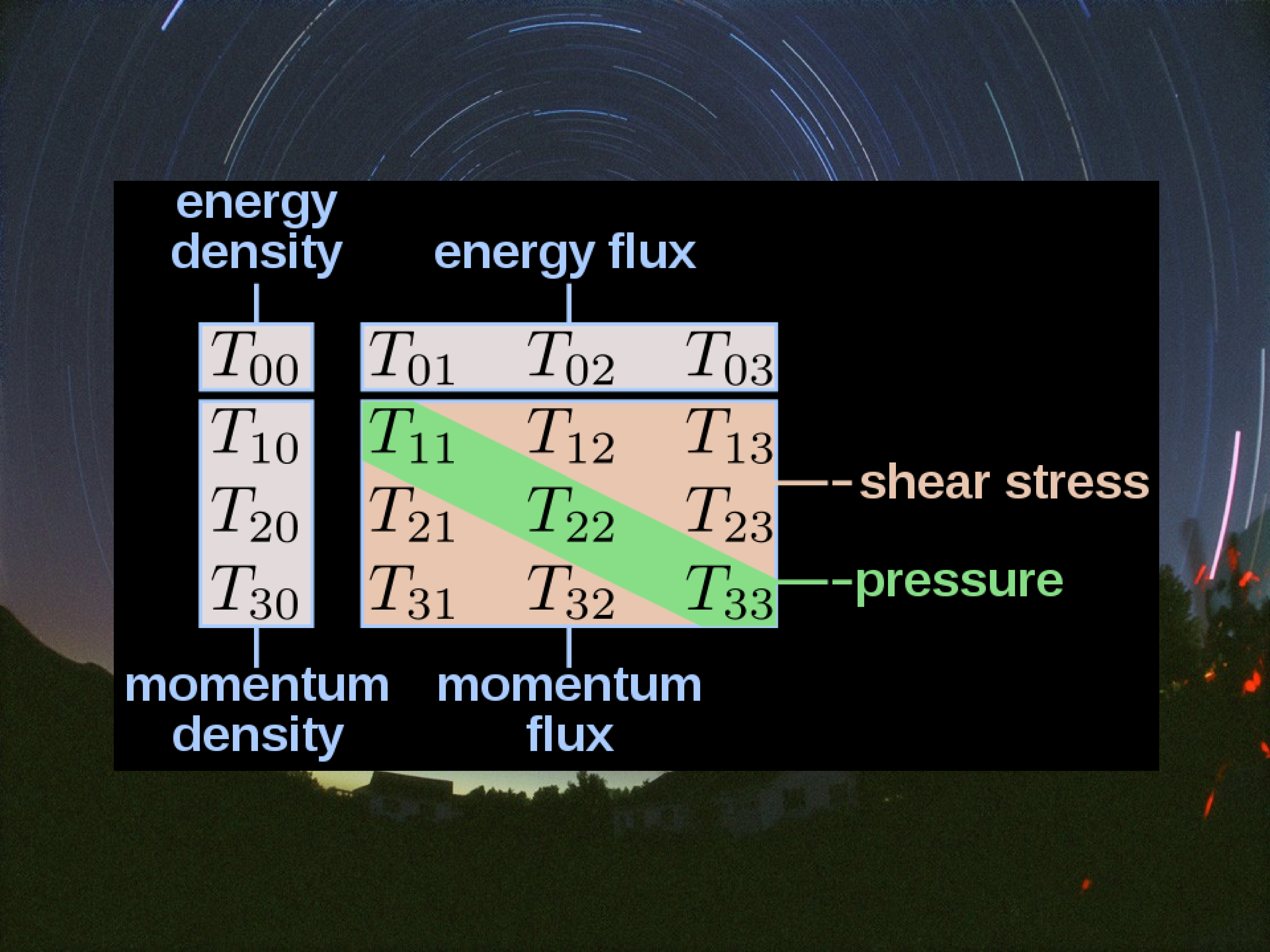
SUPERNOVA EXPLOSION

BASIC STRUCTURE

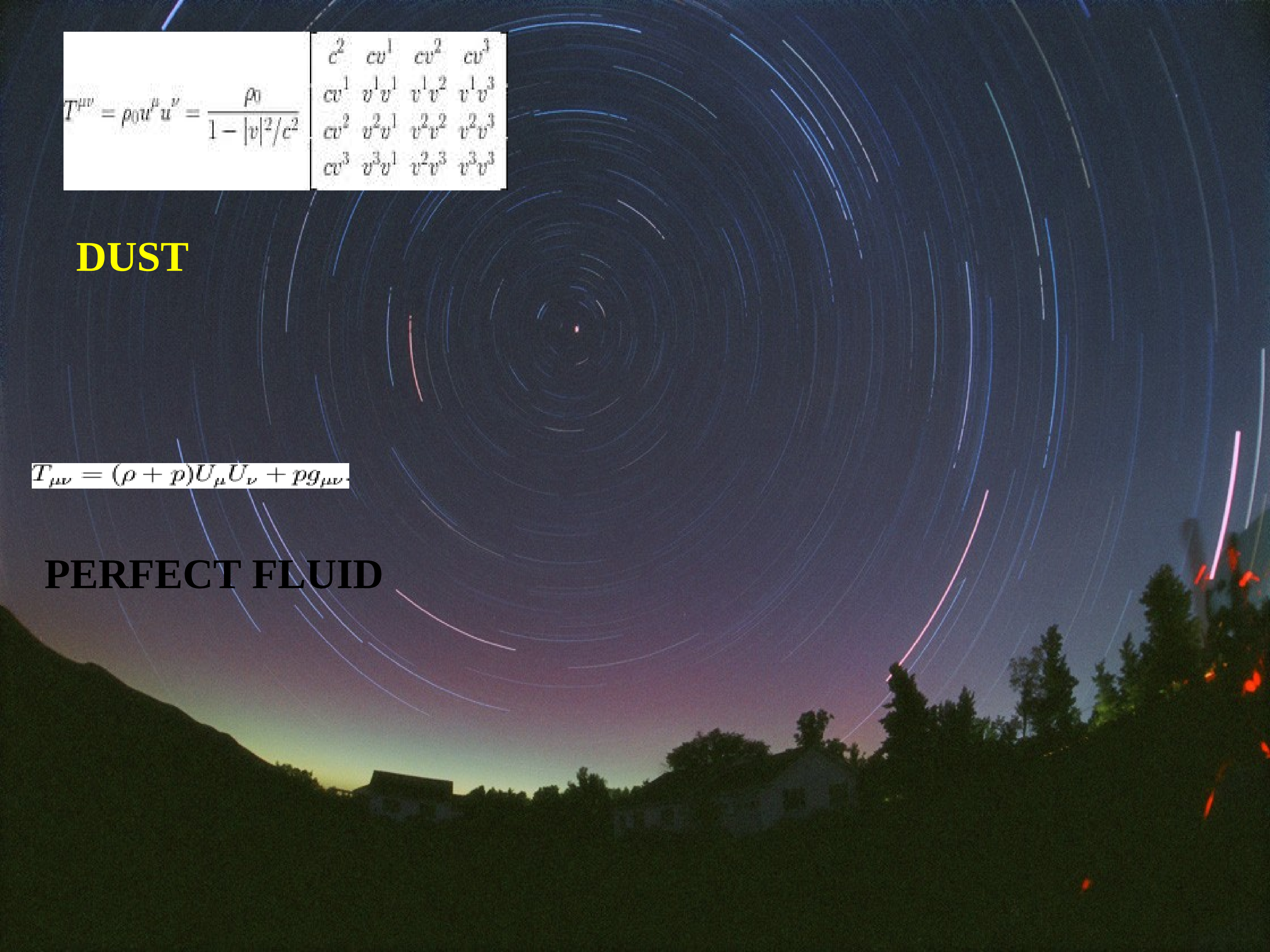


$$G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu},$$



energy density
energy flux
T_{00}
T_{01} T_{02} T_{03}
T_{10}
T_{20}
T_{30}
T_{11} T_{12} T_{13}
T_{21} T_{22} T_{23}
T_{31} T_{32} T_{33}
shear stress
pressure
momentum density
momentum flux

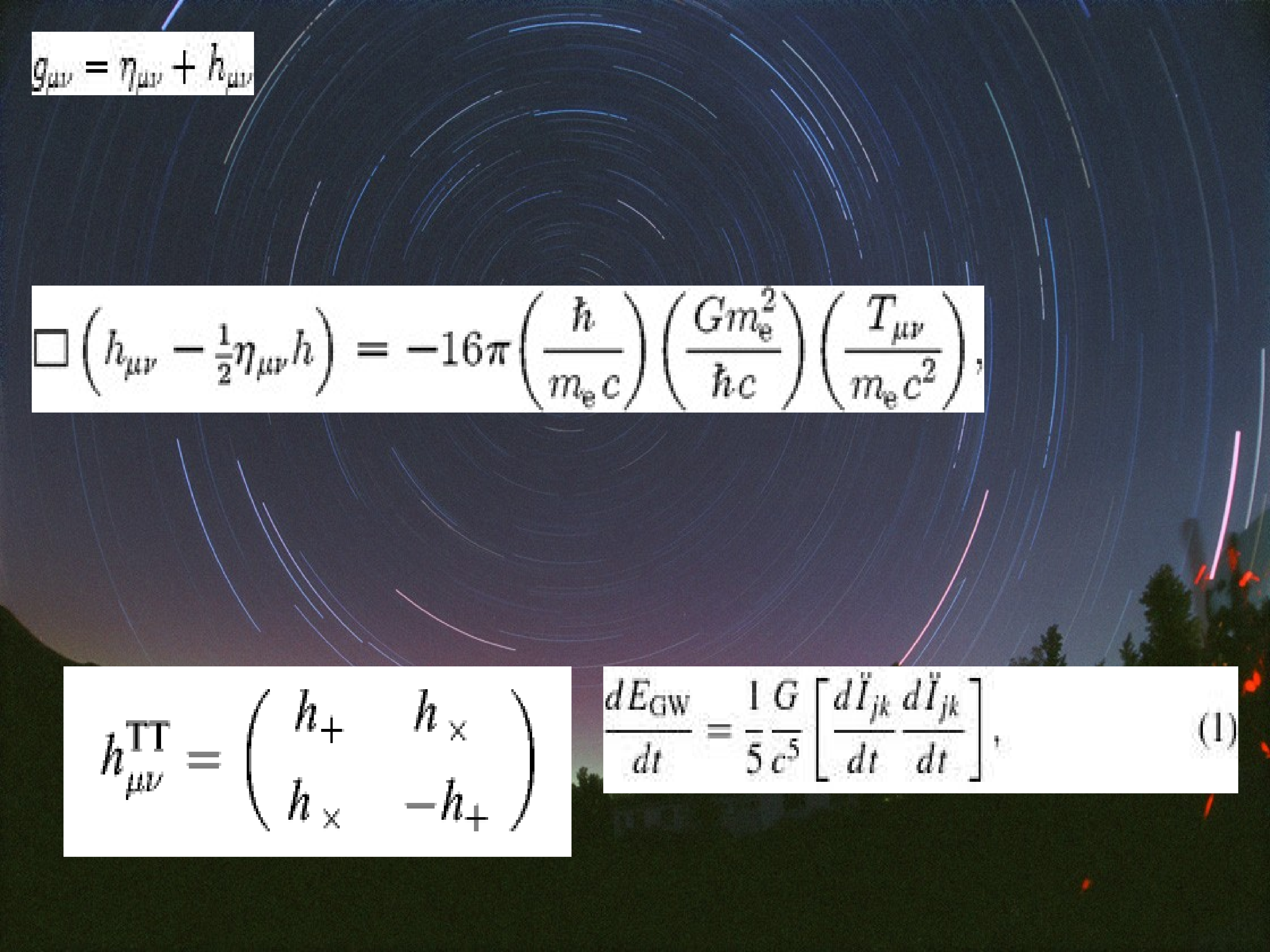


$$T^{\mu\nu} = \rho_0 u^\mu u^\nu = \frac{\rho_0}{1-|v|^2/c^2} \begin{matrix} c^2 & cv^1 & cv^2 & cv^3 \\ cv^1 & v^1v^1 & v^1v^2 & v^1v^3 \\ cv^2 & v^2v^1 & v^2v^2 & v^2v^3 \\ cv^3 & v^3v^1 & v^2v^3 & v^3v^3 \end{matrix}$$

DUST

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + pg_{\mu\nu}.$$

PERFECT FLUID

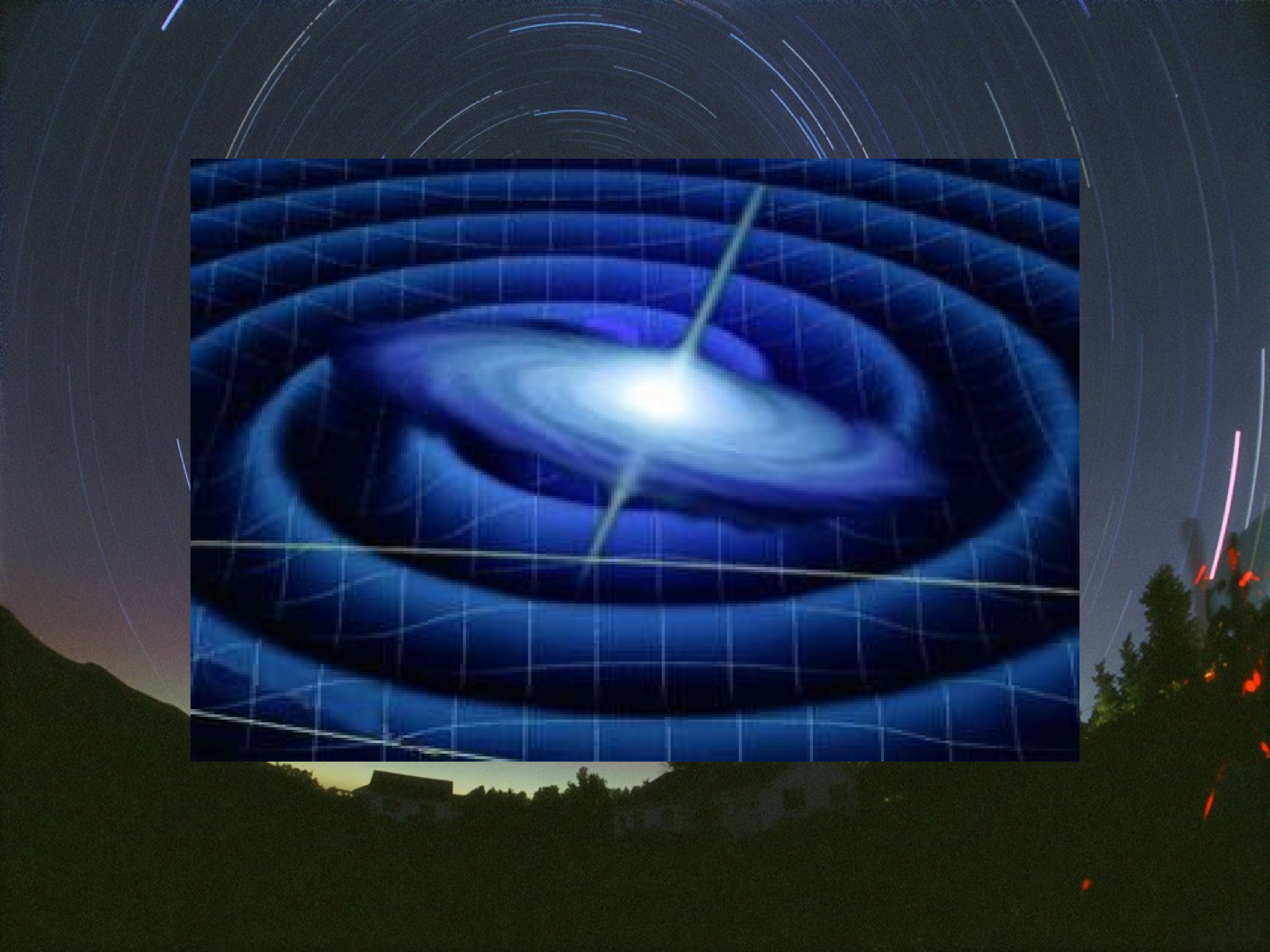


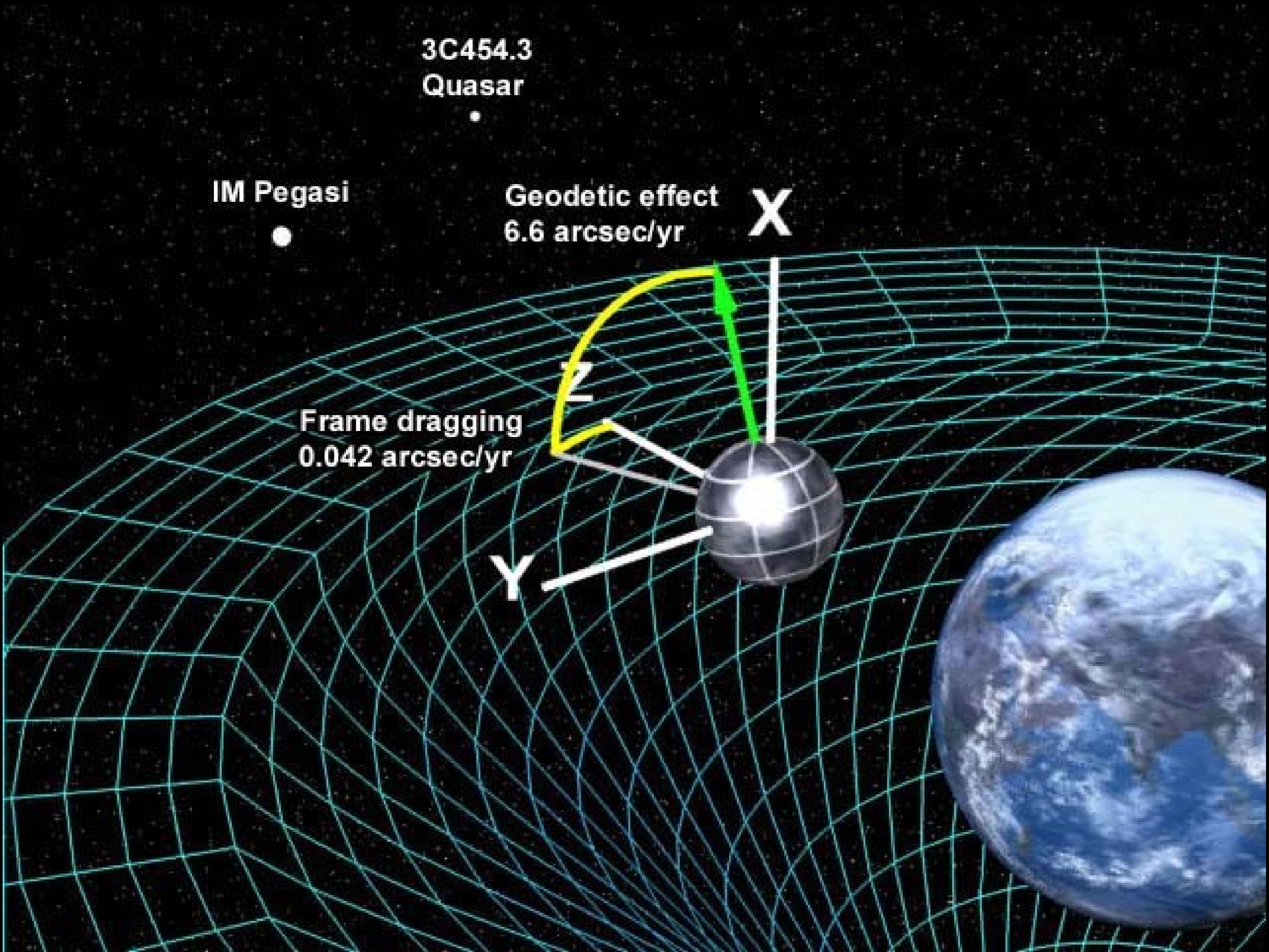
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\Box\left(h_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}h\right) = -16\pi\left(\frac{\hbar}{m_{\rm e}\,c}\right)\left(\frac{Gm_{\rm e}^2}{\hbar c}\right)\left(\frac{T_{\mu\nu}}{m_{\rm e}\,c^2}\right),$$

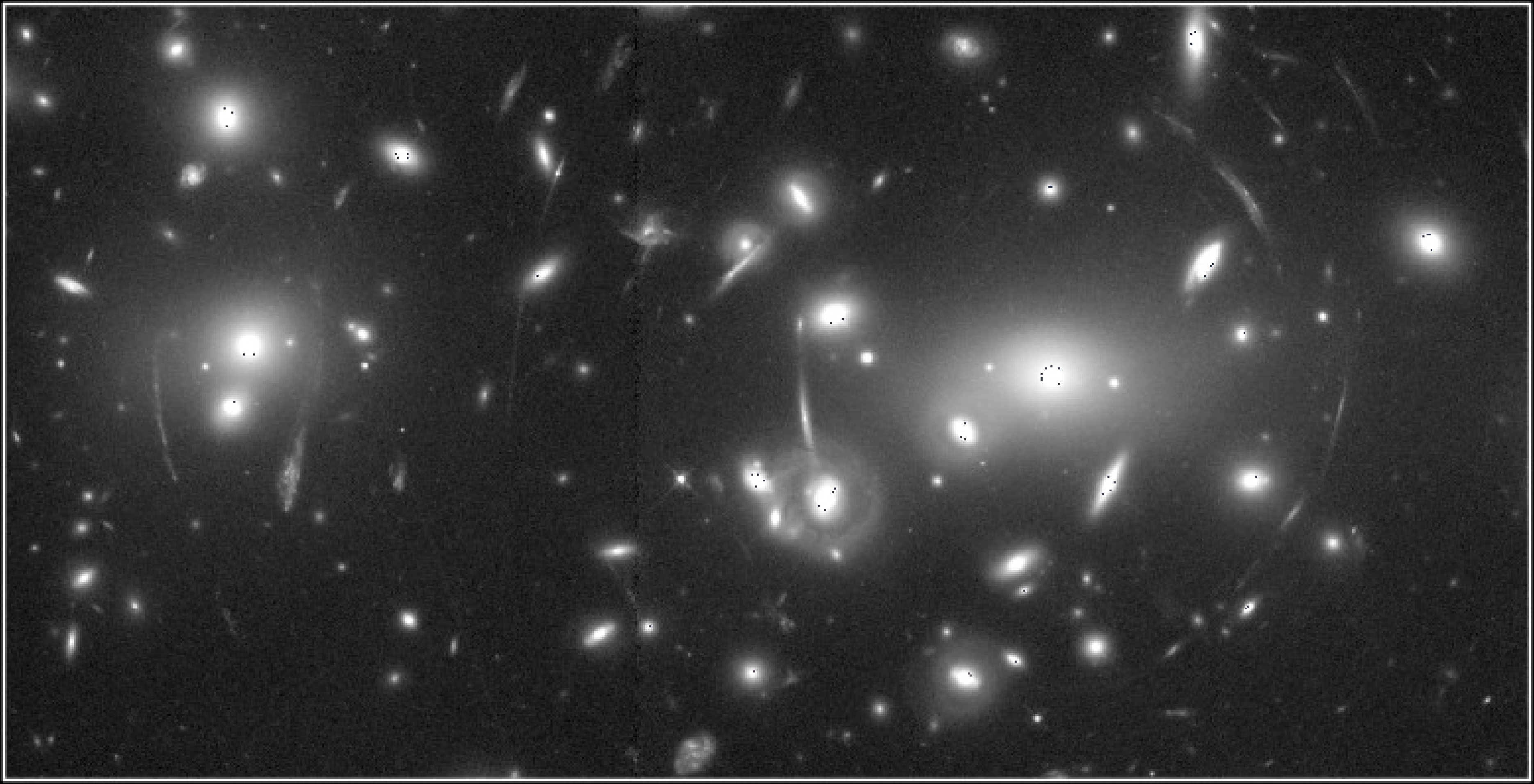
$$h_{\mu\nu}^{\rm TT} = \begin{pmatrix} h_+ & h_\times \\ h_\times & -h_+ \end{pmatrix}$$

$$\frac{dE_{\rm GW}}{dt} = \frac{1}{5}\frac{G}{c^5}\left[\frac{d\ddot{I}_{jk}}{dt}\frac{d\ddot{I}_{jk}}{dt}\right], \qquad (1)$$



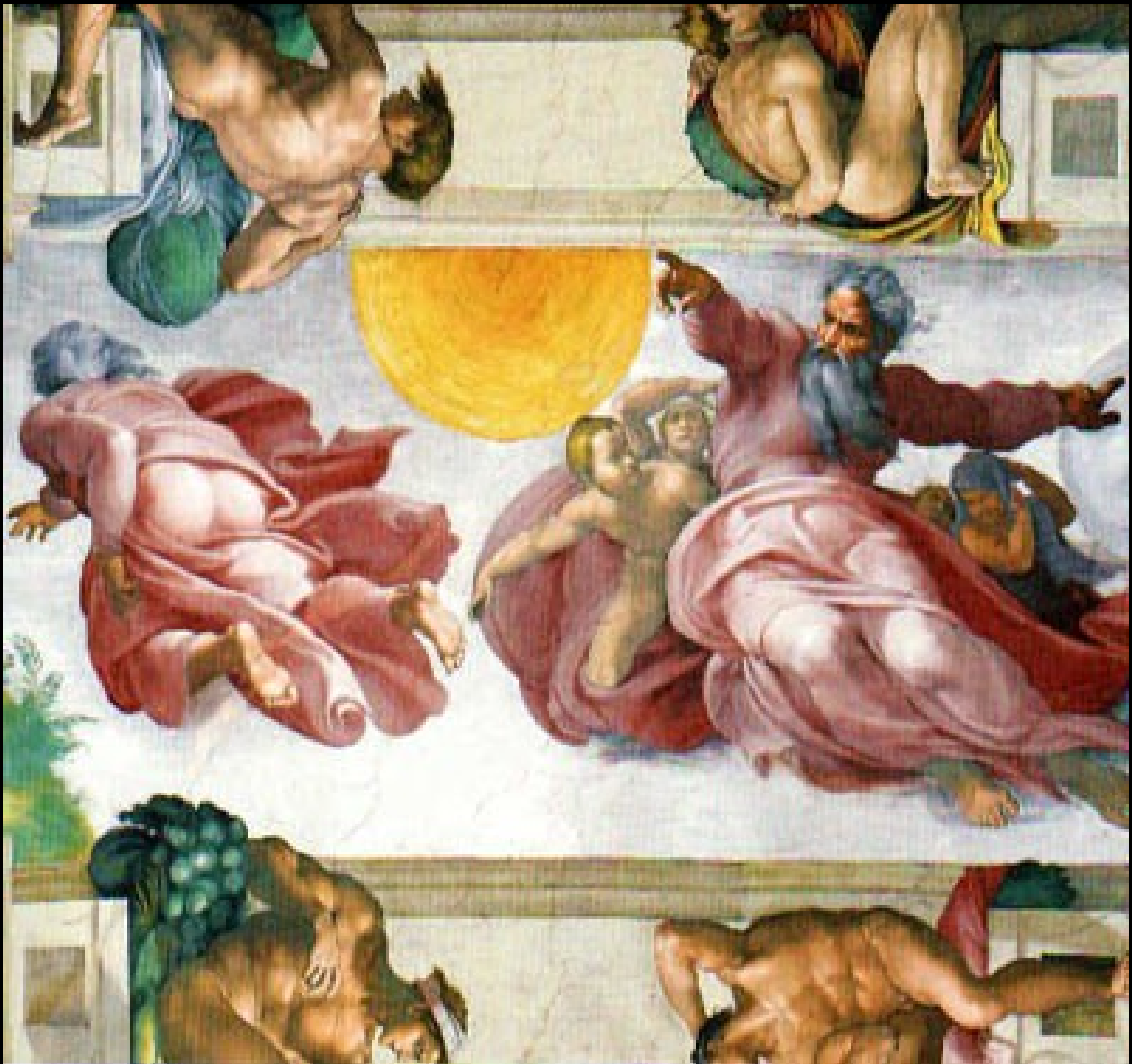


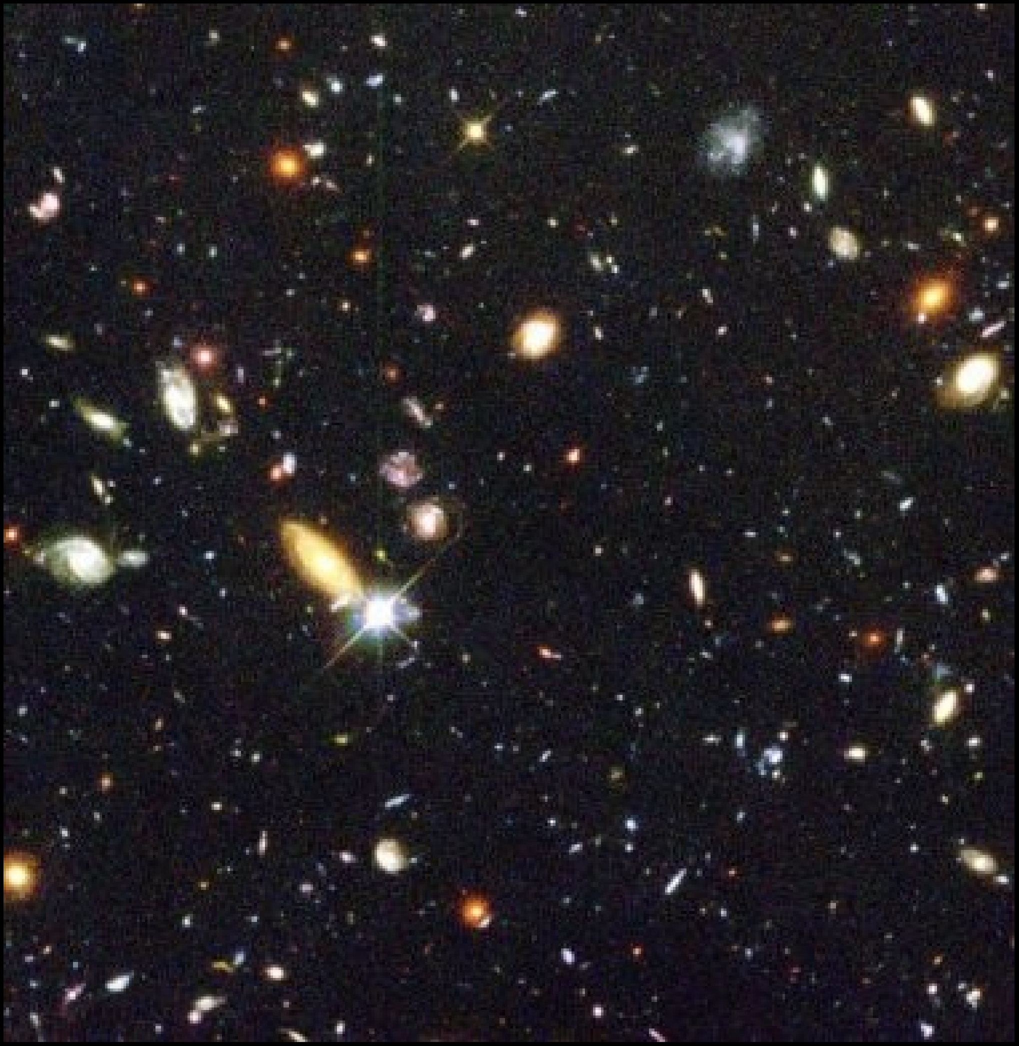
3C454.3
Quasar
IM Pegasi
Geodetic effect
6.6 arcsec/yr
X
Z
Frame dragging
0.042 arcsec/yr
Y

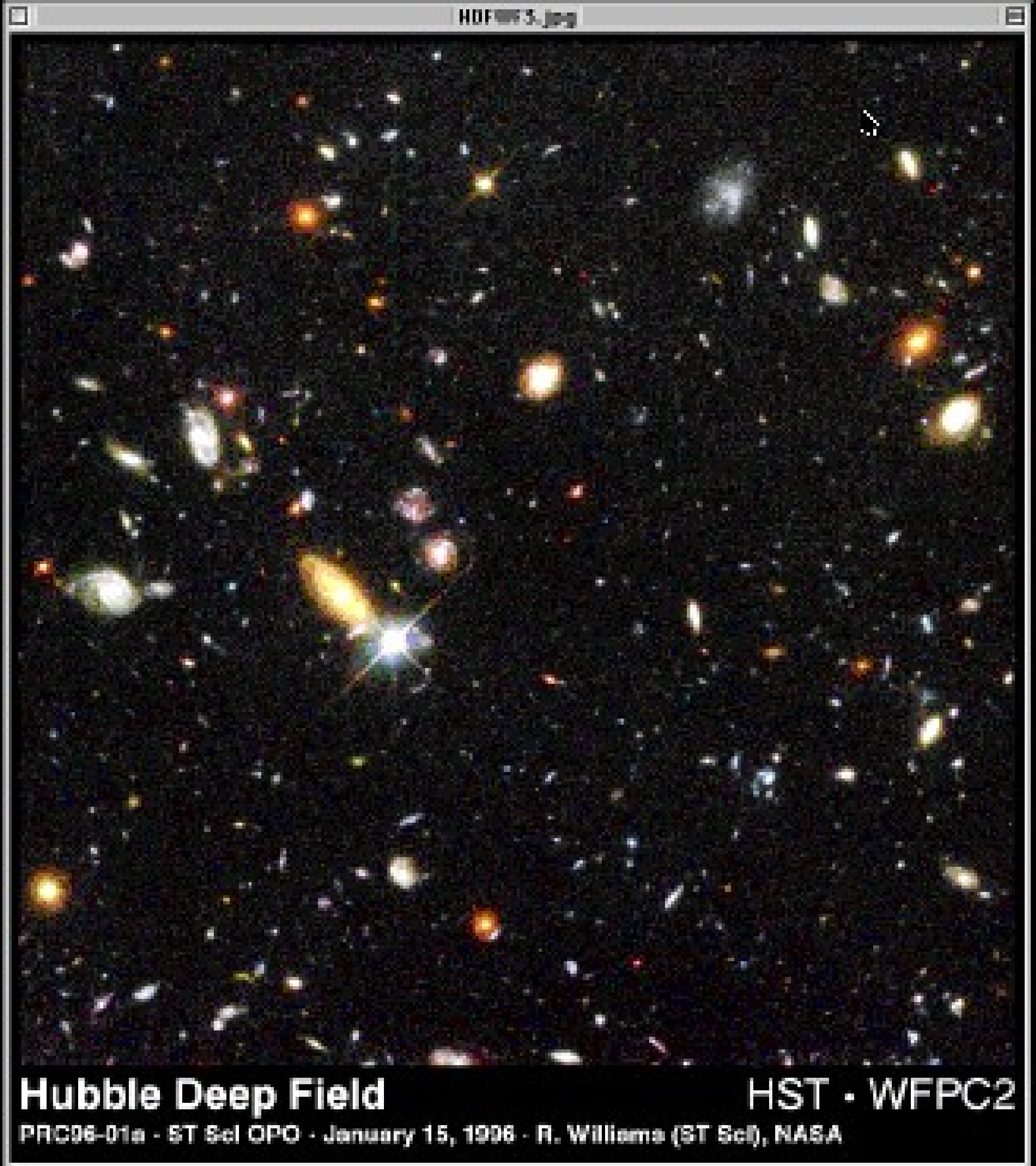


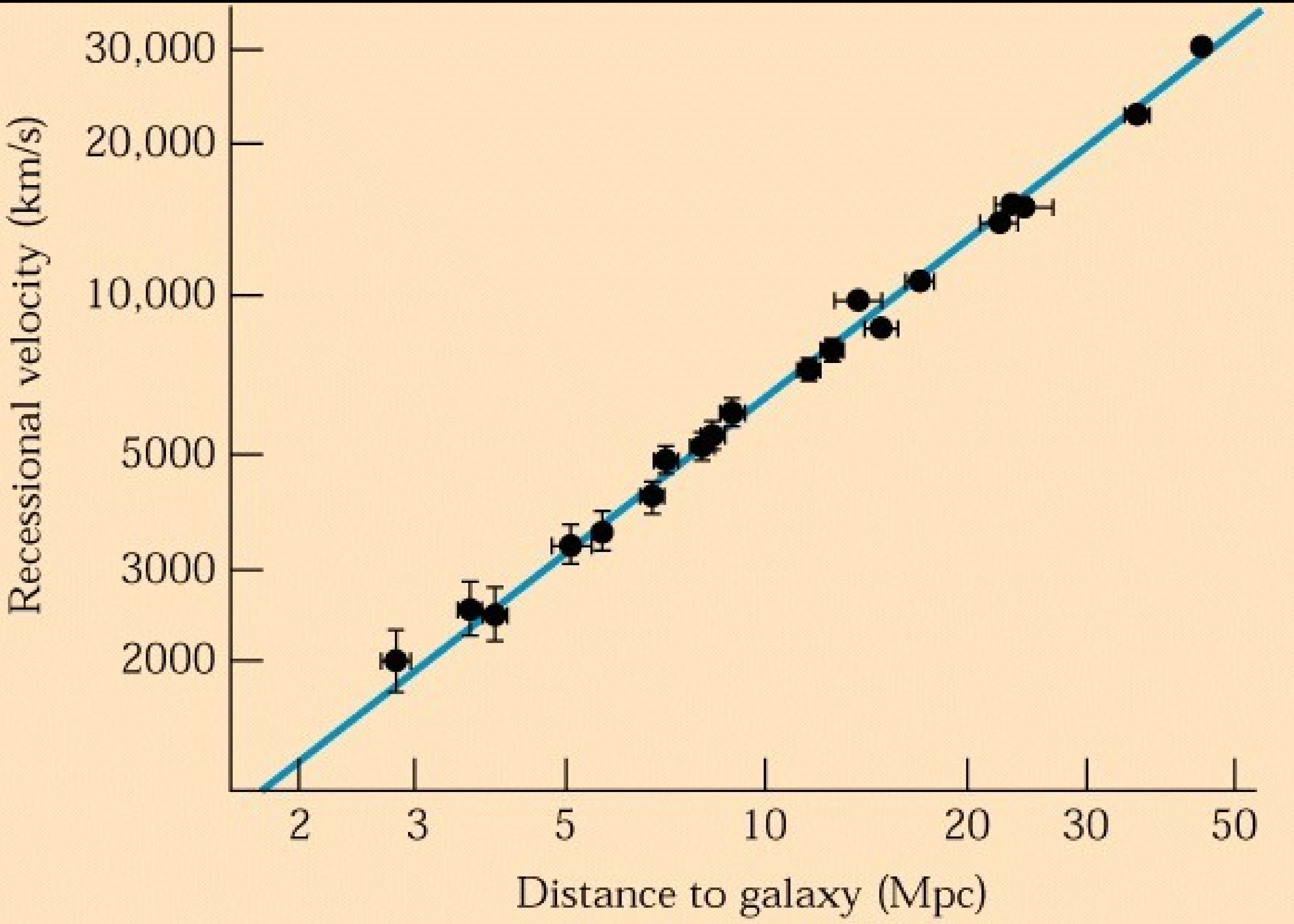
Gravitational Lens in Abell 2218 HST • WFPC2

PF95-14 · ST ScI OPO · April 5, 1995 · W. Couch (UNSW), NASA

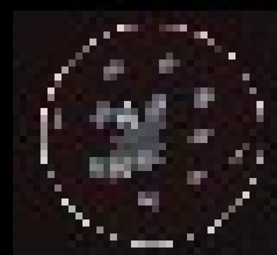





Hubble Deep Field
HST • WFPC2
PRC96-01a · ST ScI OPO · January 15, 1996 · R. Williams (ST ScI), NASA



30,000
20,000
10,000
5000
3000
2000
Recessional velocity (km/s)
2
3
5
10
20
30
50
Distance to galaxy (Mpc)



Geometry for homogeneous and isotropic space-time:

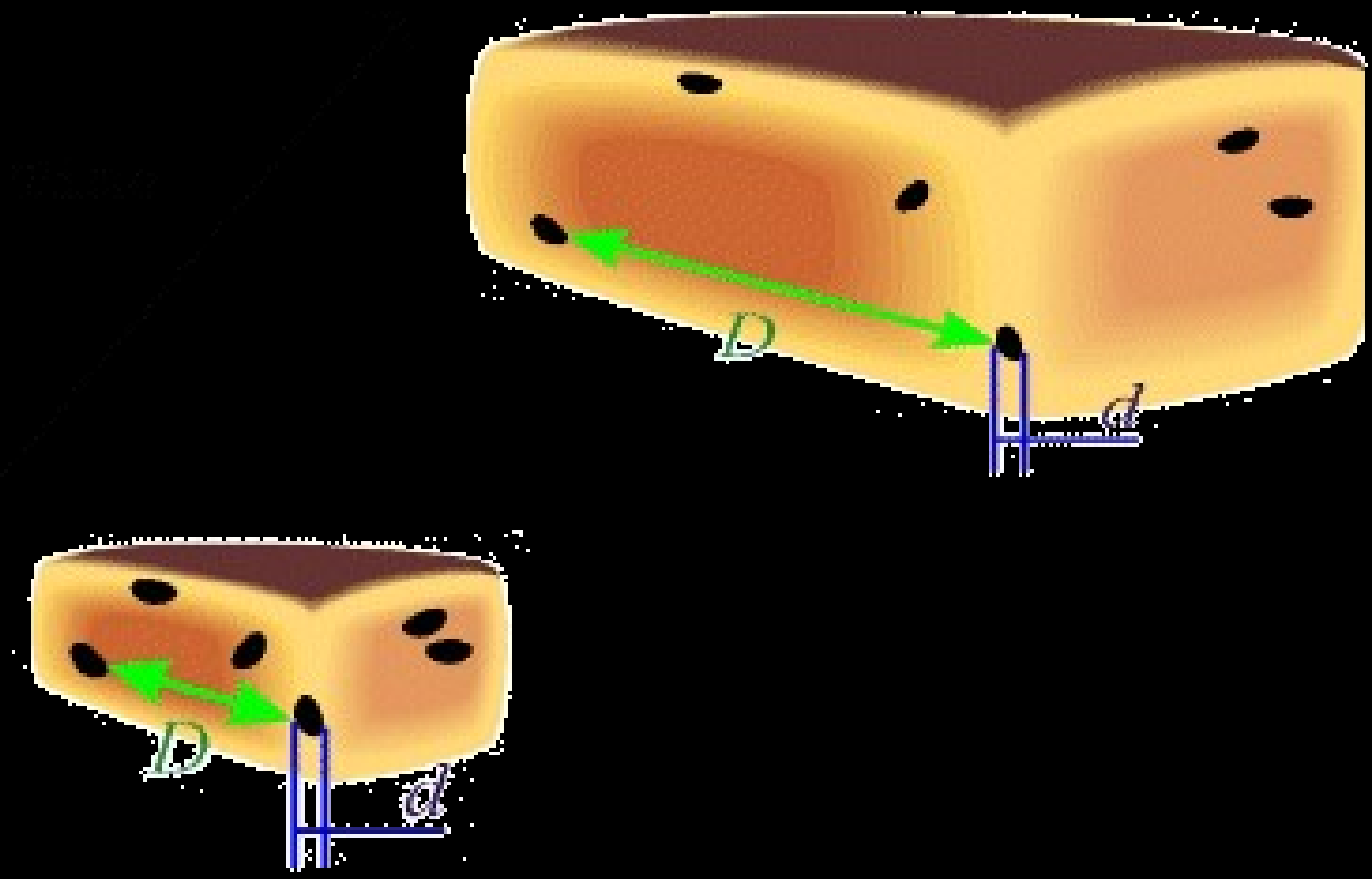
$$ds^2 = c^2 dt^2 - a^2(t)[dx^2 + f(x)^2(d\theta^2 + \sin^2\theta\, d\varphi^2)]$$

$$ds^2 = (cdt)^2 - R^2(t)\left[d\chi^2 + S_k^2(\chi)(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

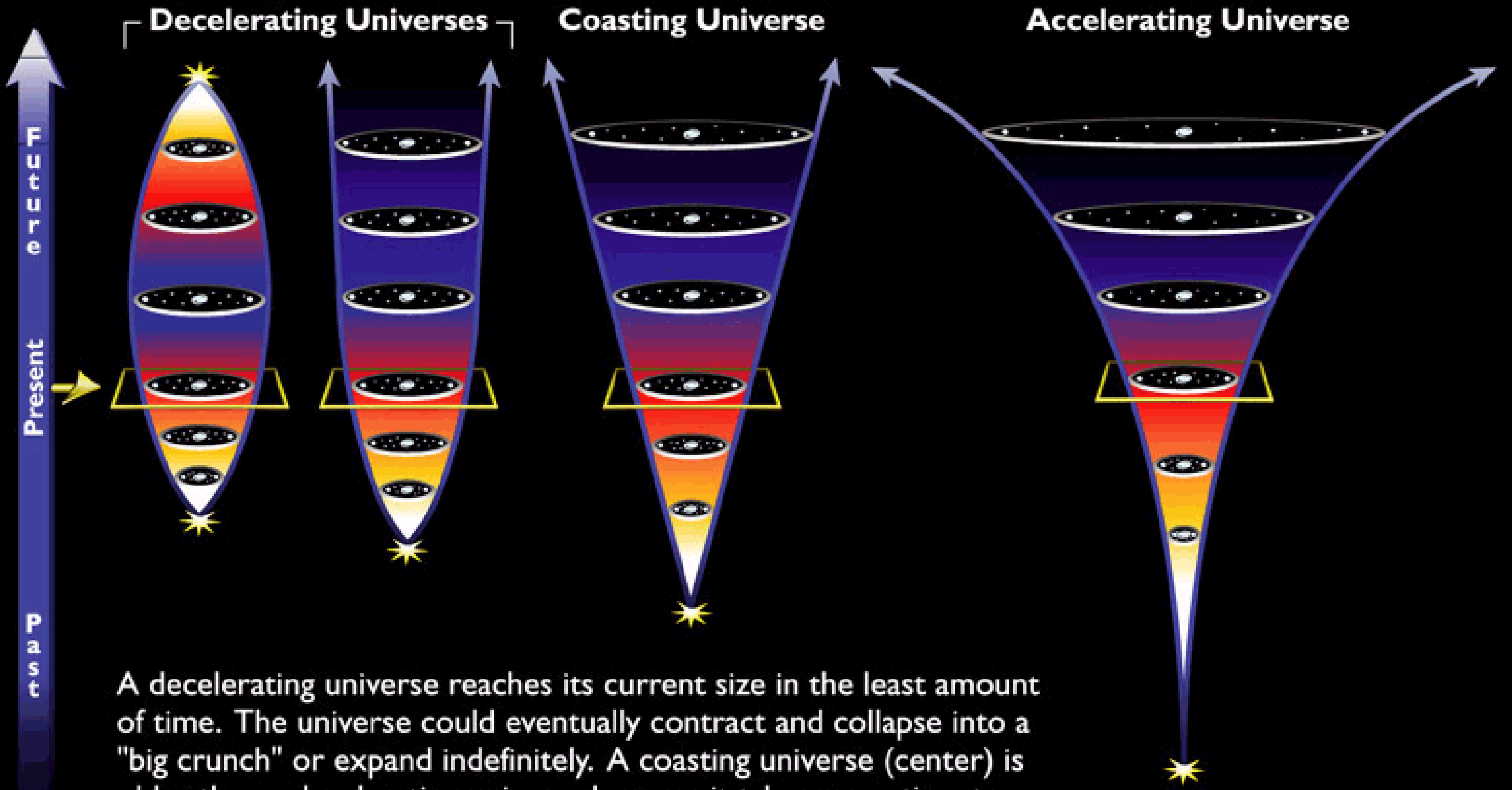
Geometry around a spherically symmetric mass distribution:

$$ds^2 = c^2\left(1 - \frac{2GM}{rc^2}\right)dt^2 - \qquad \textbf{(4-30)}$$

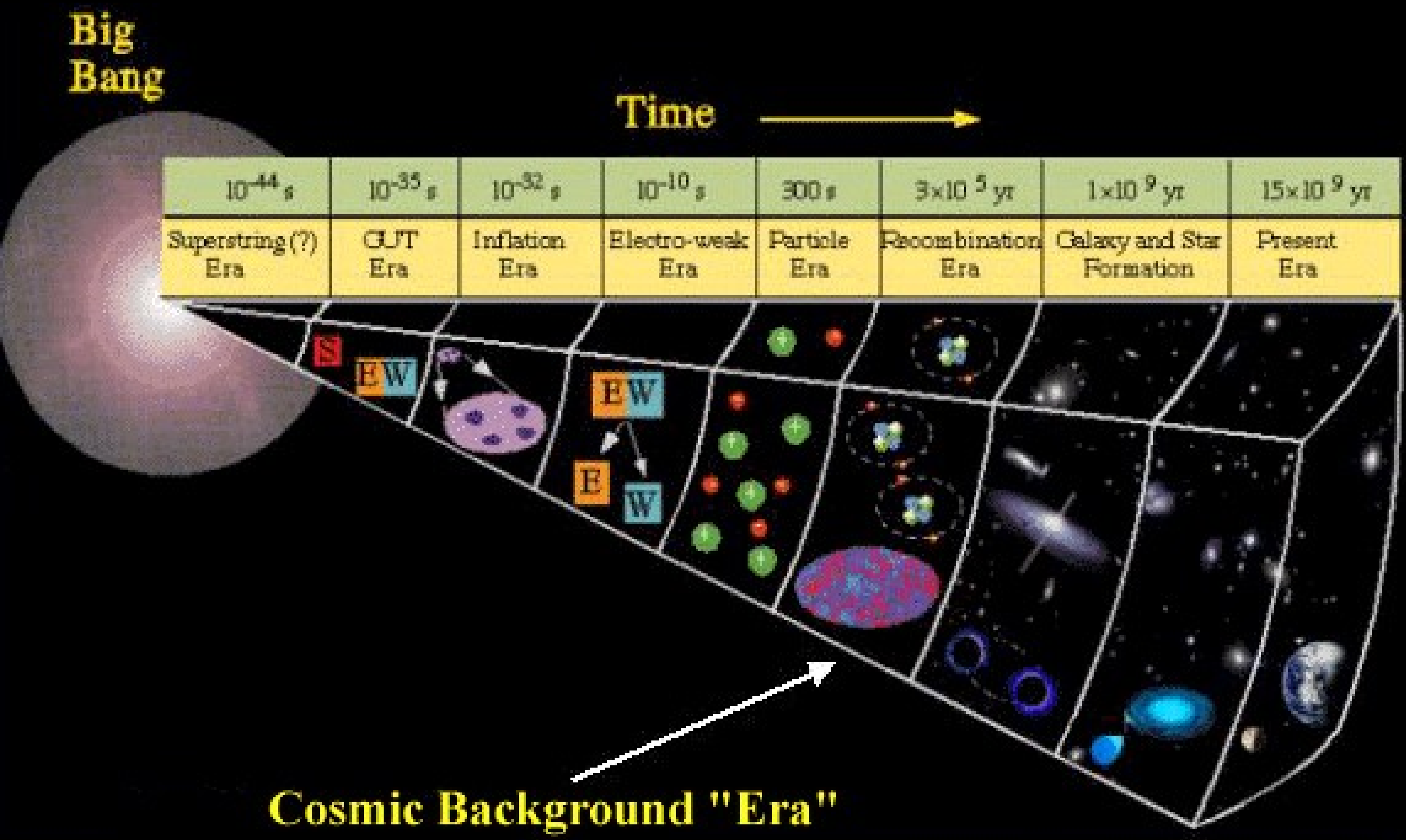
$$\frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

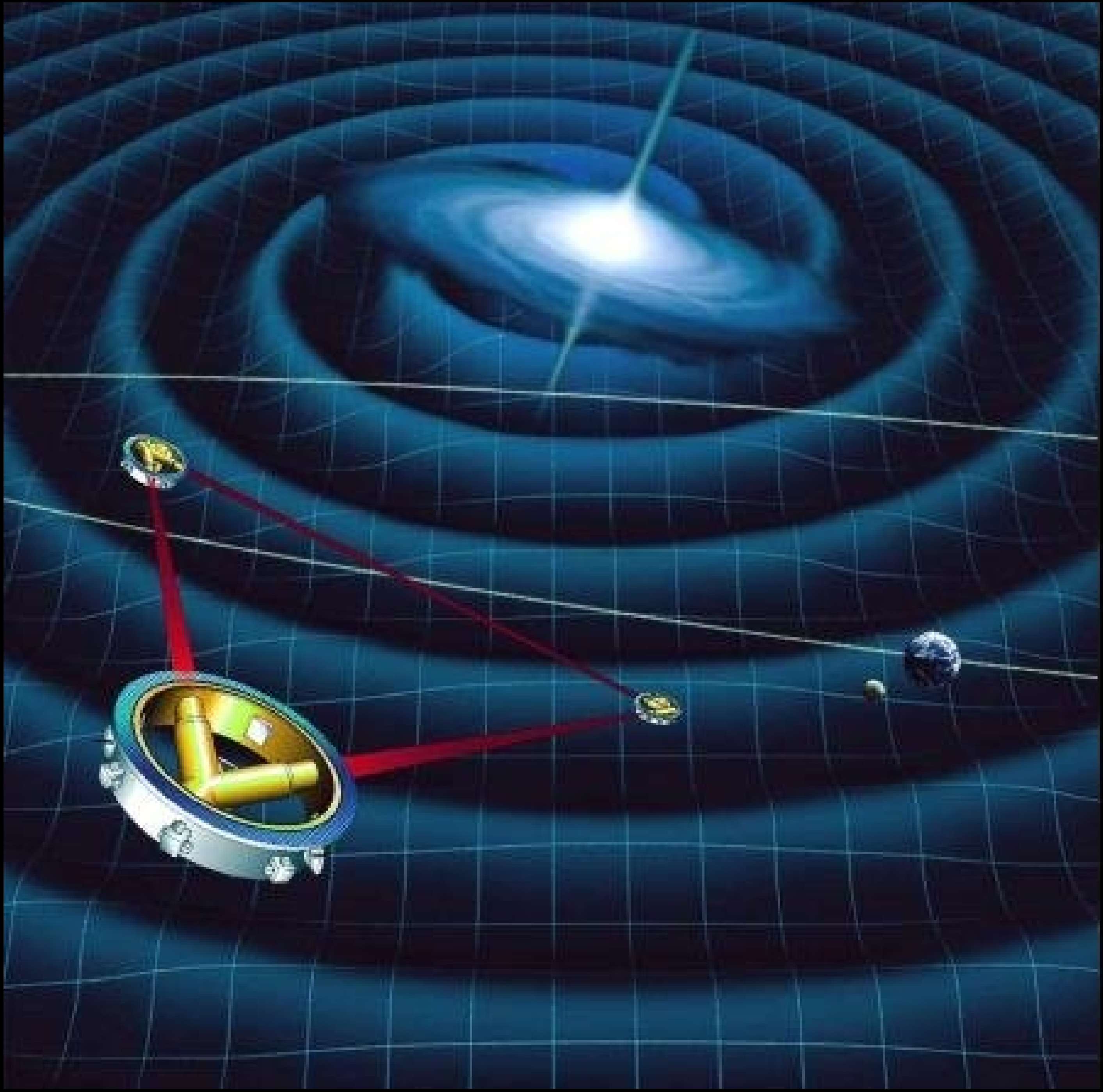

D
d
D
d

Possible Models of the Expanding Universe



A decelerating universe reaches its current size in the least amount of time. The universe could eventually contract and collapse into a "big crunch" or expand indefinitely. A coasting universe (center) is older than a decelerating universe because it takes more time to reach its present size, and expands forever. An accelerating universe (right) is older still. The rate of expansion actually increases because of a repulsive force that pushes galaxies apart.


Big Bang
Time
10^-44 s
10^-35 s
10^-32 s
10^-10 s
300 s
3×10 5 yr
1×10 9 yr
15×10 9 yr
Superstring (?) Era
GUT Era
Inflation Era
Electro-weak Era
Particle Era
Recombination Era
Galaxy and Star Formation
Present Era
S
EW
EW
E
W
Cosmic Background "Era"



ds^2 = c^2 dt^2 – (dx^2 + dy^2 + dz^2) for an empty universe. The curvature is zero every where.

BUT

If matter is distributed homogeneously and isotropically:

$$ds^2 = c^2 dt^2 - R^2(t) \times \left(\frac{d\sigma^2}{(1 - k\sigma^2)} + \sigma^2 d\theta^2 + \sigma^2 \sin^2\theta \, d\phi^2 \right), \qquad (1)$$

R(t)= Expansion scale factor.

(1) k=0 (Flat model or Einstein-de Sitter universe)
ds^2=c^2dt^2 – R^2 (dx^2 + dy^2 + dz^2)

(2) k= +1 (Closed universe)

(3) k= - 1 (Open universe)

$$H = \frac{\dot{R}}{R}$$

$$q = -\frac{R\ddot{R}}{(\dot{R})^2}$$

TIME's riddle:

Arrow of time → Entropy, Psychological, …

Is "i t" more fundamental than "t"?

Time machine → Exotic matter with negative energy density, wormholes

Is there a censorship on "closed time-like trajectories"? → Chronology Protection surmise

Time evolution and unitarity → Blackhole information puzzle

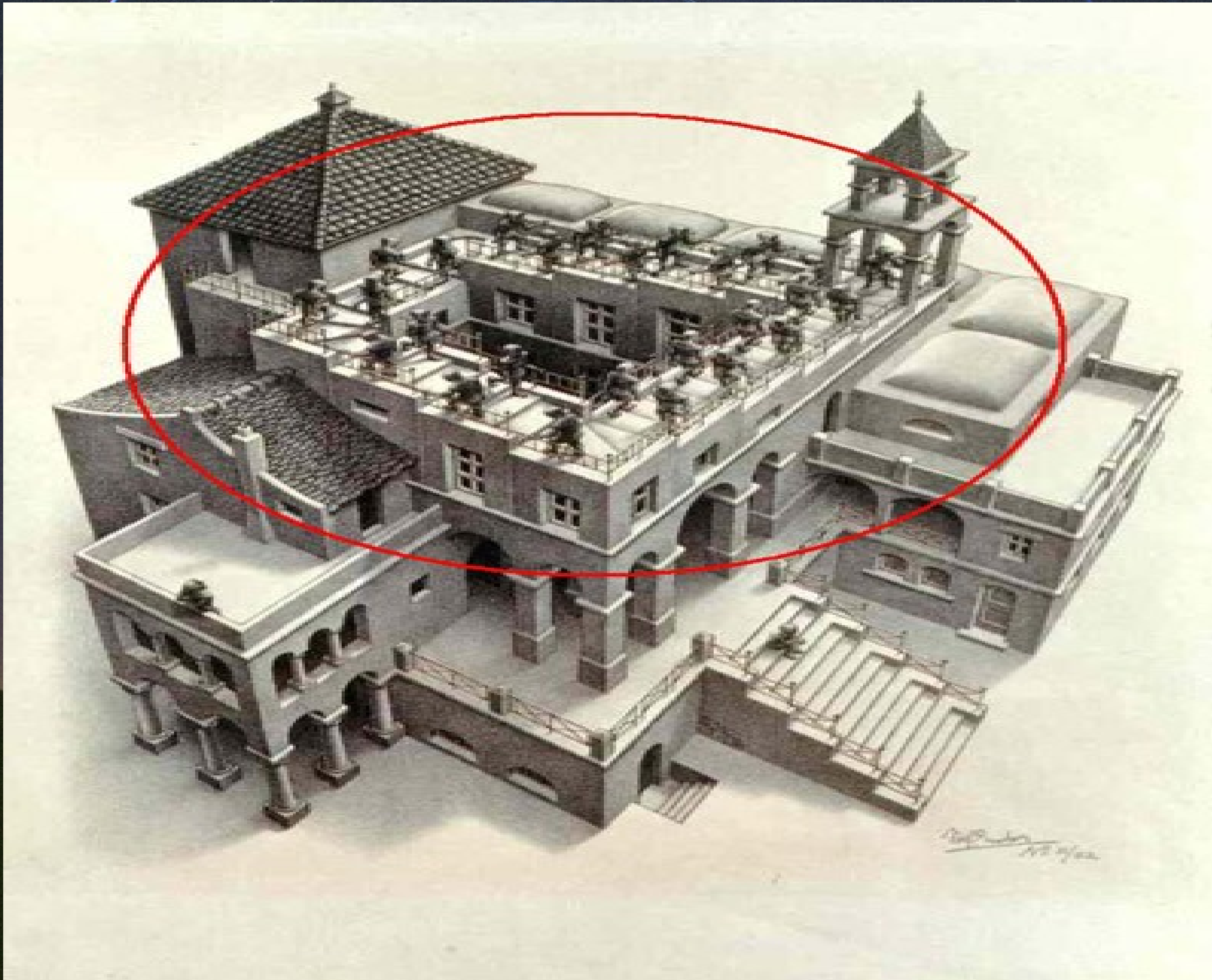
TIME'S ARROW:

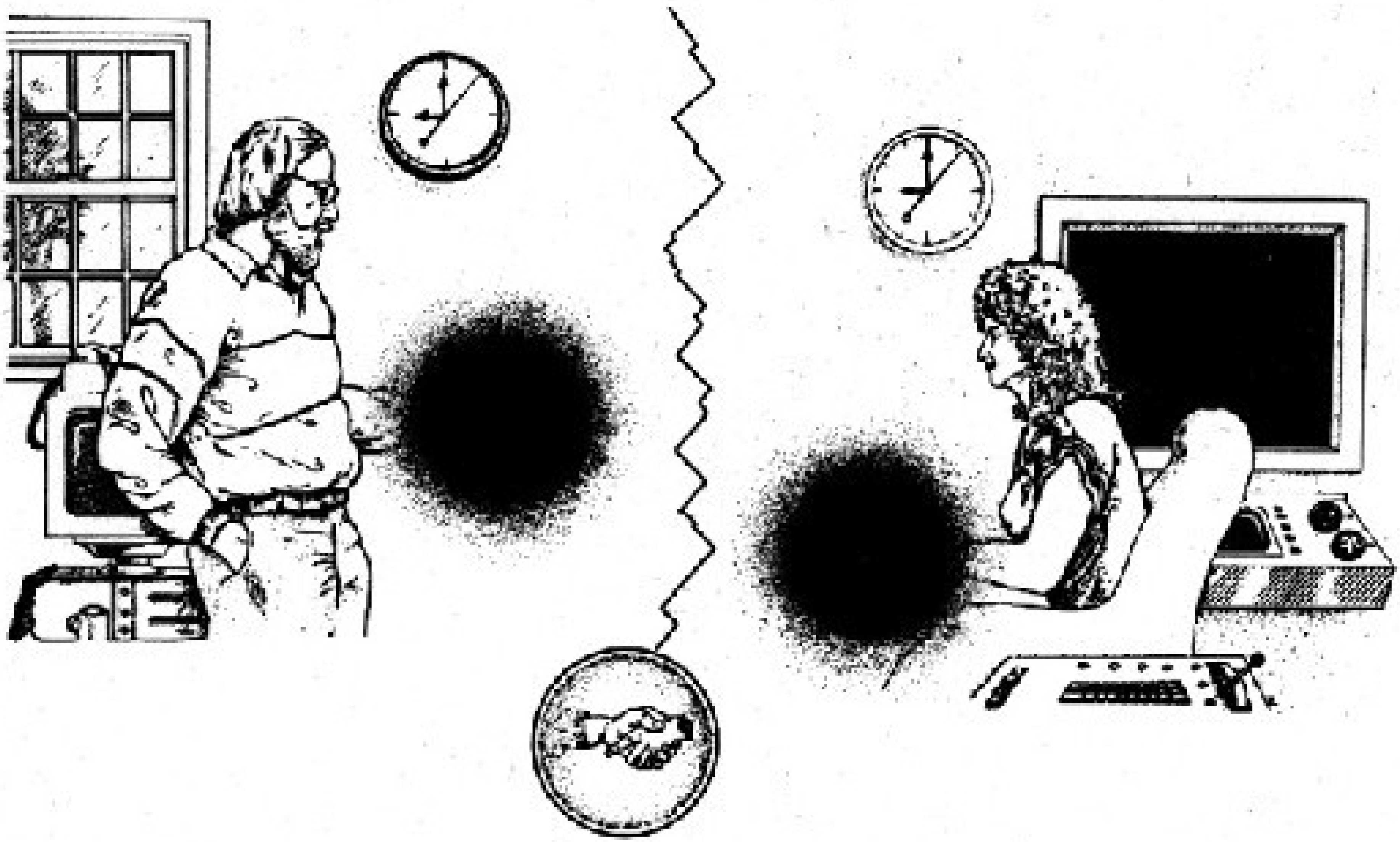
* For a system in thermodynamic equilibrium, there is NO arrow of time

* In the absence of thermodynamic equilibrium, the state of the system evolves in time, while entropy keeps increasing

* The conjecture that universe was created in a state far removed from thermodynamic equilibrium, in a low entropy state

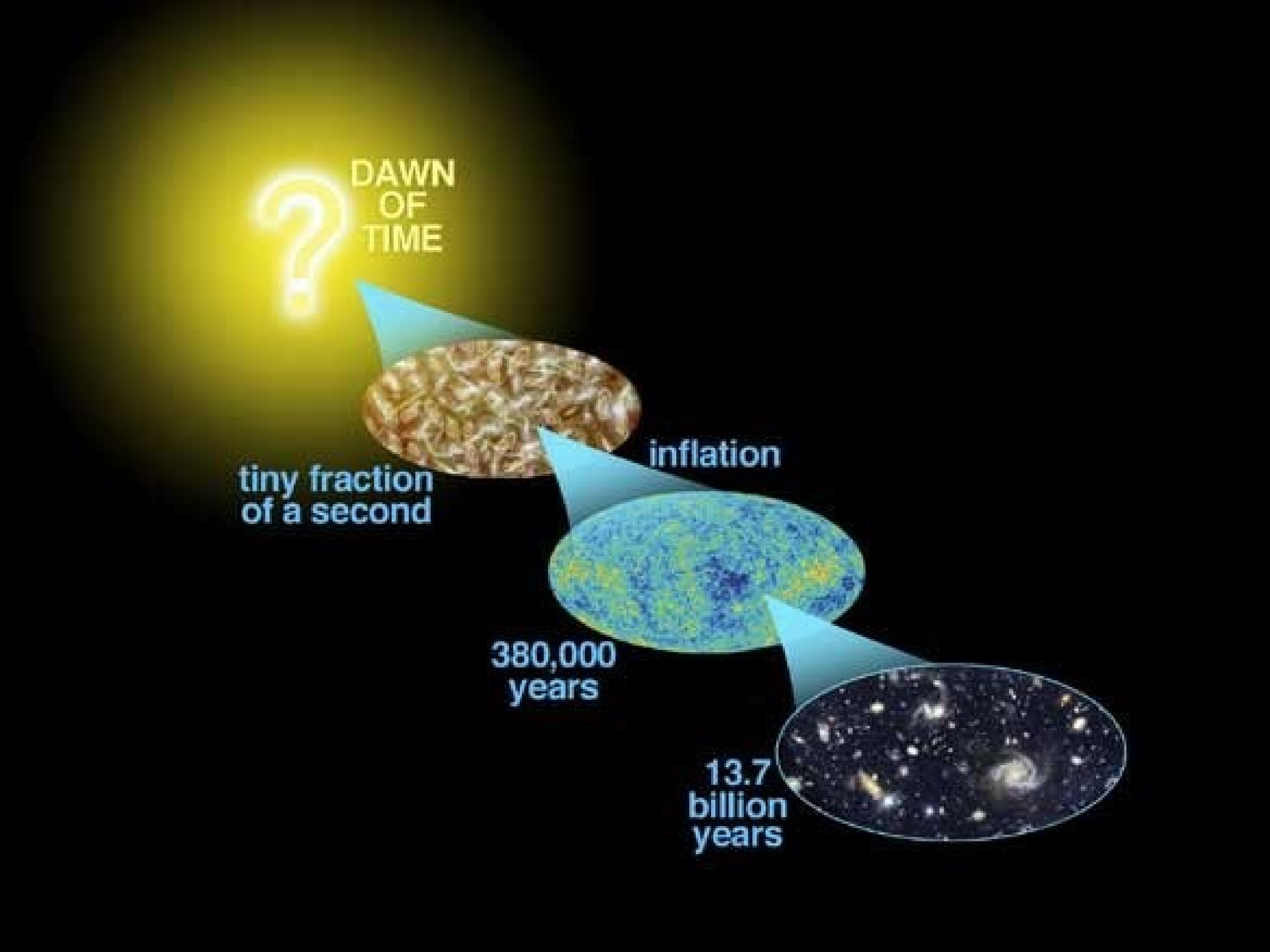
* Galaxies, stars, complex structures,........, living systems etc. form, as time evolves, to make entropy increase globally

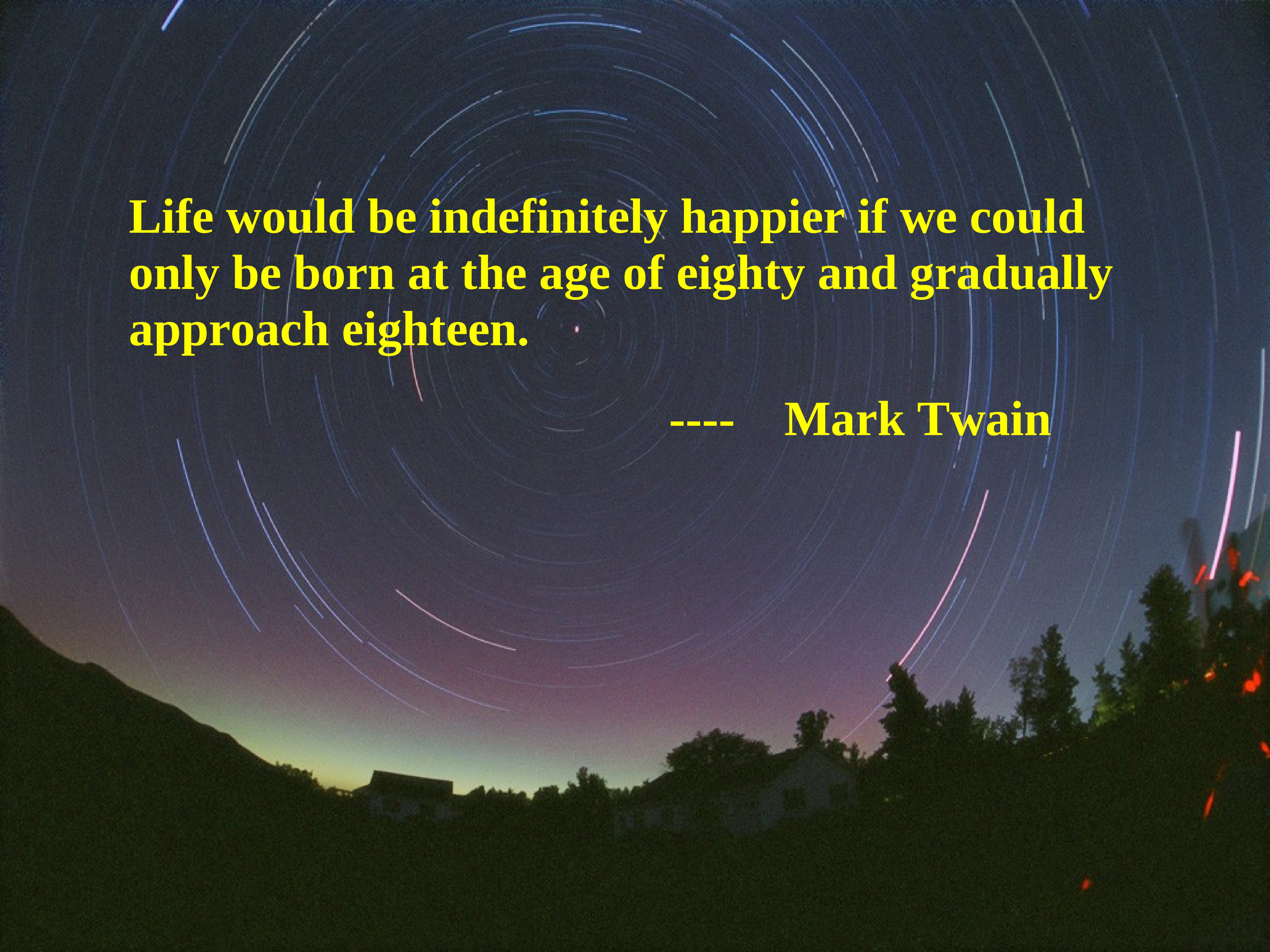




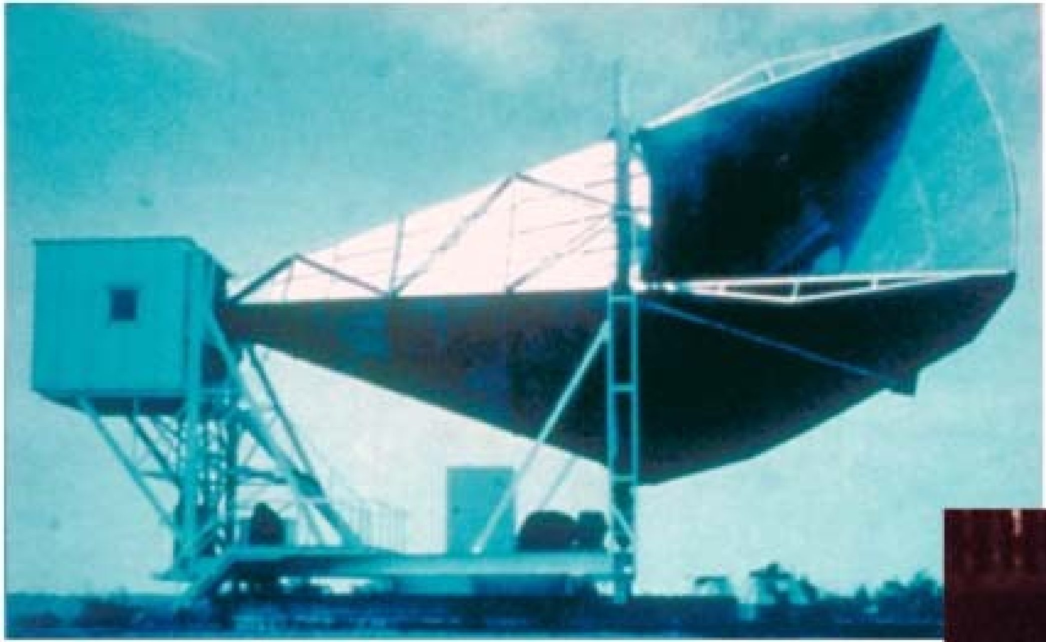
Alexander (GUY PEARCE) and the Uber-Morlock (Jeremy Irons) engage in a life-and-death struggle in DreamWorks Pictures' and Warner Bros. Pictures' THE TIME MACHINE.
Photo: Courtesy DreamWorks

THE TIME MACHINE


DAWN OF TIME
inflation
tiny fraction of a second
380,000 years
13.7 billion years

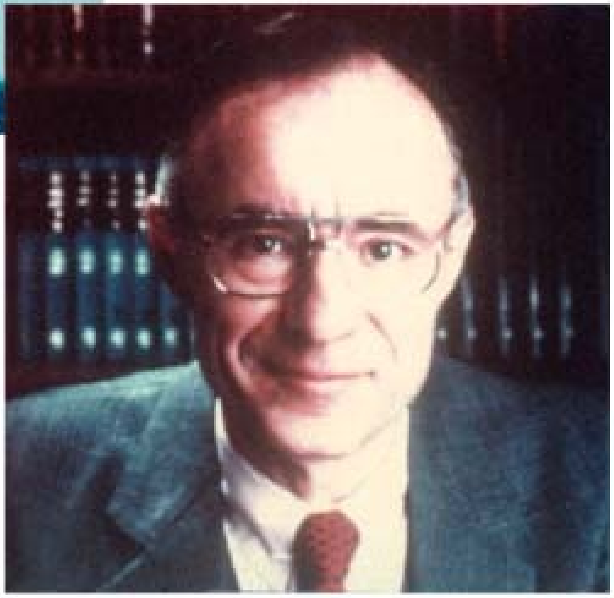

Life would be indefinitely happier if we could only be born at the age of eighty and gradually approach eighteen.
---- Mark Twain

DISCOVERY OF COSMIC BACKGROUND



Microwave Receiver

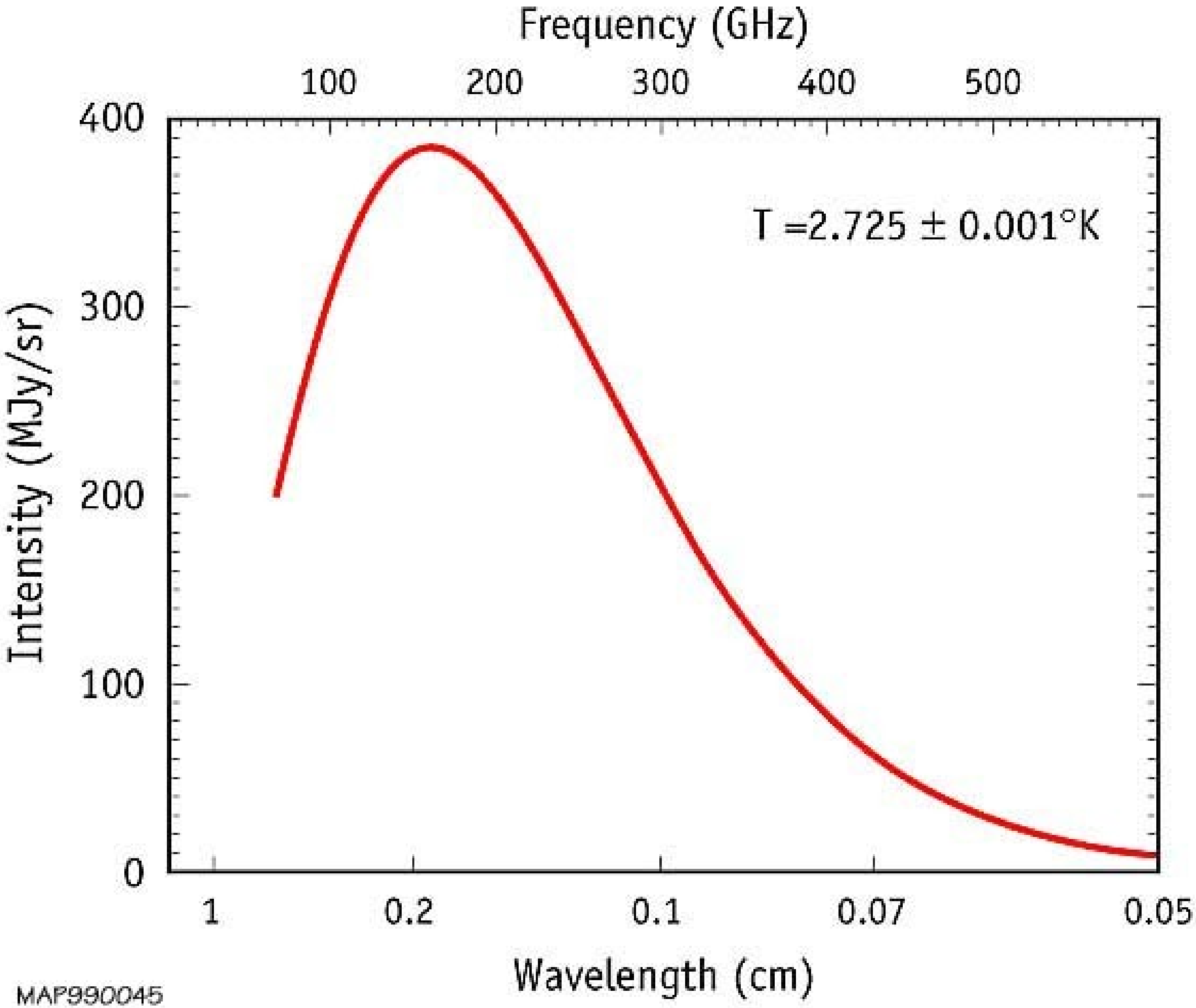
Robert Wilson

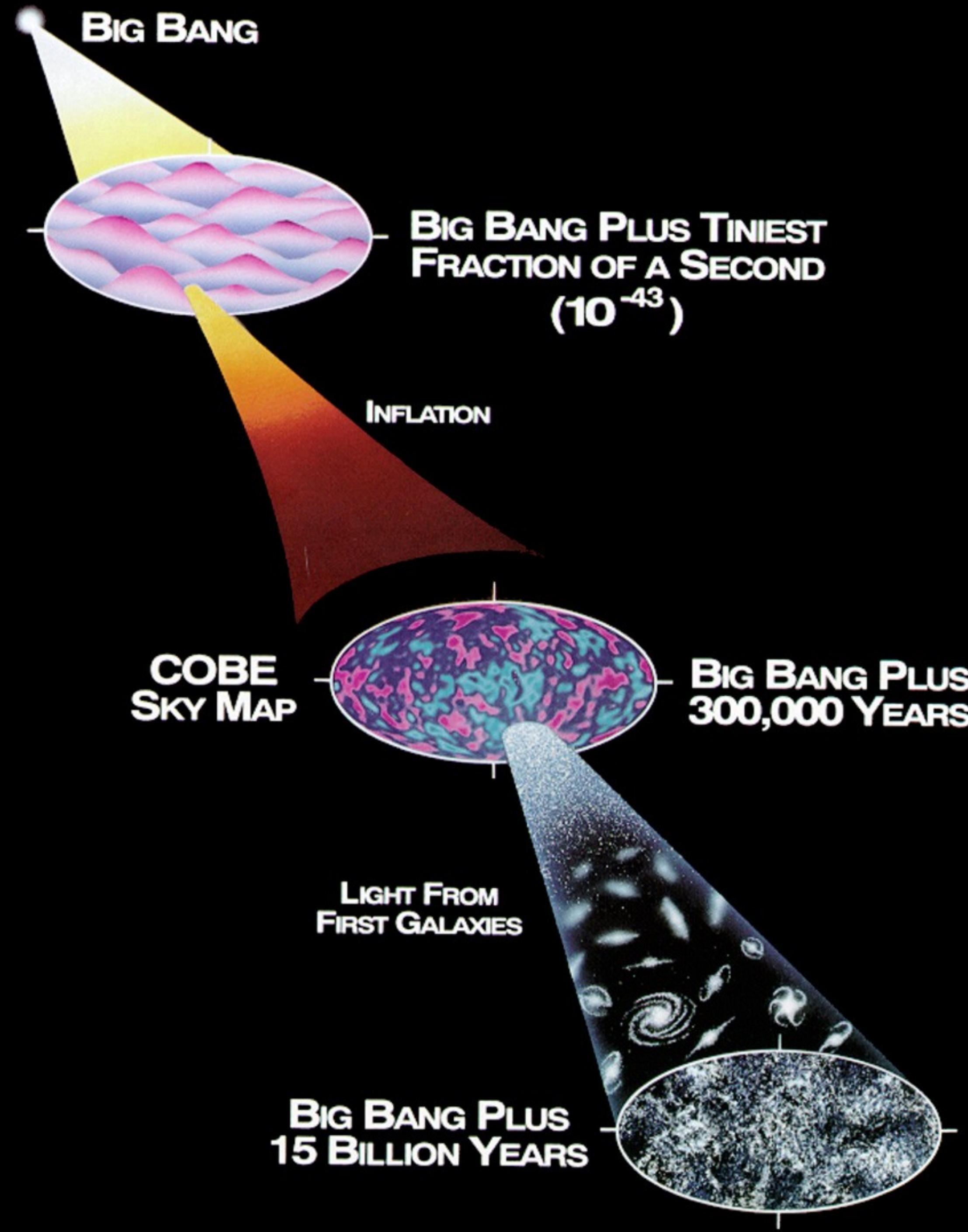


Arno Penzias

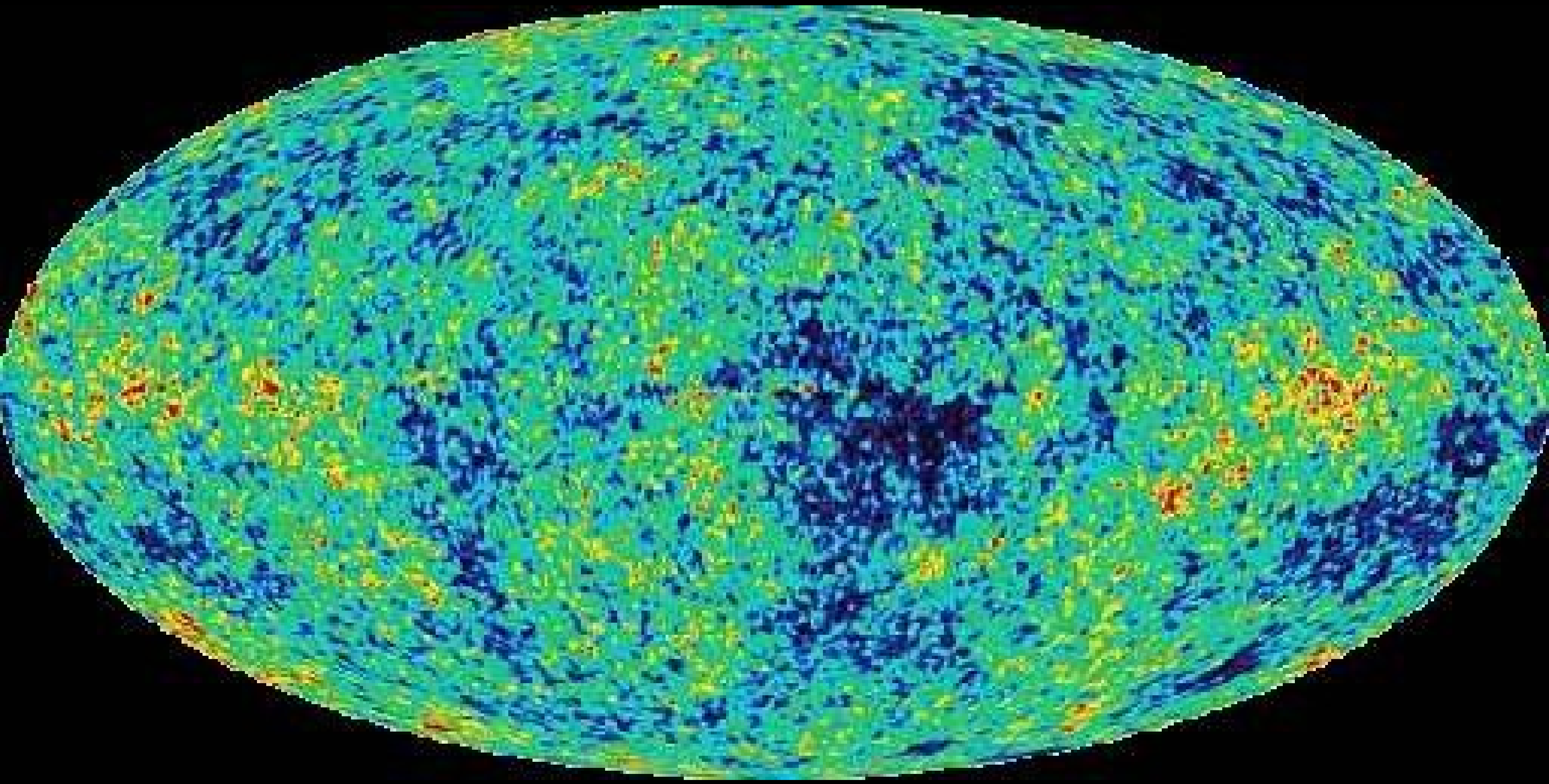
MAP990045

Spectrum of the Cosmic Microwave Background



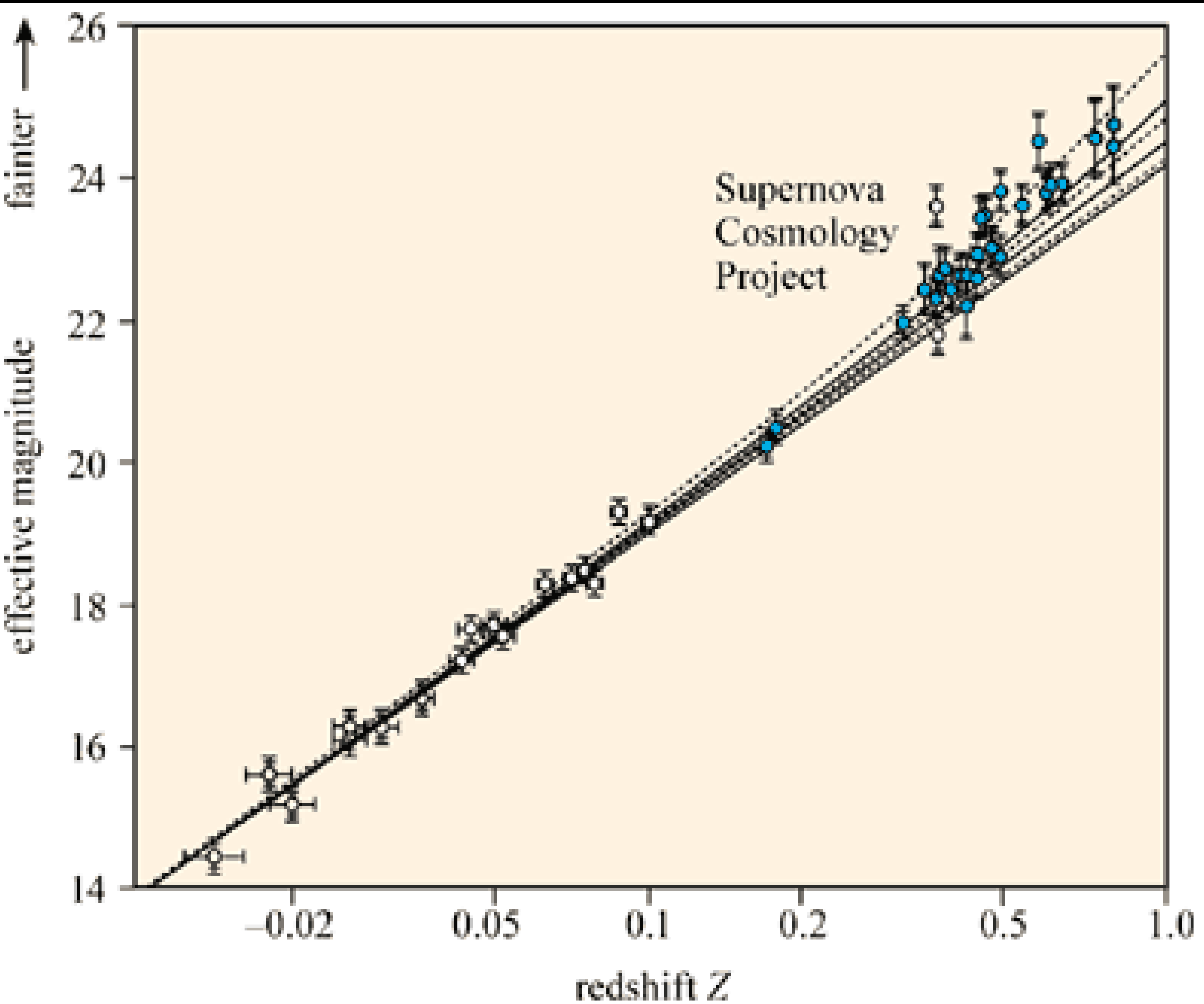


Big Bang
Big Bang Plus Tiniest Fraction of a Second (10^{-43})
Inflation
COBE Sky Map
Big Bang Plus 300,000 Years
Light From First Galaxies
Big Bang Plus 15 Billion Years



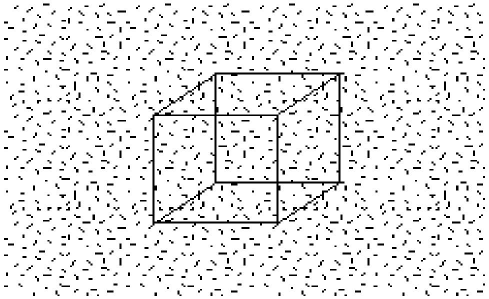
Distant Supernovae and the Accelerating Universe

- If the Universe is expanding at a constant rate, one can predict how bright distant supernovae should seem
- In 1998, astronomers discovered distant supernovae were too faint!



26
24
22
20
18
16
14
fainter
effective magnitude
Supernova
Cosmology
Project
−0.02
0.05
0.1
0.2
0.5
1.0
redshift Z

The cosmological constant, Λ, is a form of energy density which is inherent in "empty space", remaining constant as the universe expands.



The only effects of this vacuum energy are on gravity, as reflected in Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}^{\mathrm{matter}} ,$$

or

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{\mathrm{matter}} + T_{\mu\nu}^{\mathrm{vacuum}} \right) ,$$

corresponding to a vacuum energy density

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} .$$

Accelerating Universe:

\# In general relativity, pressure of matter is also a source of space-time curvature

\# Matter with “weird” equation of state: Quintessence

Pressure = w . Energy density,

with w < 0

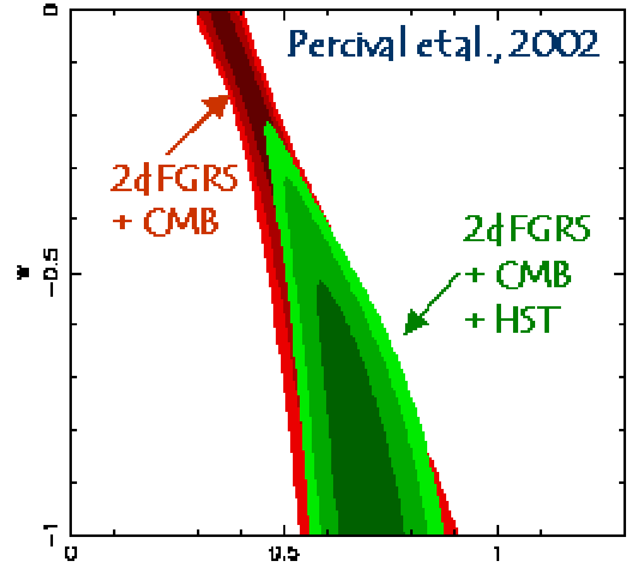
\# Negative pressure leads to repulsive gravity

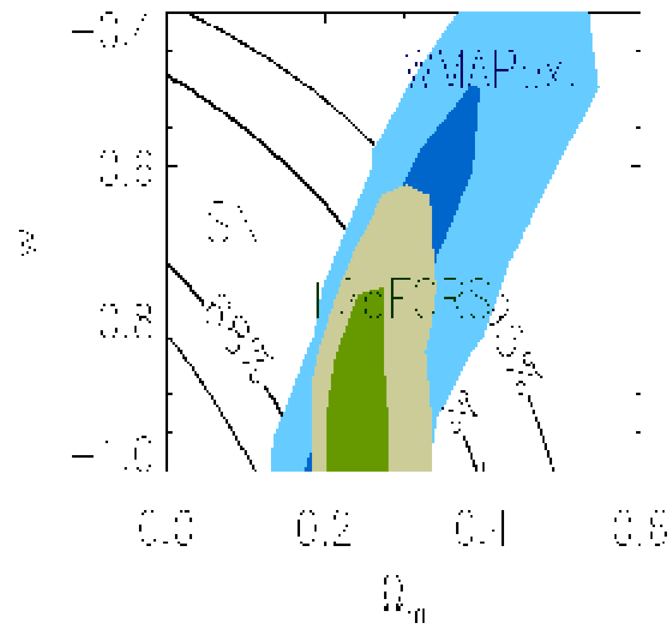
\# For the cosmological constant:

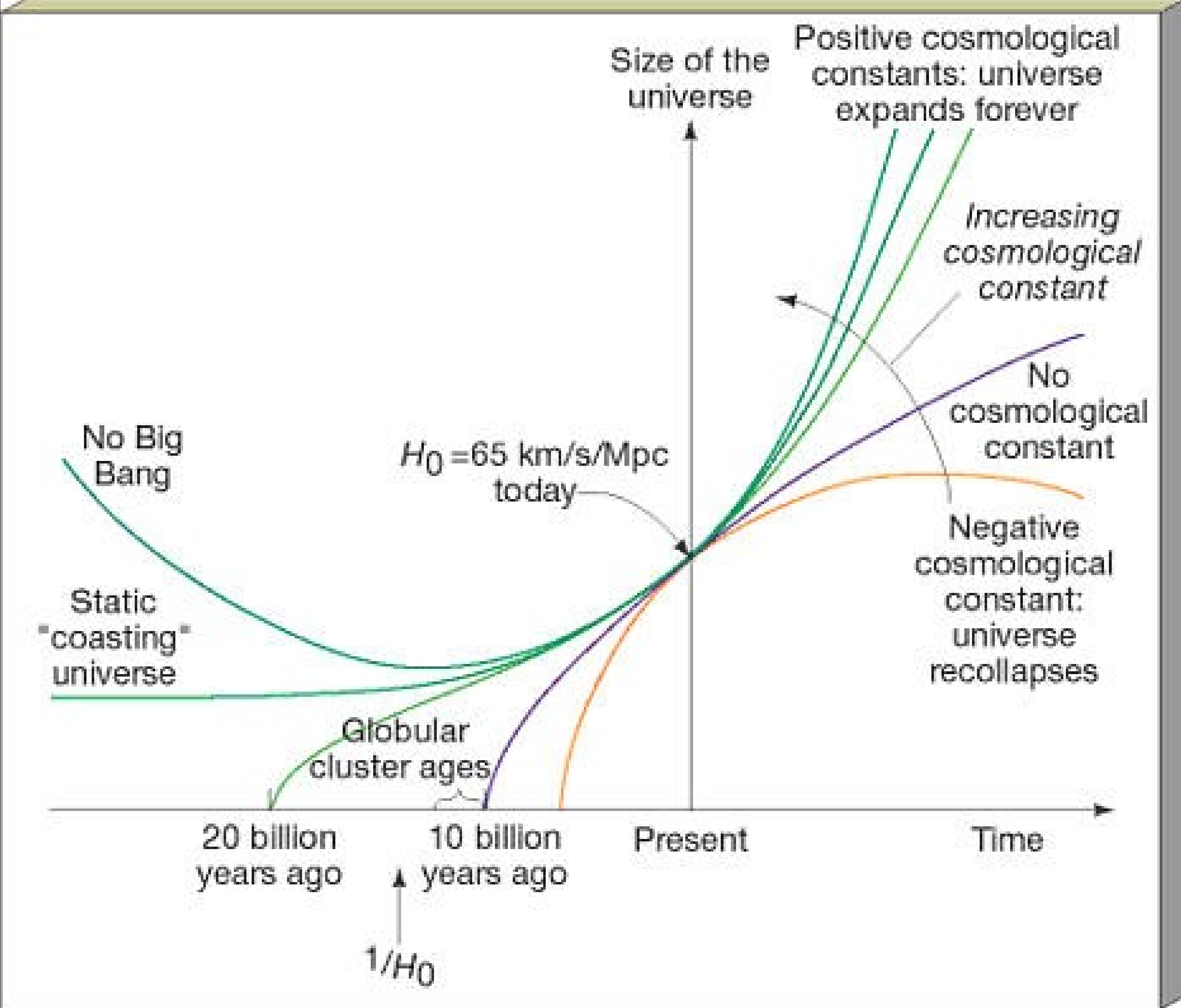
Pressure = - Energy density

The dark energy equation of state

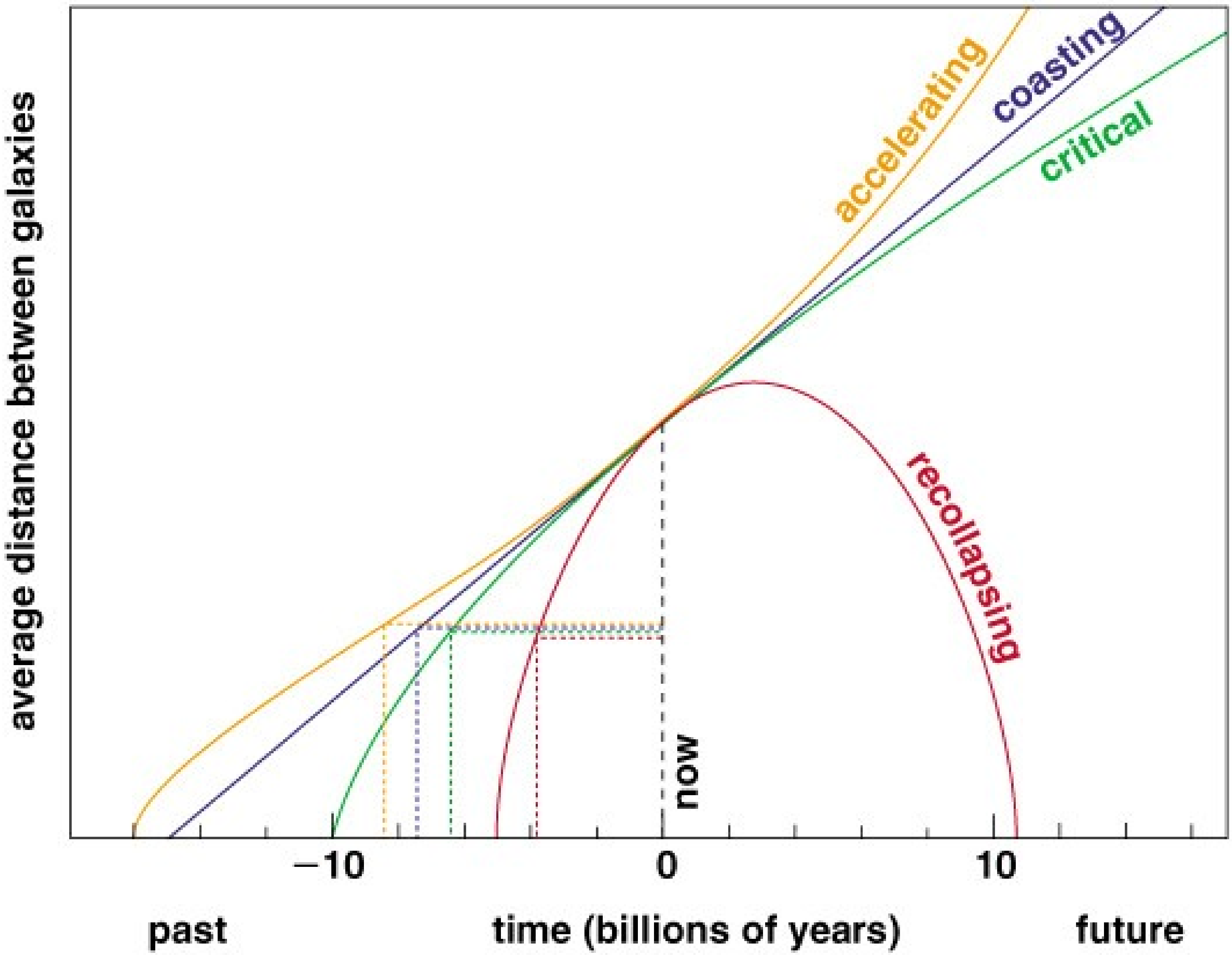
- Dark energy equation of state: $w(z) \equiv p/\rho$
  - $\rho(z) \propto (1+z)^n$ with $n \equiv 3(1+w)$;
  - cosmological constant has $w = -1$ and $n = 0$.
- The combined constraint on $\langle w \rangle$, assuming a flat universe, from the CMB and 2dFGRS power spectra plus the HST key project H_0, is:
  $\langle w \rangle \leq -0.52$ (95% c.l.)
- With the WMAP data:
  $\langle w \rangle \leq -0.78$ (95% c.l.)



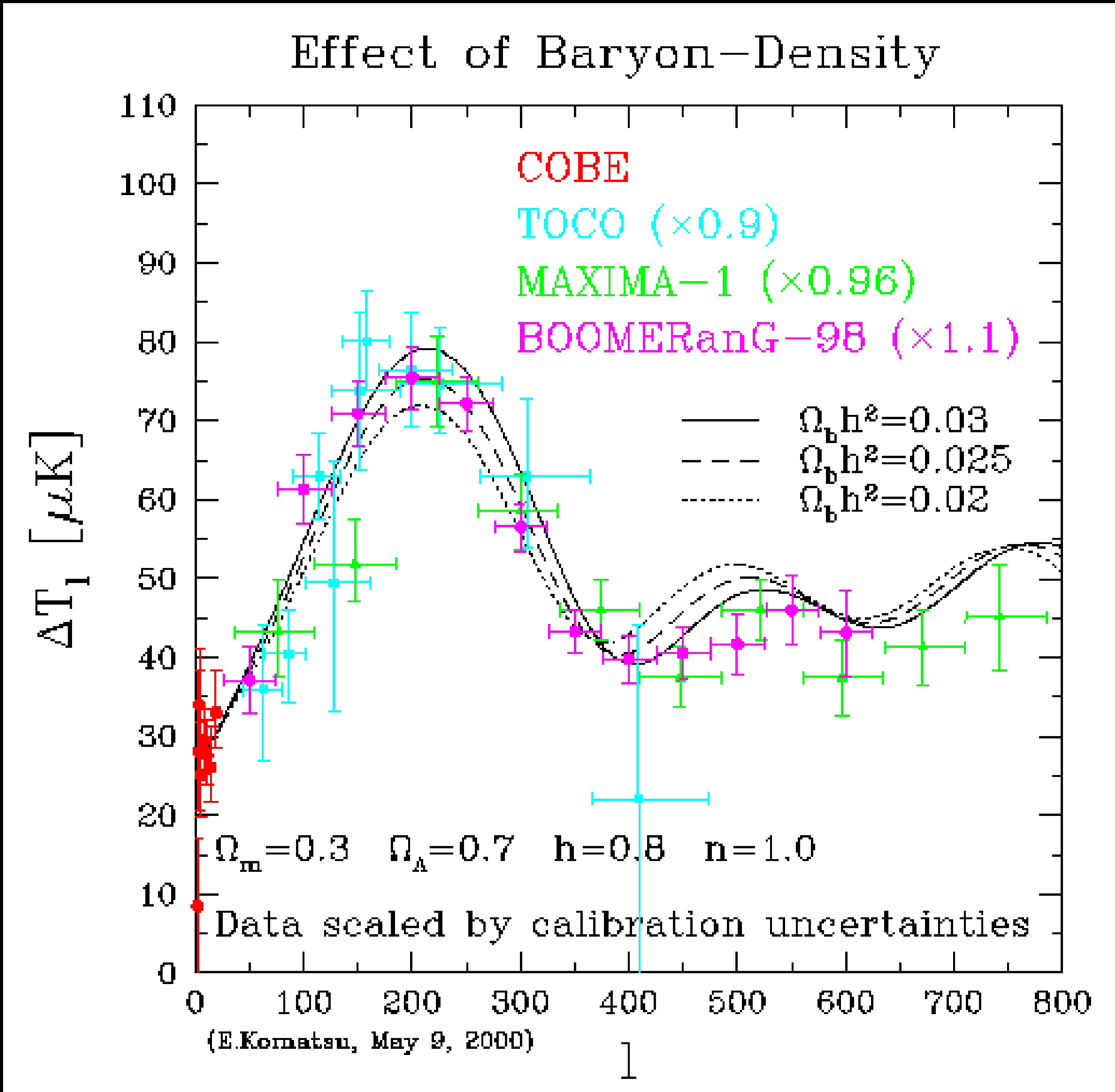




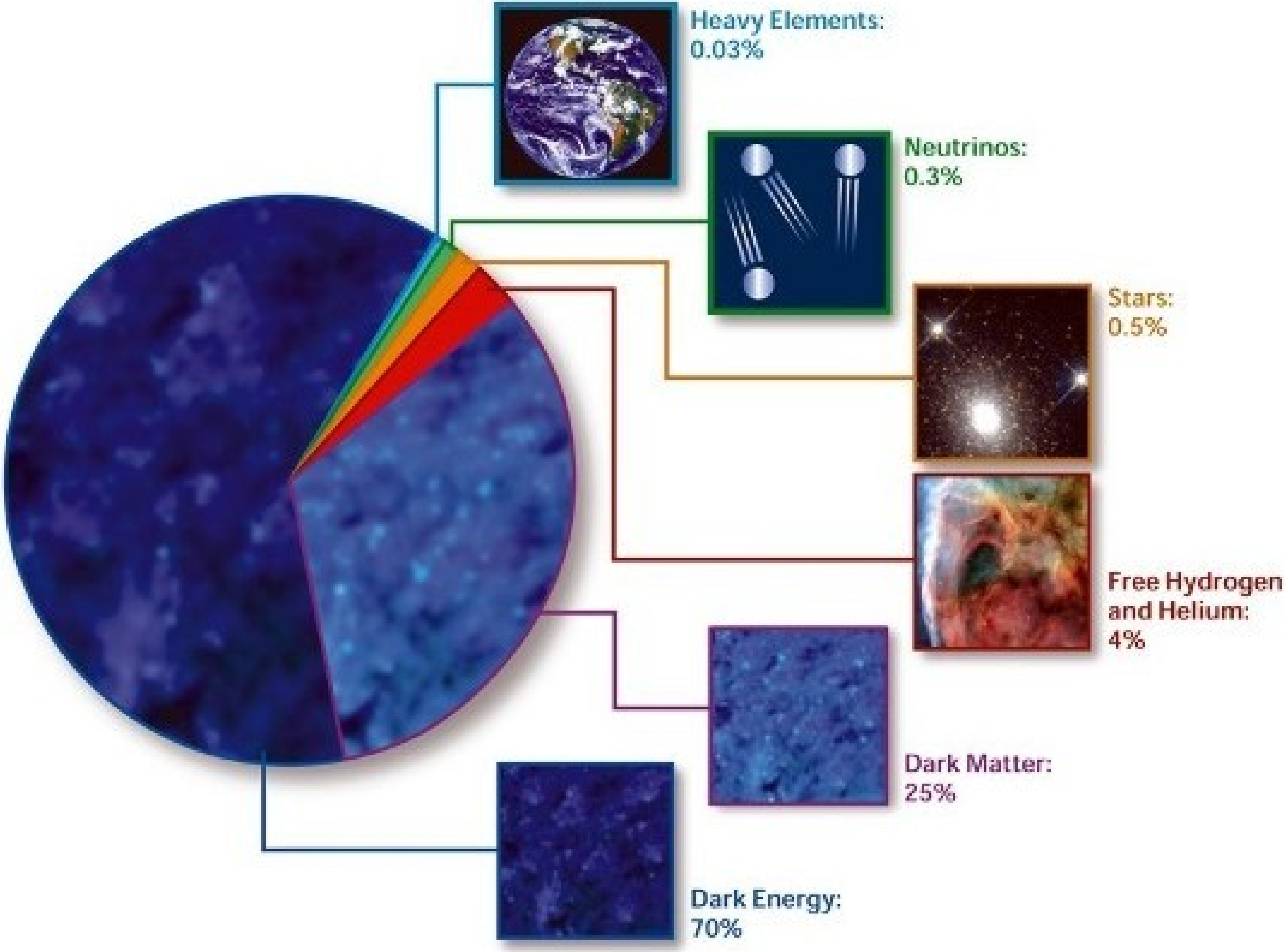
Size of the universe
Positive cosmological constants: universe expands forever
Increasing cosmological constant
No cosmological constant
Negative cosmological constant: universe recollapses
No Big Bang
H_0 =65 km/s/Mpc today
Static "coasting" universe
Globular cluster ages
20 billion years ago
10 billion years ago
Present
Time
$1/H_0$



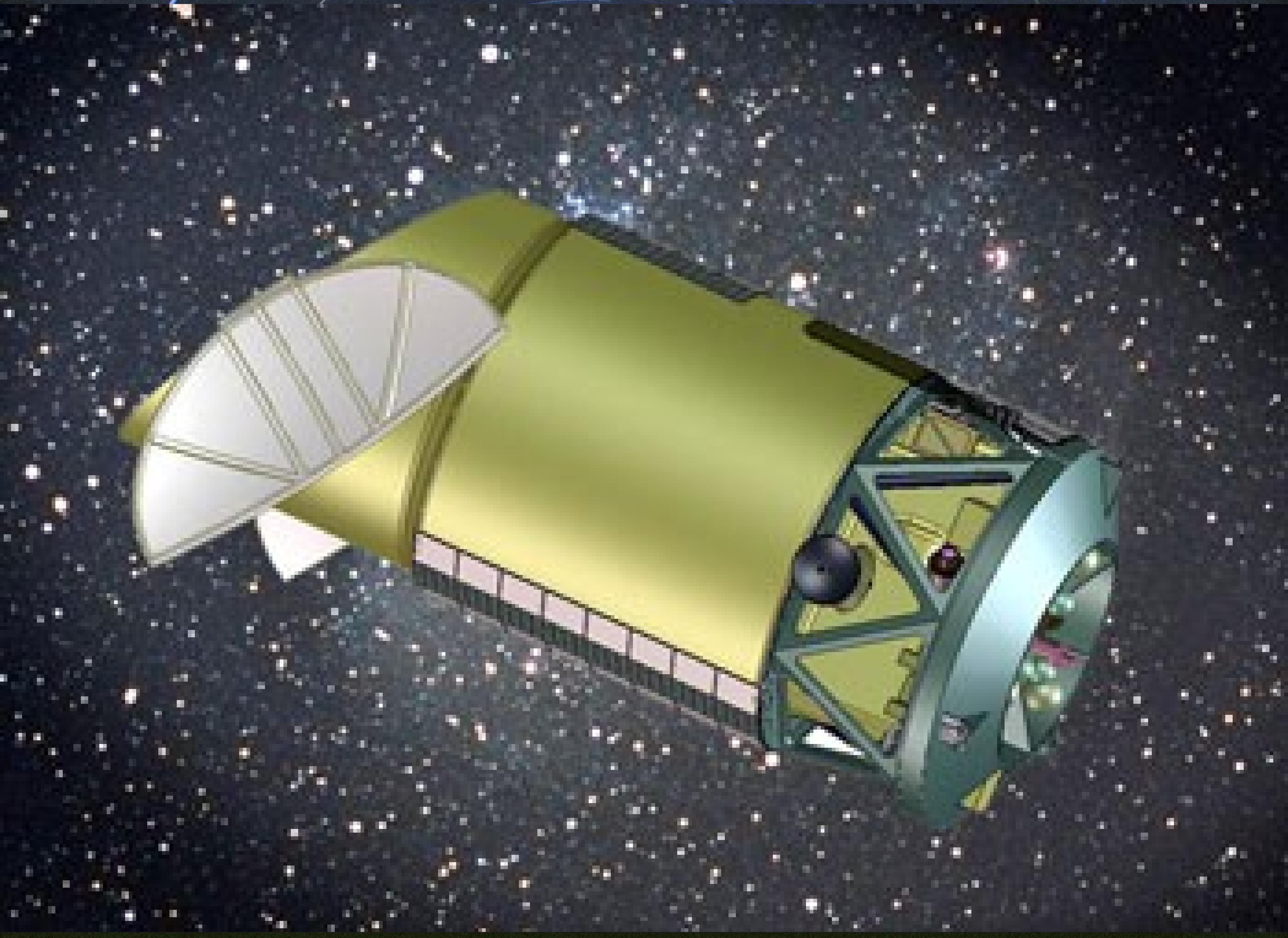
average distance between galaxies
accelerating
coasting
critical
recollapsing
now
−10
0
10
past
time (billions of years)
future



Effect of Baryon-Density
COBE
TOCO (×0.9)
MAXIMA-1 (×0.96)
BOOMERanG-98 (×1.1)
$\Omega_b h^2$=0.03
$\Omega_b h^2$=0.025
$\Omega_b h^2$=0.02
Ω_m=0.3 Ω_Λ=0.7 h=0.8 n=1.0
Data scaled by calibration uncertainties
ΔT_l [μK]
l
(E.Komatsu, May 9, 2000)

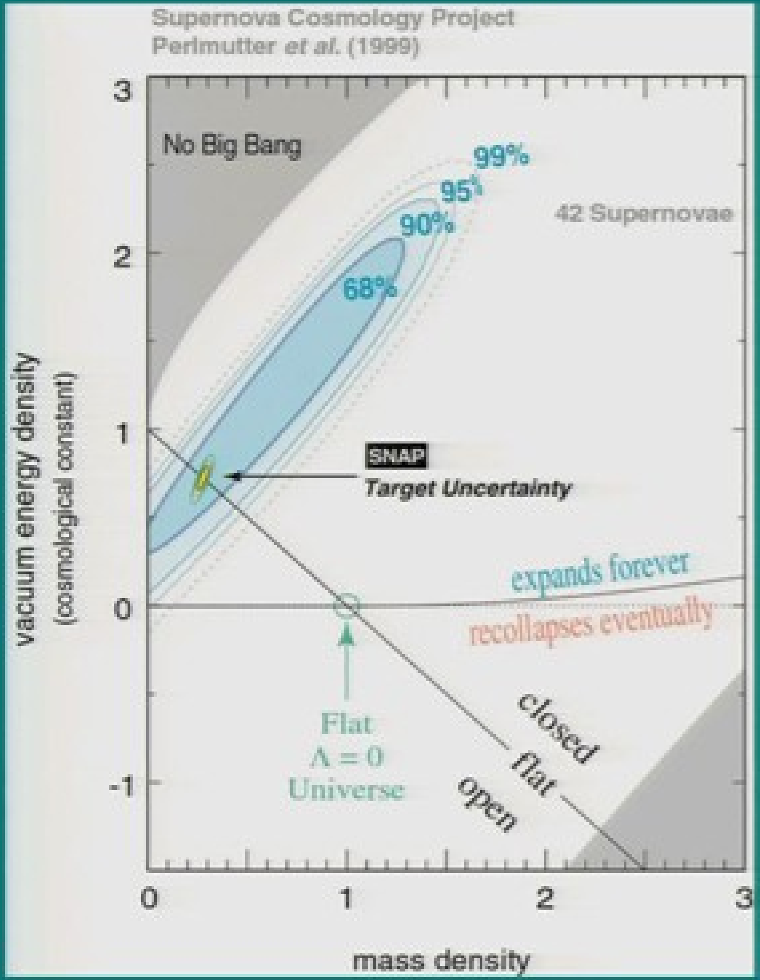


Heavy Elements:
0.03%
Neutrinos:
0.3%
Stars:
0.5%
Free Hydrogen
and Helium:
4%
Dark Matter:
25%
Dark Energy:
70%

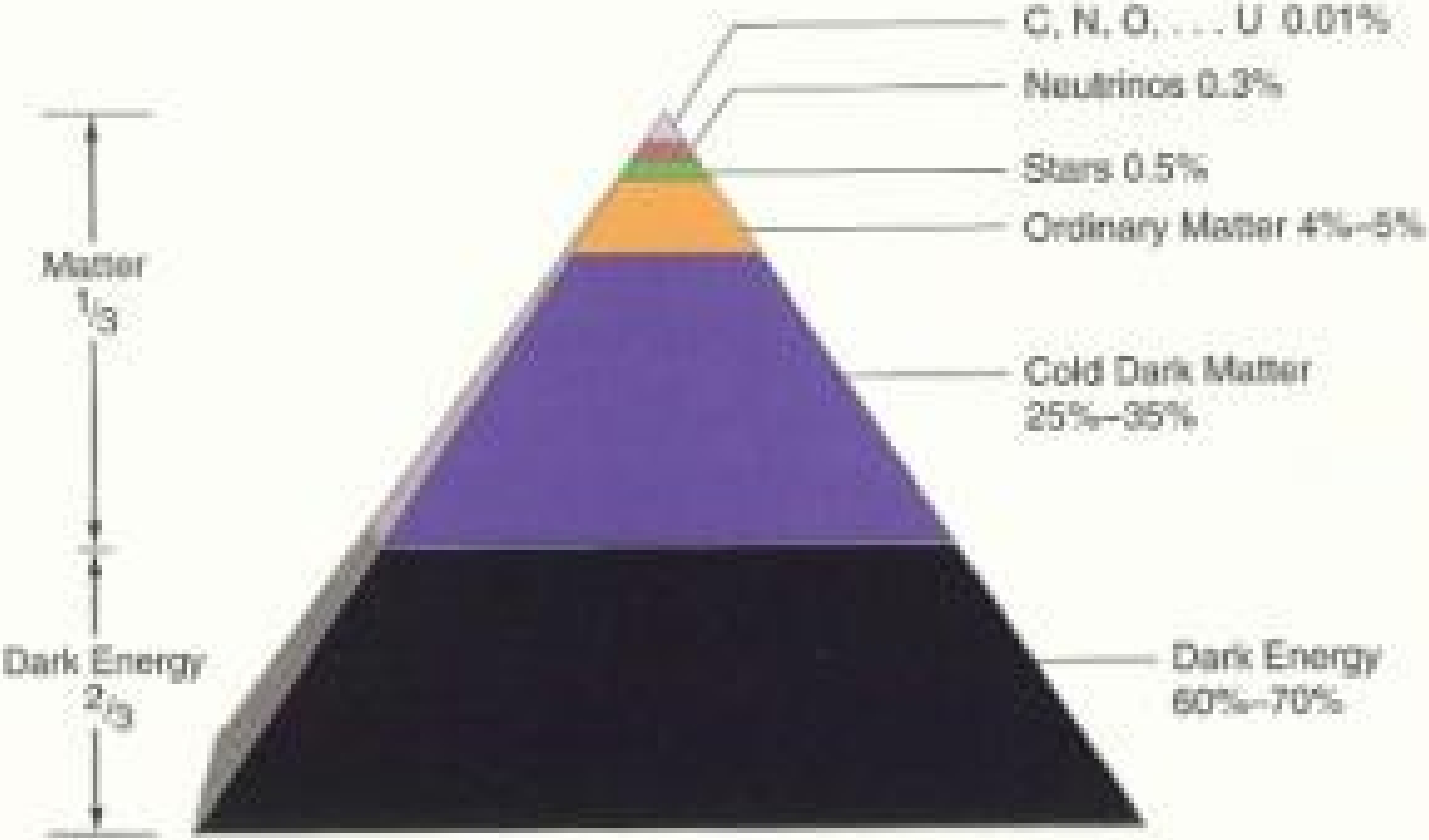


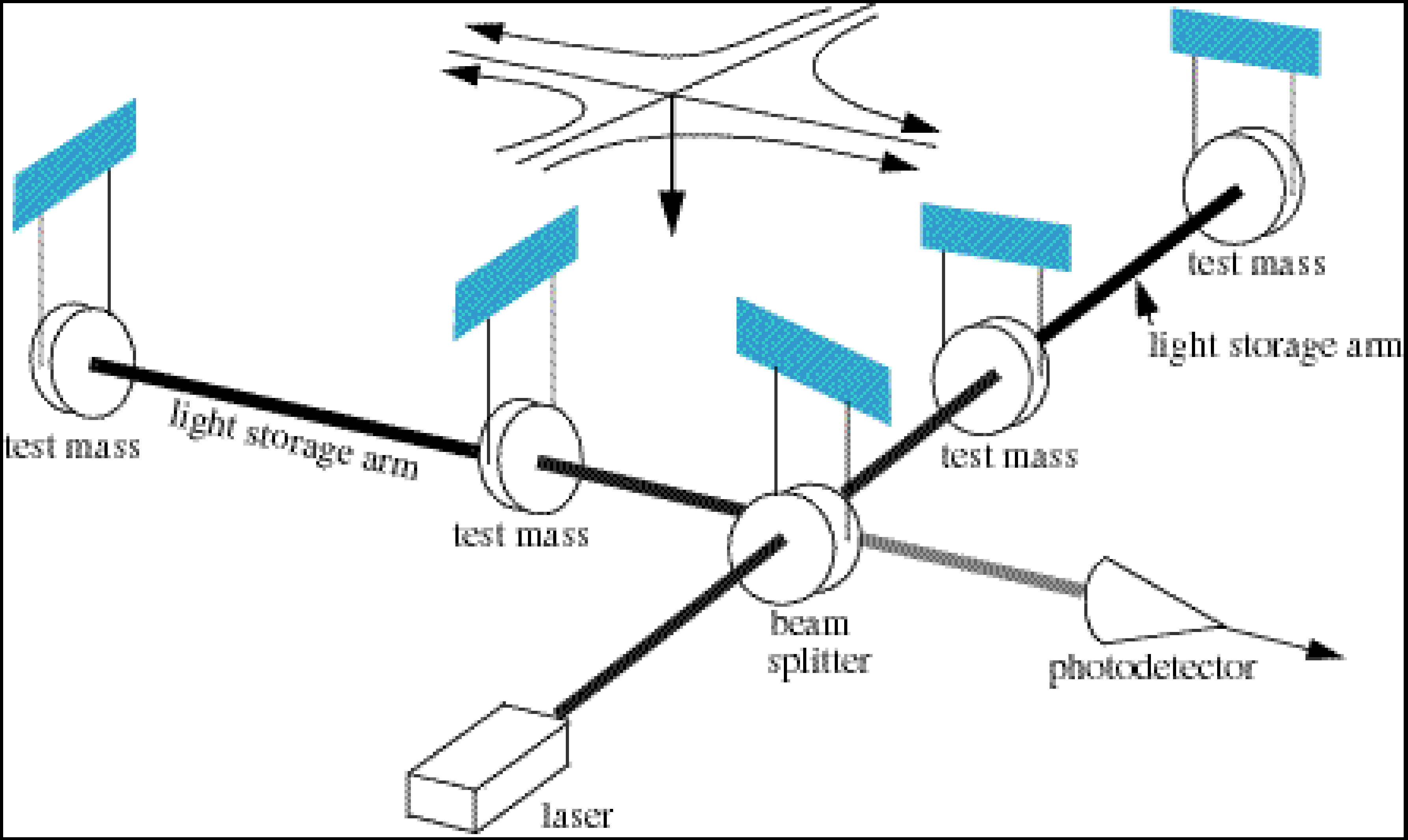
Measuring Cosmological Parameters

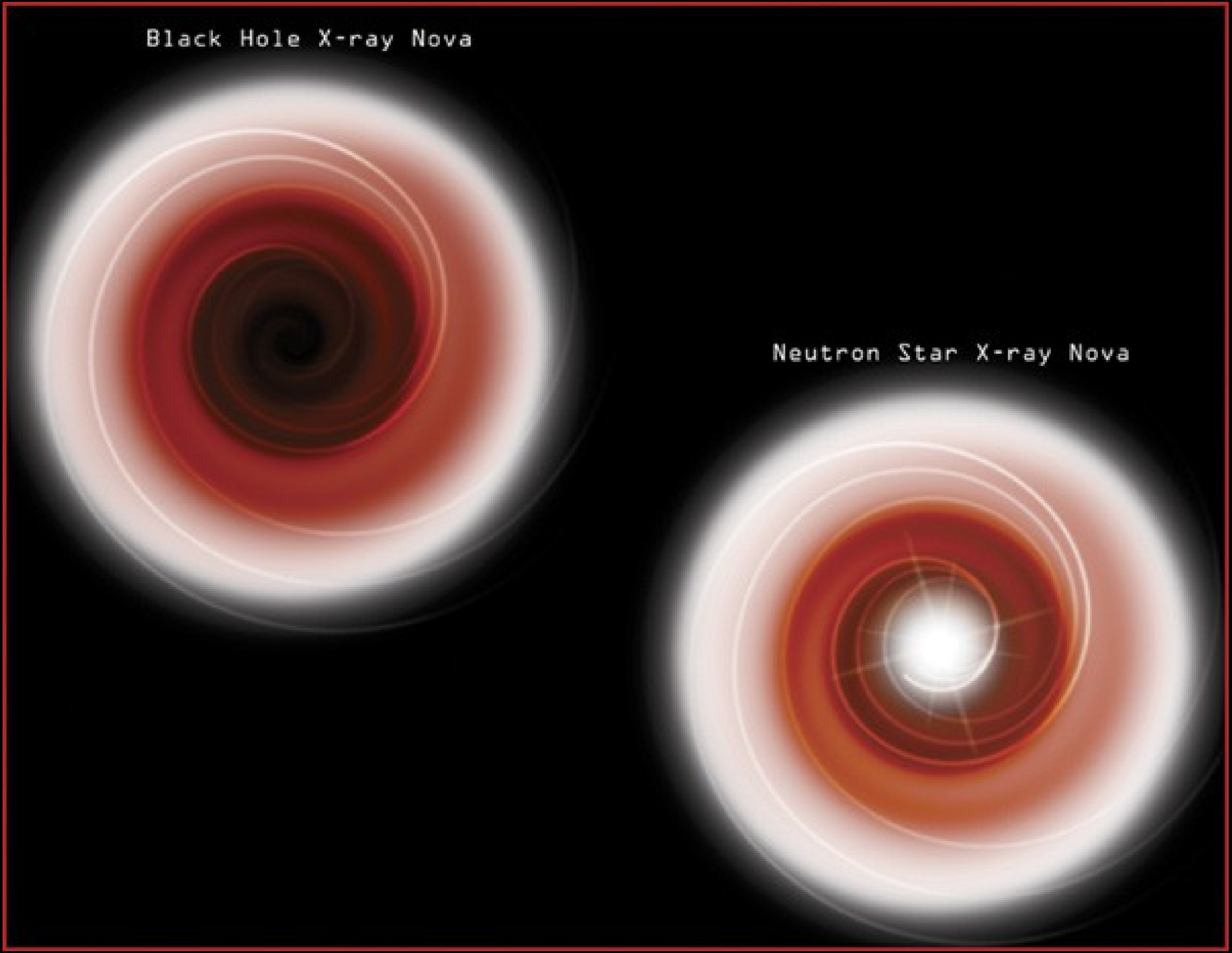
- *SNAP* will measure Ω_Λ and Ω_M to unprecedented precision.
- Together Ω_Λ and Ω_M determine Ω_T.
- SNAP will also measure the equation of state $w = p/\rho$.
- More importantly, SNAP will measure the variation of w with time (redshift)



The Cosmic Pyramid




test mass
light storage arm
test mass
test mass
test mass
light storage arm
beam
splitter
photodetector
laser


Black Hole X-ray Nova
Neutron Star X-ray Nova